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How to Value and Hedge Options on Foreign Indexes

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SUMMARY

Options on foreign stocks come in several styles, each with a different exposure to foreign currency. Each contract's value and hedge ratio depends, in a specific way, upon the stock volatility, the currency volatility, and the foreign and domestic interest rates. In this note, we review the different options, explain how to value and hedge them, and describe their risks.

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INTRODUCTION

Investors and traders increasingly use derivatives on foreign indexes to obtain exposure to global markets. There are a variety of option styles available, each of which provides different degrees of exposure to the currency of the foreign index. In this note, we provide a summary of the instruments available, and explain how to value and hedge them. To be specific, many of our illustrations will refer to U.S. dollar-based investors who participate in Japanese equity markets, but measure their profits and losses in U.S. dollars.

There are three different types of derivative instruments, each having a different exposure to the foreign stock or index level (say, the Nikkei 225) and the foreign currency (the yen).

1. Foreign-Market Derivatives.
This is the simplest case: the investor buys a yen-denominated derivative on a Japanese stock, and pays for it today by converting his dollars to yen *at the current exchange rate*. The investor is exposed to the current stock value in yen (which affects the derivative security's value in yen) and the value of the yen (which affects the option's value in dollars). An example is an investor who buys yen-denominated calls on the Nikkei, seeing great potential in the Japanese equity market and expecting the yen to remain stable or even appreciate against the dollar.

2. ADR-style/Composite Derivatives.
Here, the derivative's underlyer is the value of the Japanese stock converted to U.S. dollars at the prevailing exchange rate. The investor is exposed to the current American Depositary Receipt value of the stock in dollars, composed of the product of its value in yen and the current value of the yen in dollars. Consequently, such derivatives are sometimes called *composite* derivatives.

3. GER-style/Quanto Derivatives.
In this case, the derivative contract provides U.S. dollar exposure to Japanese stock returns with no currency risk at all. The payoff of the derivative depends only on the change in the yen value of the Japanese stock, converted to dollars at a Guaranteed Exchange Rate agreed upon at the inception of the contract.

Each of the three contracts has a value and hedge ratio that depends in different ways on the respective volatilities of the foreign stock and the currency. Similarly, each contract's value depends in different ways on the foreign and domestic interest rates. In this note, we elaborate on these contracts and explain how to value and hedge them.

NOTATION

For illustrative purposes, we refer to U.S. dollar-based investors who participate in Japanese equity index markets, but measure their profits and losses in U.S. dollars. We use the following notation:

t	current time
T	delivery time of forward contract or expiration time of option
S	index value in yen at time t
S_T	index value in yen at expiration
q	continuous dividend rate on index
X	dollar value of one yen at time t
X_T	dollar value of one yen at expiration
X_0	guaranteed exchange rate in dollars per yen
K	delivery or strike price in yen
$K_\$$	delivery or strike price in dollars
$f(S, K)$	current value of a forward contract to deliver the index at expiration for delivery price K
F_S	forward price or the fair delivery value of forward contract on foreign index in yen
F_X	forward price or the fair delivery value of forward contract on yen in dollars
$r_\$$	U.S. riskless interest rate
$l_\$$	U.S. stock loan rate
r_Y	Japanese riskless interest rate
l_Y	Japanese stock loan rate
b	U.S. or Japanese rebate rate earned by lending the stock
σ_S	estimated return volatility of index in yen
σ_X	estimated return volatility of yen in dollars
σ_{XS}	estimated covariance between returns on X and S
ρ_{XS}	estimated correlation coefficient between returns on X and S

DEFINING THE CONTRACTS

The following table summarizes the payoffs and properties of the various contracts.

Table 1: Foreign stock index contracts as defined by their payoffs at expiration

Derivative style	Forward payoff in $	Call option payoff in $	Comments
Foreign-Market	$X_T(S_T - K)$	max $[0, X_T(S_T - K)]$	• sign of payoff/moneyness is independent of X_T • magnitude of payoff in $ depends upon X_T
ADR-style	$X_T S_T - K_\$$	max $[0, X_T S_T - K_\$]$	• sign of payoff/moneyness depends upon X_T • magnitude of payoff in $ depends upon X_T
GER-style	$X_0(S_T - K)$	max $[0, X_0(S_T - K)]$	• sign of payoff/moneyness is independent of X_T • magnitude of payoff in $ is independent of X_T

FORWARD CONTRACTS ON FOREIGN STOCK

We begin with the valuation of forward contracts because they are simpler than options, but still capture important features and subtleties involved. A forward contract on a foreign index is an agreement to buy the index on a certain date, at a certain delivery price, in a specified currency. The *forward value* is the delivery price that makes the forward contract worth zero today.

We will summarize the valuation of forward contracts of three different styles defined in Table 1. Our discussion closely follows the earlier publications on this topic, which are listed under **References** (see page 21).

Foreign-Market Forwards

We begin with a standard forward contract $f(S, K)$ on the yen-denominated stock worth S today. On the delivery date, an investor long the forward receives S_T yen and pays K, so that

$$\text{payoff in yen} = S_T - K \quad \text{(EQ 1)}$$

The forward value, F_S, in yen is the delivery price that makes the forward contract worth zero today.

You can replicate the payoff S_T by investing today in $e^{-(b+q)(T-t)}$ shares of the stock; by reinvesting both the future dividend yield q on the stock and the future rebate rate b, you will end up with exactly one share worth S_T on the delivery date. You can replicate the payment of K yen by selling a zero-coupon bond of face K maturing at time T. The current replication value of the forward $f(S, K)$ is

$$f(S, K) = e^{-(b+q)(T-t)}S - e^{-r_Y(T-t)}K \quad \text{(EQ 2)}$$

The delivery value K that makes $f(S, K)$ have zero value is the forward value

$$F_S = Se^{(r_Y - b - q)(T-t)}$$

The rate earned on shorting the borrowed stock after paying the rebate is the yen stock loan rate l_Y given by

$$l_Y = r_Y - b \quad \text{(EQ 3)}$$

so that the forward value can be written as

$$F_S = Se^{(l_Y - q)(T - t)} \text{ yen} \quad \textbf{(EQ 4)}$$

The value of the payoff in Equation 1 in dollars is

$$\text{payoff in dollars} = X_T(S_T - K) \quad \textbf{(EQ 5)}$$

where X_T is the exchange rate at delivery. The current value of the forward contract in dollars is simply its value in yen given by Equation 2 multiplied by the current exchange rate:

$$f(S, K) = Xe^{-r_Y(T - t)}[e^{(l_Y - q)(T - t)}S - K]$$

ADR Forwards

This is a forward contract on the ADR value of the Japanese stock in dollars with payoff given by

$$\text{payoff in dollars} = X_T S_T - K_\$ \quad \textbf{(EQ 6)}$$

If tradable securities on deposit in the U.S. do exist, the ADR forward can be replicated in the same way as the ordinary forward -- by holding a portfolio that contains:

- a long position in a fraction $e^{-(b + q)(T - t)}$ of one ADR "share" worth XS;
- a short position in a U.S. zero-coupon bond with face value $K_\$$.

The fair value of the forward contract today is

$$f(S, K_\$) = XSe^{-(b + q)(T - t)} - K_\$ e^{-r_\$(T - t)} \text{ dollars}$$

The forward price, F_{ADR}, is the delivery price that makes this contract worth zero today, that is

$$F_{\text{ADR}} = XSe^{(l_\$ - q)(T - t)} \text{ dollars} \quad \textbf{(EQ 7)}$$

where $l_\$ = r_\$ - b$ is the ADR's loan rate.

You may, however, want to create an ADR-like contract on a foreign security which does not have deposits in U.S. In this case, you cannot easily buy a security that has the value $X_T S_T$ at maturity. Instead, you can replicate the payoff in Equation 6 by investing in the following portfolio:

- a long position in one forward contract on foreign index in yen
- a long position in forward contract to buy F_S yen
- a long position in a U.S. zero-coupon bond with face value $F_S F_X$
- a short position in a U.S. zero-coupon bond with face value $K_\$$

The dollar value of this portfolio at time T is

$$X_T(S_T - F_S) + F_S(X_T - F_X) + F_S F_X - K_\$ = X_T S_T - K_\$$$

which is exactly the payoff at delivery we want to replicate. Since the forward contracts in this portfolio are currently worth zero, the value of the whole portfolio today is

$$f(S, K_\$) = F_S F_X e^{-r_\$(T-t)} - K_\$ e^{-r_\$(T-t)}$$

Again, the forward value of the ADR contract, F_{ADR}, is the delivery price that makes this contract worth zero today:

$$F_{\mathrm{ADR}} = F_S F_X \qquad \text{(EQ 8)}$$

Using the value for F_S given by Equation 4 together with

$$F_X = X e^{(r_\$ - r_Y)(T-t)} \qquad \text{(EQ 9)}$$

and

$$r_\$ - r_Y = l_\$ - l_Y$$

we find that the value given by Equation 8 agrees with the previously derived result in Equation 7. It is interesting to note that the forward value of the ADR forward is simply the product of forward price of the index in yen and the forward price of yen in dollars.

Finally, as an alternative, we describe the pricing of ADR forwards using the *risk-neutral* method. Note that the payoff in Equation 6 is given completely in dollars. To avoid arbitrage, all investments in a dollar-based risk-neutral world should earn the dollar riskless rate. The value of the ADR forward is then the discounted expected value of the ADR forward's payoff at maturity, i.e.,

$$f(S, K_\$) = e^{-r_\$(T-t)}(E[X_T S_T] - K_\$) \quad \text{dollars} \qquad \text{(EQ 10)}$$

Here, $E[\ ]$ denotes an expected value at time T in a dollar-based risk-neutral world. The expected value of index in dollars (whose current value is XS) is

$$E[X_T S_T] = XSe^{(l_\$ - q)(T-t)} \quad \text{(EQ 11)}$$

where q is the annual dividend yield. Equation 10 now gives

$$f(S, K_\$) = e^{-r_\$(T-t)}(XSe^{(l_\$ - q)(T-t)} - K_\$) \quad \text{dollars}$$

which leads to the ADR forward price in Equation 7.

GER Forwards

The guaranteed exchange rate forward contract has a payoff at expiration given by

$$\text{payoff in dollars} = X_0(S_T - K) \quad \text{(EQ 12)}$$

where X_0 is the multiplier that determines how many dollars the contract pays per foreign index point. A contract similar to this one is the Chicago Mercantile Exchange-traded futures contract on the Nikkei 225 index, in which case $X_0 = 5$ dollars per yen.

Unfortunately, there is no simple buy and hold strategy that guarantees a payoff of $X_0 S_T$ dollars at expiration. The reason is that the final dollar value of any static position in Nikkei is unknown, since the dollar value of yen at expiration is unknown. The only strategy for replicating the GER payoff in dollars is a dynamic one, and involves creating a synthetic fund portfolio, Π, of traded securities such that it always has the dollar value $X_0 S$, an exposure of X_0 dollars to the Nikkei, and zero exposure to the yen. This is achieved by:

- investing $X_0 S$ dollars;
- borrowing $\frac{X_0}{X} S$ yen; and
- using the borrowed yen to buy $\frac{X_0}{X}$ shares of Nikkei.

These three components must be adjusted as S and X change and as we pay and receive interest and dividends.

The GER forward can be replicated in the same way as the ordinary forward -- by holding a portfolio that contains:

- a long position in a fraction $e^{-(b + q_{syn})(T - t)}$ of one synthetic "share" of the above fund, where q_{syn} is the dividend yield of a share of this fund worth $X_0 S$; and
- a short position in a U.S. zero-coupon bond with face value $X_0 K$.

The fair value of the forward contract today is

$$f(S, K) = X_0 S e^{-(b + q_{syn})(T - t)} - X_0 K e^{-r_\$ (T - t)} \text{ dollars}$$

It is shown in the appendix that the dividend yield of synthetic security is

$$q_{syn} = q + r_\$ - r_Y + \sigma_{XS}$$

which implies

$$f(S, K) = X_0 e^{-r_\$ (T - t)} [S e^{(l_Y - q - \sigma_{XS})(T - t)} - K]$$

The forward price, F_{GER}, is the delivery price that makes this contract worth zero today, that is

$$F_{GER} = S e^{(l_Y - q - \sigma_{XS})(T - t)} \text{ yen} \qquad \textbf{(EQ 13)}$$

This is similar to the forward value F_S in Equation 4, except that there is an additional term $\sigma_{XS}(T - t)$ in the exponent. F_{GER} in Equation 13 can be written as

$$F_{GER} = S e^{(l_Y - q')(T - t)} \text{ yen} \qquad \textbf{(EQ 14)}$$

where

$$q' = q + \sigma_{XS} = q + \rho_{XS} \sigma_X \sigma_S \qquad \textbf{(EQ 15)}$$

is an "effective dividend" comprised of the dividend yield q and an additional "induced dividend yield" σ_{XS}. This shows that a positive correlation ρ_{XS} increases the effective dividend and in turn lowers the GER forward value relative to an ordinary forward contract. We will present an intuitive explanation of this effect below after we summarize the valuation of a GER forward contract using the risk-neutral method.

In a risk-neutral world, the value of the GER forward is the discounted expected value of the payoff at time T:

$$f(S, K) = e^{-r_\$(T-t)}(E[X_0 S_T] - X_0 K) \text{ dollars} \quad \textbf{(EQ 16)}$$

The value of $E[X_0 S_T]$ cannot be calculated directly because $X_0 S_T$ is not the price of tradable security. However, we can use the following mathematical result:

If S_T and X_T are lognormally distributed random variables with return covariance σ_{XS} then at time t

$$E[X_T S_T] = E[X_T]E[S_T]e^{\sigma_{XS}(T-t)} \quad \textbf{(EQ 17)}$$

Using the result in Equation 11 together with

$$E[X_T] = Xe^{(r_\$ - r_Y)(T-t)} \quad \textbf{(EQ 18)}$$

we find that

$$E[S_T] = Se^{(l_Y - q - \sigma_{XS})(T-t)} \quad \textbf{(EQ 19)}$$

Equation 16 now gives

$$f(S, K) = e^{-r_\$(T-t)} X_0 (Se^{(l_Y - q - \sigma_{XS})(T-t)} - K) \text{ dollars}$$

from which we read off the GER forward value as given in Equation 13.

An intuitive explanation of the effect of correlation goes as follows. Suppose that over a short time period all securities can either move up or down. Consider the extreme case where Nikkei index S and the dollar value of yen X are 100% positively correlated, that is $\rho_{XS} = 1$. This means that an up (down) move in the index is theoretically always accompanied by an up (down) move in the yen. In the following table, we illustrate their evolution together with the corresponding evolution of a GER version of the forward contract, G, with the guaranteed exchange rate set to the initial dollar/yen rate, $X_0 = X$.

Table 2: Evolution of the Nikkei, the yen and the GER Nikkei index over a short time period

	Index (yen)	**Yen (dollars)**	**Index (dollars)**	**GER (dollars)**
Initial	S	X	SX	SX
UP move	S_u	X_u	$S_u X_u$	$S_u X$
DOWN move	S_d	X_d	$S_d X_d$	$S_d X$

To highlight the essence of the argument, we assume that all securities have zero interest and dividends over this short time period. This means that the forward value of the standard contract equals the index value. Suppose that the initial dollar value of GER forward is also SX and consider an initial investment in the following hedged portfolio, Π:

- a long position in one GER forward contract with $X_0 = X$
- a short position in one share of Nikkei index converted to dollars
- a long position in S yen

The initial value of this portfolio is

$$\Pi_i = SX - SX + SX = SX$$

The final value of the portfolio in the up state is

$$\Pi_f^{up} = S_u X - S_u X_u + S X_u$$

and in the down state

$$\Pi_f^{down} = S_d X - S_d X_d + S X_d$$

The net P&L after the up move is

$$\Pi_f^{up} - \Pi_i = -(S_u - S)(X_u - X) < 0$$

and after the down move

$$\Pi_f^{down} - \Pi_i = -(S - S_d)(X - X_d) < 0$$

where we used the obvious relation $S_u > S > S_d$. If the net P&L is negative for both up and down moves in the index, someone taking the opposite position from the one in our portfolio can theoretically earn riskless profit. Therefore, we conclude that the fair initial value of GER forward should be less than the assumed value SX. This is equivalent to saying that the GER forward will appreciate more slowly which, in turn, means that the effective dividend is higher than the regular dividend. In short, positive correlation implies a positive dividend. Similar arguments can be used to show that negative correlation leads to negative effective dividend and to a higher GER forward price.

Summary of Results for Forwards

Table 3 summarizes the main result for various types of forward contracts considered here.

Table 3: Summary of results for forward contracts

Style	Payoff in \$	Forward price	Current value of forward in \$
Foreign-Market	$X_T(S_T - K)$	$Se^{(l_Y - q)(T-t)}$ yen	$Xe^{-r_Y(T-t)}[Se^{(l_Y - q)(T-t)} - K]$
ADR-style	$X_T S_T - K_\$$	$XSe^{(l_\$ - q)(T-t)}$ \$	$e^{-r_\$(T-t)}[XSe^{(l_\$ - q)(T-t)} - K_\$]$
GER-style	$X_0(S_T - K)$	$Se^{(l_Y - q - \sigma_{XS})(T-t)}$ yen	$X_0 e^{-r_\$(T-t)}[Se^{(l_Y - q - \sigma_{XS})(T-t)} - K]$

EUROPEAN OPTIONS ON FOREIGN INDEX

We now turn our discussion to options. For European options, we can write analytic expressions assuming constant volatility and interest rates. Although not realistic, the results in this simplified world will help us understand generic properties of options on foreign indexes and strategies for their hedging. To be specific, we will discuss call options. The results for put options can be derived in a similar way.

Foreign-Market Options

We first consider standard options which are traded on Japanese exchanges and which can be hedged by shorting the foreign index. Therefore, they can be valued by a risk-neutral method. The payoff is given by

$$C(T) = \max[0, X_T(S_T - K)] \qquad \text{(EQ 20)}$$

and the present value of the call option in dollars is simply the option's value in yen converted to dollars at the current exchange rate

$$C = Xe^{-r_Y(T-t)}[F_S N(d_1) - KN(d_2)] = XC_Y \qquad \text{(EQ 21)}$$

Here F_S is the forward value of the index (see Equation 4),

$$F_S = Se^{(l_Y - q)(T-t)}$$

and

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(l_Y - q + \frac{\sigma_S^2}{2}\right)(T-t)}{\sigma_S\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_S\sqrt{T-t}$$

The price of a call option depends on the Japanese discount and stock loan rate, the index volatility and dividend yield. These parameters affect the price in the same way as in the case of an ordinary option. In addition, the price is proportional to the current exchange rate. The correlation between S and X is irrelevant.

A long call option can be hedged in the Japanese market in the same way as an ordinary option -- by shorting $n_S = \Delta_Y$ shares of Nikkei index where

$$\Delta_Y = \frac{\partial C_Y}{\partial S} = e^{-(q+b)(T-t)} N(d_1)$$

The dollar value of the hedged position is

$$X C_Y - X(\Delta_Y S) = X(C_Y - \Delta_Y S)$$

which clearly depends on the exchange rate. The exchange rate risk can be hedged by shorting

$$n_X = C_Y - \Delta_Y S \text{ yen}$$

It is easy to see that a portfolio consisting of a call option and short positions in Δ_Y shares of Nikkei and n_X yen

$$\Pi = C - n_S\{XS\} - n_X\{X\}$$

is insensitive to small changes in S and X.

ADR Options

The dollar payoff for these options is

$$C_{\text{ADR}}(T) = \max[0, X_T S_T - K_\$]$$

This is just a U.S.-traded option on the foreign index denominated in dollars that can be hedged by shorting the index and converting payment to dollars. This can be thought of as an option with strike price $K_\$$ on an asset XS which is denominated in dollars, pays a continuous dividend yield q and has volatility

$$\sigma = \sqrt{\sigma_S^2 + \sigma_X^2 + 2\rho_{XS}\sigma_S\sigma_X}$$

The option value in dollars is given by Black-Scholes formula with the adjusted volatility:

$$C_{\text{ADR}} = e^{-r_\$(T-t)}[F_{\text{ADR}} N(d_1) - K_\$ N(d_2)] \qquad \textbf{(EQ 22)}$$

where

$$F_{\text{ADR}} = XSe^{(l_\$ - q)(T-t)}$$

is the ADR forward value, and

$$d_1 = \frac{\log\left(\frac{XS}{K_\$}\right) + \left(l_\$ - q + \frac{\sigma^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}}$$

$$d_2 = d_1 - \sigma\sqrt{T - t}$$

The value of the ADR option is independent of the Japanese discount and stock loan rates. The effects of index price, current exchange rate, the U.S. discount and stock loan rates and time to maturity are as expected. The value of the ADR option increases with increasing overall volatility σ. The effects of individual volatilities are no longer clear-cut. If S and X are positively correlated, then the volatilities affect the option's value in a conventional way. In the case of sufficiently negative correlation, it is possible that increasing individual volatilities may cause a decrease in option price. Specifically, for a negative ρ_{XS}, when $\rho_{XS}\sigma_X > \sigma_S$, the value of the ADR option decreases as the index volatility increases. However, since the exchange rate volatility is usually smaller than index volatility, this situation does not happen very often. Similarly, for a negative ρ_{XS}, when $\rho_{XS}\sigma_S > \sigma_X$, the value of the ADR option increases as the index volatility increases.

Theoretically this option has no currency risk and a long call should be hedged by shorting

$$\Delta = \frac{\partial C_{\mathrm{ADR}}}{\partial(XS)} = e^{-(q+b)(T-t)}N(d_1)$$

shares of Nikkei. No holding of yen is necessary, since the index held also hedges the exchange rate risk.

GER (Quanto) Options

GER or quanto options have payoff

$$C_{\mathrm{GER}}(T) = \max\,[0, X_0(S_T - K)]$$

The difficulty in pricing this option comes from the fact that there is no traded security whose dollar value is X_0S at any time t. Since you cannot hedge directly with the underlyer, you cannot price by arbitrage. As we described in the discussion of valuing a GER forward, it is possible to construct a portfolio, Π, of traded securities such that it always has the value X_0S.

The GER option can then be understood as an ordinary option with strike KX_0 dollars on an imaginary stock whose price distribution is that of $X_0 S_T$ with volatility σ_S and mean growth rate $l_Y - q - \sigma_{XS}$ (see the discussion of GER forwards). The call option price is now simply

$$C_{\text{GER}} = X_0 e^{-r_\$(T-t)} [F_{\text{GER}} N(d_1) - KN(d_2)] \qquad \text{(EQ 23)}$$

where

$$F_{\text{GER}} = S e^{(l_Y - q - \sigma_{XS})(T-t)}$$

is the GER forward value, and

$$d_1 = \frac{\log\left(\frac{S}{K}\right) + \left(l_Y - q - \sigma_{XS} + \frac{\sigma_S^2}{2}\right)(T-t)}{\sigma_S \sqrt{T-t}}$$

$$d_2 = d_1 - \sigma_S \sqrt{T-t}$$

There are three interesting features of this result:

- The value of the GER option depends on both the U.S. discount rate and the Japanese stock loan rate. The Japanese stock loan rate affects the option's price in a conventional way, while increasing the U.S. discount rate decreases the option price.
- The value of the GER option does not depend on the prevailing exchange rate. Instead, it depends on the correlation between the index level and the exchange rate. Note that even in the case of zero correlations, the price of the GER option is not exactly equal to the price of an ordinary option, since the rates in two different currencies are involved.
- Increasing exchange rate volatility increases the call option value if the correlation between X and S is negative, and decreases the option value if the correlation is positive. The effect of change in index volatility depends, among other things, on the sign and size of the correlation coefficient and the degree to which the option is in the money. (Typically, only when the option is deep-in-the-money, increasing the index volatility will cause the price to decrease.)

In order to understand how to hedge a GER option, it is useful to write Equation 23 in the following form

$$C_{\mathrm{GER}} = X_0 e^{(r_Y - r_\$)(T-t)} C_{Y'} \qquad \text{(EQ 24)}$$

where $C_{Y'}$ is the price of a standard option on the Nikkei in yen, but with an effective dividend yield

$$q' = q + \rho_{XS}\sigma_X\sigma_S$$

Since the price of a GER option does not depend on the exchange rate, you may naively think that you only need to hedge the risk associated with the index price. This can be done in the Japanese market by shorting n_S shares of the index where

$$n_S = \frac{X_0}{X} e^{(r_Y - r_\$)t} \Delta_{Y'}$$

and

$$\Delta_{Y'} = \frac{\partial C_{Y'}}{\partial S}$$

is the option's delta in the Japanese market with a modified dividend yield. Although the price does not depend on the exchange rate, holding n_S Nikkei shares in your hedging portfolio introduces currency risk, which can be offset by going long an equivalent amount of yen, i.e.,

$$n_X = n_S S \quad \text{yen}$$

Thus, as you adjust your delta-hedge of the option you need to change your long position in yen depending on the number of shares in your hedge and the prevailing index price.

One can check that a portfolio consisting of a GER call option, a short position in n_S shares of Nikkei and a long position in n_X yen

$$\Pi = C_{\mathrm{GER}} - n_S\{XS\} + n_X\{X\}$$

is insensitive to small changes in S and X.

Summary of Results for Options

In Table 4, we summarize the dependence of option prices on various parameters.

Table 4: Dependence of option prices on various parameters

Style	r_Y	l_Y	$r_\$$	$l_\$$	q	X	σ_S	σ_X	ρ_{XS}
Foreign-Market	yes	yes	no	no	yes	yes	yes	no	no
ADR-style	no	no	yes	yes	yes	yes	yes	yes	yes
GER-style	no	yes	yes	no	yes	no	yes	yes	yes

Using the rules summarized in Table 5, you can find the price of options on a foreign index, even if you only have a standard Black-Scholes calculator that allows you to input the current price of the underlyer, strike, discount rate, loan rate, dividend yield and volatility

Table 5: How to use a Black-Scholes calculator to price options on foreign index

Style	underlyer	strike	discount rate	loan rate	dividend yield	volatility
Foreign-Market	XS	XK	r_Y	l_Y	q	σ_S
ADR-style	XS	$K_\$$	$r_\$$	$l_\$$	q	$\sqrt{\sigma_S^2 + \sigma_X^2 + 2\rho_{XS}\sigma_S\sigma_X}$
GER-style	$X_0 S$	$X_0 K$	$r_\$$	l_Y	$q + \rho_{XS}\sigma_X\sigma_S$	σ_S

Note that the parameters specified in this table can be used to build the binomial tree which you can use to price American options.

Finally, we summarize the rules for delta hedging a long call position in Table 6:

Table 6: Hedging a long call position

Style	Number of shares	Amount of yen
Foreign-Market	***short*** Δ_Y shares Δ_Y = delta of the option in Japanese market	***short*** $C_Y - \Delta_Y S$ yen C_Y = option price in Japanese market S = current index level
ADR-style	***short*** Δ ADR shares Δ = delta of the option on ADR with modified volatility $\sigma = \sqrt{\sigma_S^2 + \sigma_X^2 + 2\rho_{XS}\sigma_S\sigma_X}$	***none***
GER-style	***short*** $n_S = \frac{X_0}{X} e^{(r_Y - r_\$)t} \Delta_{Y'}$ shares $\Delta_{Y'}$ = option's delta in Japanese market with a modified dividend yield $q' = q + \rho_{XS}\sigma_X\sigma_S$	***long*** $n_S S$ yen n_S = number of shares S = current index level

APPENDIX

In this Appendix, we derive the effective dividend yield on a "share" of a portfolio which at any time has value $X_0 S$ dollars. As discussed in the text, such a portfolio consists of the following three components:

- investing $n_\$ = X_0 S$ dollars
- borrowing $n_Y = -\frac{X_0}{X} S$ yen
- using the borrowed yen to buy $n_S = \frac{X_0}{X}$ shares of Nikkei.

At any time, the dollar value of this portfolio is

$$\Pi = n_\$ + n_Y X + n_S S X$$

The change in value of one synthetic "share" of this portfolio during a small time period Δt comes from a change in the exchange rate ΔX and a change in the foreign index price ΔS, as well as the interest earned and paid:

- interest earned on invested dollars: $n_\$ r_\$ \Delta t$
- interest paid on borrowed yen converted to dollars: $(n_Y r_Y \Delta t) X$
- the profit/loss from borrowed yen: $n_Y \Delta X$
- the change in value of Nikkei shares in dollars: $n_S \Delta(XS) = n_S(X\Delta S + S\Delta X + \Delta X \Delta S)$
- the dividend earned on Nikkei shares in dollars: $(n_S q S \Delta t) X$

The total change is simply the sum of all these contributions. Taking into account the amounts of each of the portfolio components, this can be written as

$$\Delta\Pi = X_0 \Delta S + X_0 S\left(q + r_\$ - r_Y + \frac{1}{\Delta t}\frac{\Delta X}{X}\frac{\Delta S}{S}\right)\Delta t$$

The first term of this total change is due to the change in price of the synthetic share. The second term is the dividend yield. If we assume that the returns of the yen value in dollars and of the Nikkei price in yen are jointly normally distributed and correlated, as the time interval Δt approaches zero we have

$$\frac{1}{\Delta t}\frac{\Delta X}{X}\frac{\Delta S}{S} = \rho_{XS}\sigma_X\sigma_S$$

where σ_X and σ_S are the annualized volatilities of returns and ρ_{XS} is the correlation coefficient of these two returns. Putting this all together, we find the dividend yield of one synthetic share to be

$$q_{\mathrm{syn}} = q + r_{\$} - r_Y + \rho_{XS}\sigma_X\sigma_S$$

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