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SOME STATISTICAL PROPERTIES OF THE MINI FLASH CRASHES

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Abstract. We present some properties of the data from the recent mini flash crashes occurring in individual stocks of the Dow Jones Industrial Average. The top five are: 1) Gaussianity is absent in data; 2) the tail decay of the return distributions follow power laws; 3) chaos and logperiodicity cannot be dismissed at first; 4) chaos and logperiodicity are not good models for the data on second thoughts; and 5) a threshold GARCH fit can also describe the data well, but fails to detect the power law tail decay of most distributions of returns.

Keywords: mini flash crashes; flash crashes; chaos; log-periodicity; threshold GARCH; financial time series.

2010 AMS Subject Classification: 91G80.

1. Introduction

Recently, stock markets have been subjected to episodes of “flash crashes.” The first occurred on May 6, 2010 when the Dow Jones Industrial Average plunged by nearly 1,000 points in a matter of minutes. A report from the Securities and Exchange Commission [1], the American financial regulator, concluded that, “on May 6, when markets were already under stress, the sell algorithm chosen by a large trader to only target trading volume, and neither price nor time, executed its sell program extremely rapidly in just 20 minutes. At a later date, the large fundamental trader executed trades over the course of more than six hours to offset the net short position accumulated on May 6.” The report then concluded, “one key lesson is that under stressed market conditions, the automated execution of a large sell order can trigger extreme price movements, especially if the automated execution algorithm does not take prices into

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account." The Securities and Exchange Commission then suggested a marketwide system of "circuit breakers," which would require all exchanges to stop or slow down for a few minutes if the market experienced a certain threshold rate of decline. Of note, the SEC does not aim to act pre-emptively, but only to react in the aftermath. It is no surprise then that another flash crash occurred in the DJIA on April 22, 2013. The DJIA dropped about 150 points in a matter of seconds before bouncing back when traders realized that a tweet was false from a hacked Associated Press account.

Such rapid crashes occurred in individual stocks as well as in the stock index. Stocks that experienced "mini flash crashes," or rapid plunges followed by rapid rebounds, on particular days in 2011 and 2012 were Abott Labs, Apple, Cisco Systems, Citigroup, Core Molding, Enstar, Jazz Pharmaceuticals, Micron, Progress Energy, Pfeizer, Pall Corporation, RLJ Equity Partners, Thermo Fischer Co., and Washington Post. Stock exchanges do not publicly release data about these mini crashes, but most active traders say there are currently at least a dozen a day [2]. This study collects data from the days of the crashes for the aforementioned stocks, with the aim of uncovering some of their properties.

The researcher familiar with complex systems is willing to correct the SEC report by adding that the DJIA had to be in a critical state on May 6 for that sell order to trigger the flash crash. Otherwise, the crisis never would have materialized. There are already claims that the first flash crash of the DJIA index showed the footprints of a complex system at work [3]. Furthermore, the build up of correlations brought by high-frequency trading [4] may shape the nature of the flash crashes as "log-periodic phenomena" [5], in that after a critical time, a crash may suddenly occur without any earlier warning signs. For this reason, this study considers models of complexity, such as log-periodic and chaotic.

The remaining of this paper is organized as follows: Section 2 presents the data for the mini flash crashes, shows some descriptive statistics, and performs selected tests. Section 3 concludes the study.

2. Materials and methods

The data were collected from a private database using Bloomberg. They were sampled at trade time (9:30 a.m. to 4 p.m. EST-USA) from the New York Stock Exchange. The tic-by-tic frequency yielded roughly 370 daily observations. Table 1 shows sample size, time period, mean return and variance for each of the 14 stocks as well as the DJIA index. Figure 1 shows a plot of the time series, including the crash episodes, for both prices and returns.

A particular characteristic of the data is an excess kurtosis incompatible with a Gaussian distribution. Table 2 shows this measure for all the stocks experiencing the crashes are well above three. It varies from seven for Pfeizer to an amazing 473 for Jazz Pharmaceuticals. Table 2 also shows the drawdowns, which refer to the largest accumulated drop in percentage terms (times 100). All these facts suggest departures from Gaussianity.

Table 3 shows Gaussianity is indeed rejected using Lilliefors, Cramer-Von Mises and Jarque-Bera tests. Figure 2 shows *Q*-*Q* plots of the stock returns, which track the two-tailed frequency distribution. The red lines show the Gaussian behavior for comparison. Tail decays following power laws are then expected. Figure 3 shows the estimated power laws for the stock returns. None shows the exponential decay typical of a Gaussian. The DJIA index along with the stocks of Cisco, Progress Energy, and Thermo Fischer show power law decays compatible with a Lévy-stable regime, where the scale factor $\alpha \in (0,2)$. The remaining stocks fall outside the Lévy regime, Washington Post is compatible with a cubic law and Enstar overshoots the scale factor of three. All the time series present non-Gaussian scaling ($\alpha \neq 2$).

Figure 4 shows the largest Lyapunov exponents $\lambda_{\max}$ of the stock series. Stable plateaus occur for all of them. Table 4 shows all the largest exponents to be negative. Because such properties are not enough for concluding the presence of chaos [6], we apply a recent methodology [6] using a test of linear independence through ordinary least squares

$$\langle \lambda_{\max}(T_i) \rangle = \beta_0 + \beta_1 T_i + \varepsilon_{T_i} \qquad (1)$$

for subsamples of several bootstrap block sizes $T_i = T_1, ..., T_n = n$, where β_0 is a constant and the error term $\varepsilon_{T_i} \sim N(0, \sigma^2)$. The stability of $\lambda_{\max}(T_i)$ takes place if $\hat{\beta}_1 \leq 0$, and the null of chaos is rejected if $\hat{\beta}_1 > 0$. We ran 250 regressions as in (1) for four subsamples T_i of each stock time series. Table 4 shows we cannot reject the null of chaos at the 5 percent significance level. Regardless of the block size considered, the average path of the estimated negative largest Lyapunov exponents tends to stabilize as the number of observations increases.

Log-periodic fits for the time series data are attempted [7] using

$$\ln F_p(t) = A + B(\tau)^{m^2} + C(\tau)^{m^2}\cos(\omega \log\tau + \pi)\,, \tag{2}$$

where $\ln F_p(t)$ is the natural log of the series in levels of the random variables $X \sim (0,\sigma^2)$, and *A*, *B*, and *C* are the constants resulting after the fit of the remaining parameters: τ (critical time), m^2 (exponent controlling the acceleration near the critical time), ω (log-periodic frequency), and π (phase parameter). The *m* and *w* parameter values are first derived through nonlinear least squares using the Levenberg-Marquardt search algorithm and then fed back into the objective function. The remaining parameters are first enslaved into the nonlinear ones and then obtained using a standard *LU* decomposition method. Figure 5 shows the best fit of Eq. (2) as a continuous red line. Table 5 shows the estimated parameters.

Table 6 shows the Shannon entropy and long memory tests. We take a rescaled range analysis (*R/S*) to calculate the Hurst exponents, along with the method of the detrended fluctuation analysis (*DFA*). Apart from Cisco, Citigroup and Core Molding, long memory is not unequivocal for the remaining stocks. Here, one has to bear in mind the fact that some studies have shown that long memory can be generated spuriously due to the presence of small level shifts in the time series of returns [10]. This result casts doubt on the presence of chaos and logperiodicity for most of the series. The observed log-periodic fluctuations may also have been created synthetically because of the sampling method [11]. Furthermore, the values of Shannon entropy are more akin to those of a stochastic series than to those of the deterministic ones. We also calculated the values in Table 6 by excluding the data on the very date of the crashes only to realize our results did not change a great deal.

We then go back to stochastic models and revisit the data aggregated through the series using econometricians' favorites: threshold GARCH and skewed-*t* GARCH. The family of GARCH models can describe data locally nonstationary and asymptotically stationary. The processes present nonconstant variances conditional on the past, and constant "unconditional" variances (observed on a long time interval). The models enable one to control the amount and the nature of the variance memory.

Using Monte-Carlo simulations we find the threshold GARCH to present a good fit to the data. This confirms that, in general, models of the GARCH family are successful in describing distributions of price changes at a given time horizon. However, Table 7 shows the models cannot replicate the entropy of the individual stocks and long memory cannot be dismissed by detrended fluctuation analysis. This also conflicts with the findings for most of the time series. Moreover, a linear regression shows both models outside the power law regime. This replicates the finding that the GARCH family of models fails to properly describe the scaling properties of the distribution of tail returns [8, 9].

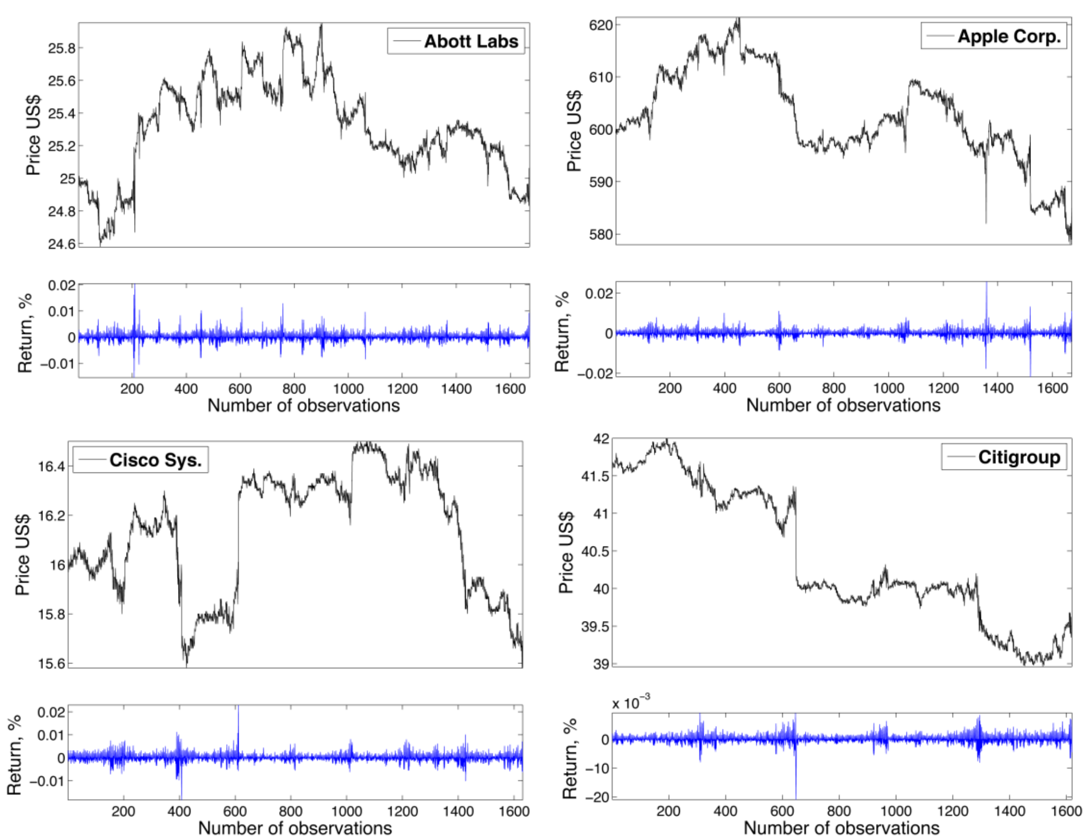
3. Concluding remarks

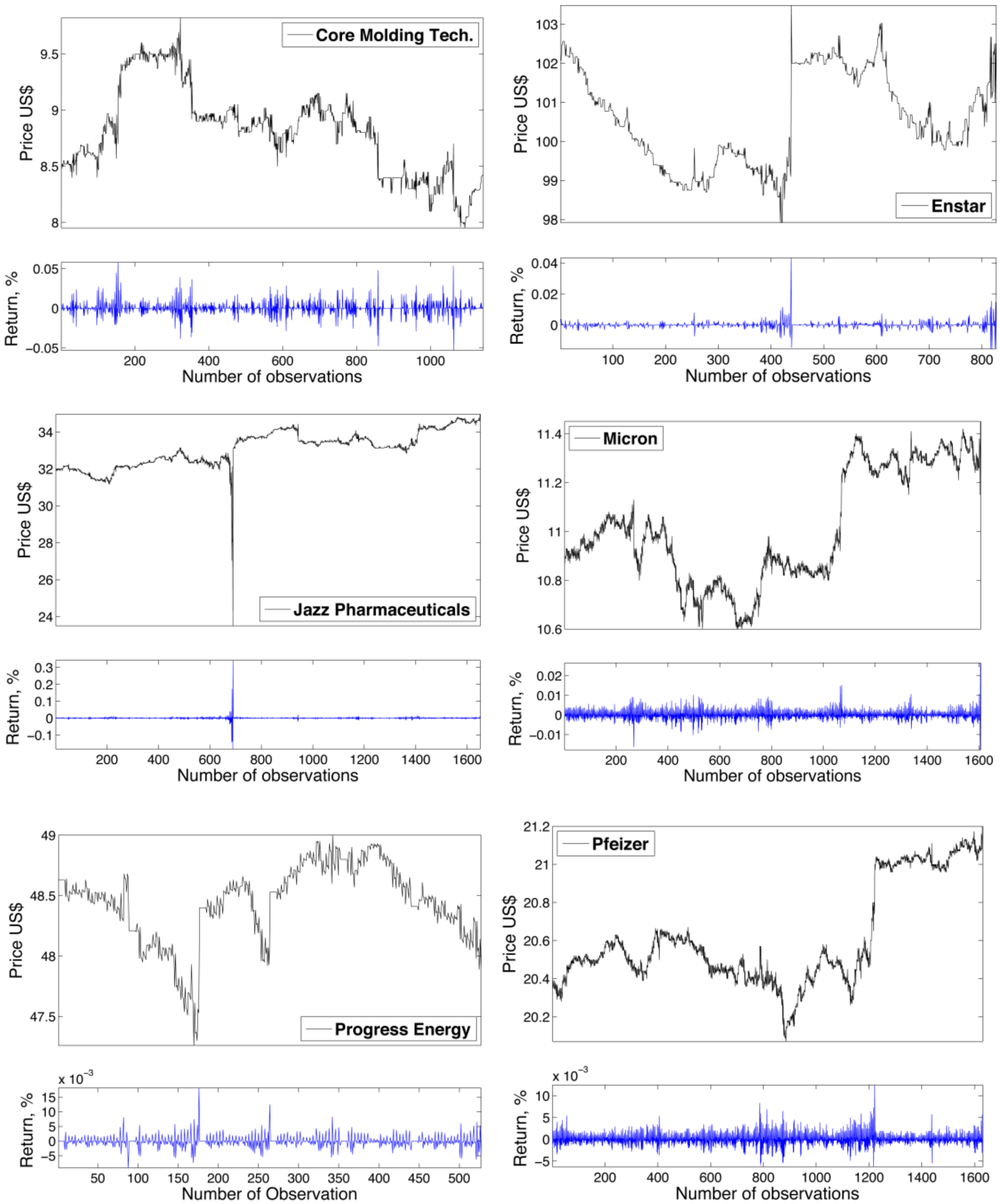
This study of tic-by-tic, high frequency data for the 14 stocks that experienced mini flash crashes, as well as the DJIA index, found clear characteristics. The first and unequivocal one being that Gaussianity is absent in the data. This is largely expected for time series that exhibit extreme datapoints. A second result was that tail decay of the return distributions follow power laws. This is a possible consequence of the absence of Gaussianity. Indeed, we found no stock returns to show the exponential decay typical of a Gaussian. In particular, the stocks of Cisco, Progress Energy and Thermo Fischer, along with the DJIA index itself, showed power law decays compatible with a Lévy-stable regime, where the scale factor $\alpha \in (0,2)$. The remaining stocks fell outside the Lévy regime; Washington Post was compatible with a cubic law; and Enstar overshot the scale factor of three. In short, all the series presented non-Gaussian scaling ($\alpha \neq 2$).

Scaling suggests a complex time series thus, we investigated whether the hypothesis of complex determinism could be rejected. We then tested for the presence of chaos and logperiodicity. Using a recent method for detecting chaos, we could not reject the null of chaos at the 5 percent significance level. We then adjusted a logperiodic formula to the data for each stock and showed that logperiodic fits. Thus, our third finding was that chaos and logperiodicity could not be dismissed at first.

This prompted us to investigate the Shannon entropy and test for long memory for each series, as chaos and logperiodicity implicitly assume long memory. We failed to find long memory for most stocks and the values of Shannon entropy were more akin to those of a stochastic series than to those of deterministic ones. Such findings cast doubt on the presence of chaos and logperiodicity for most series. Therefore, our fourth finding was that chaos and logperiodicity are not good models for the data on second thoughts.

We then returned to stochastic models and considered threshold GARCH and skewed-*t* GARCH, which enable one to control for the amount of memory. These models were adjusted to the data aggregated through all the series. Monte-Carlo simulations showed the threshold GARCH to have a good fit to the data. But both models could not replicate the entropy of the individual stocks and their long memory. But, such models were also found outside the power law regime. Thus, our fifth finding was that, although a good description of the data was possible using the threshold GARCH, this model could not detect the scaling properties of the tail decay of most stock returns.




Core Molding Tech.
Price US$
Return, %
Number of observations
Enstar
Price US$
Return, %
Number of observations
Jazz Pharmaceuticals
Price US$
Return, %
Number of observations
Micron
Price US$
Return, %
Number of observations
Progress Energy
Price US$
x 10^-3
Return, %
Number of Observation
Pfeizer
Price US$
x 10^-3
Return, %
Number of observations

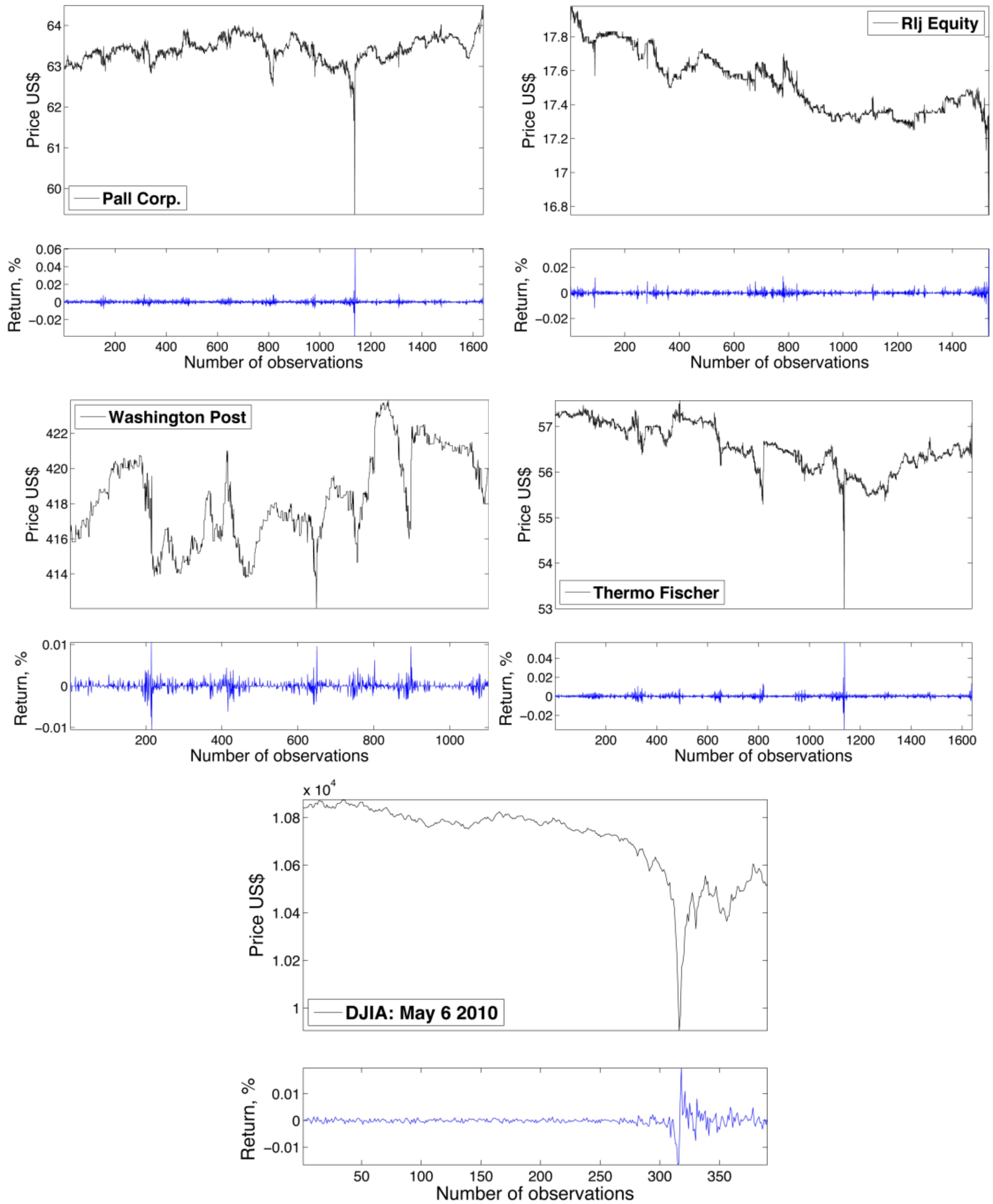
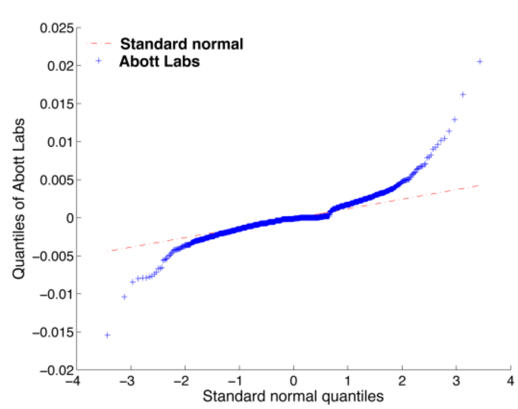
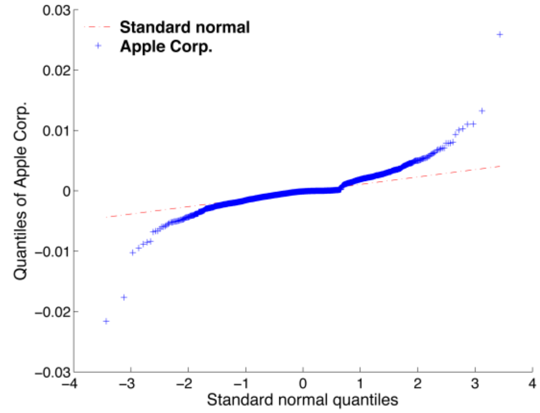
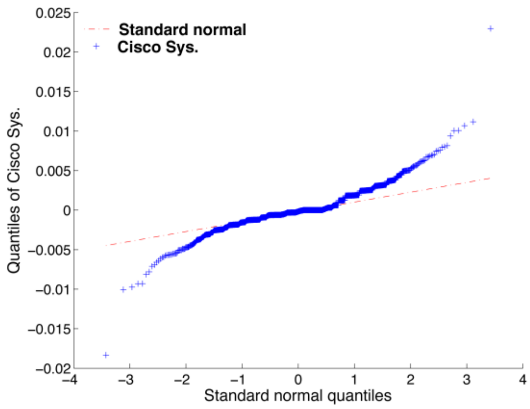


Figure 1. Stock prices and returns during the mini flash crashes. *Source*: Bloomberg.


Standard normal
Abott Labs
Quantiles of Abott Labs
Standard normal quantiles


Standard normal
Apple Corp.
Quantiles of Apple Corp.
Standard normal quantiles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Cisco Sys.
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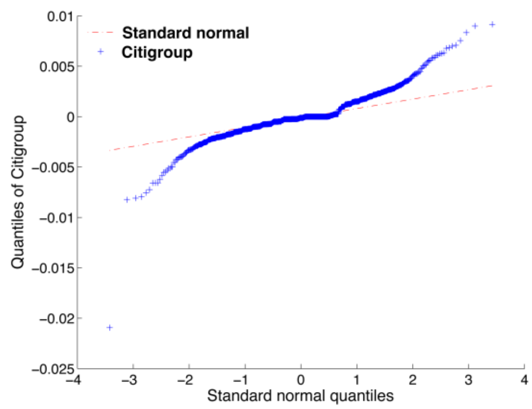
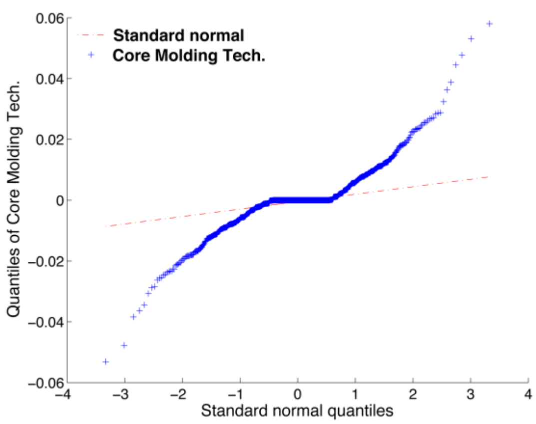
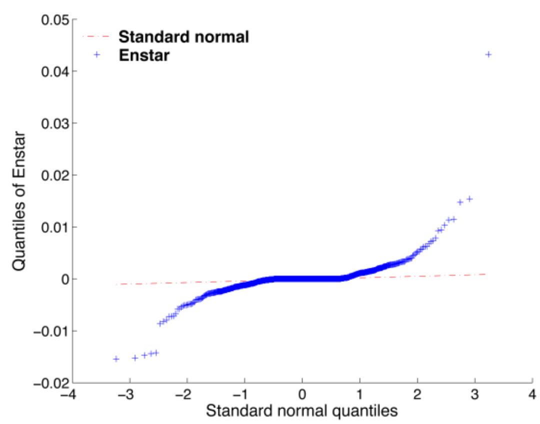
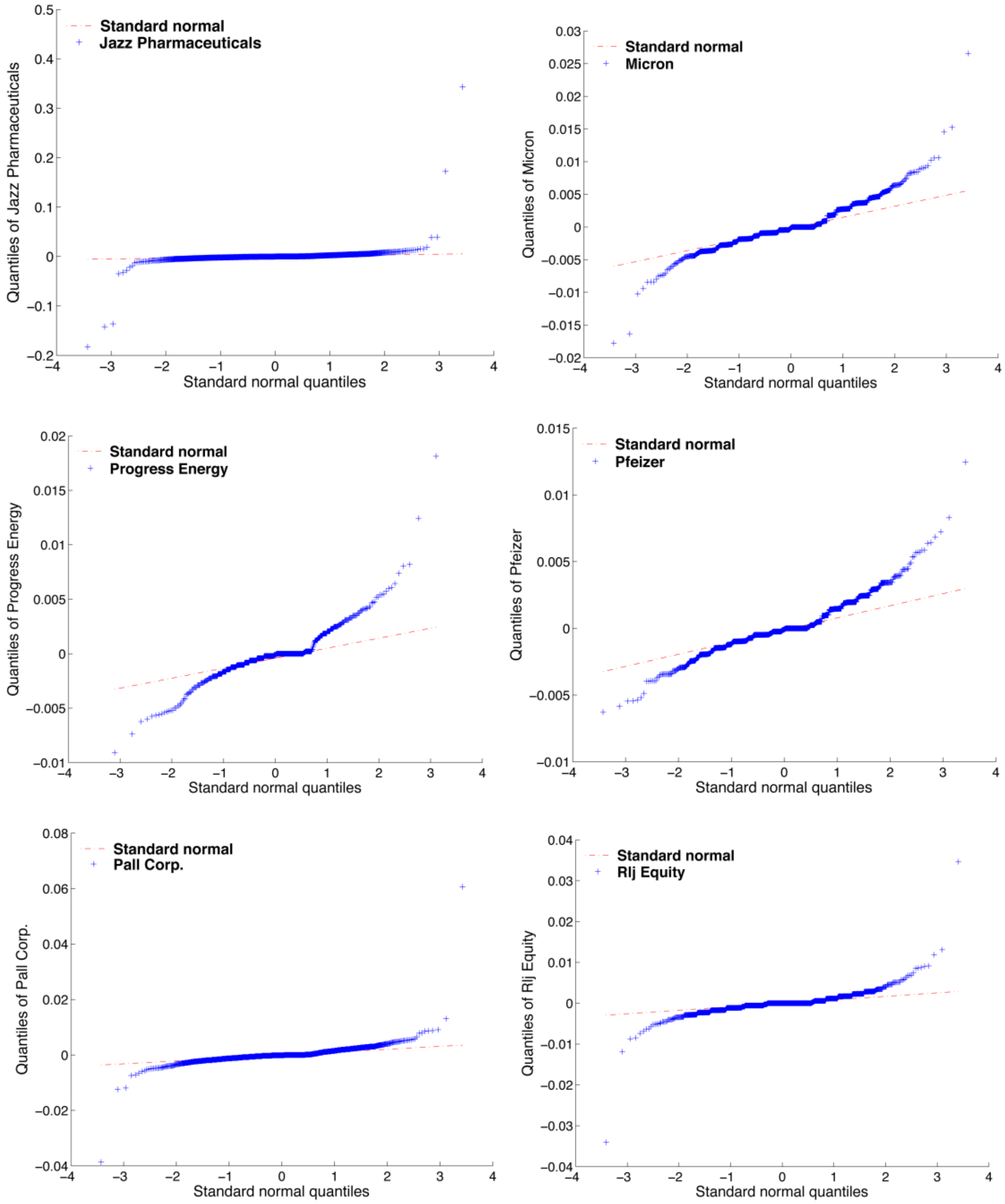

Standard normal
Citigroup
Quantiles of Citigroup
Standard normal quantiles


Standard normal
Core Molding Tech.
Quantiles of Core Molding Tech.
Standard normal quantiles


Standard normal
Enstar
Quantiles of Enstar
Standard normal quantiles


Standard normal
Jazz Pharmaceuticals
Quantiles of Jazz Pharmaceuticals
Standard normal quantiles
Standard normal
Micron
Quantiles of Micron
Standard normal quantiles
Standard normal
Progress Energy
Quantiles of Progress Energy
Standard normal quantiles
Standard normal
Pfeizer
Quantiles of Pfeizer
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Standard normal
Pall Corp.
Quantiles of Pall Corp.
Standard normal quantiles
Standard normal
Rlj Equity
Quantiles of Rlj Equity
Standard normal quantiles

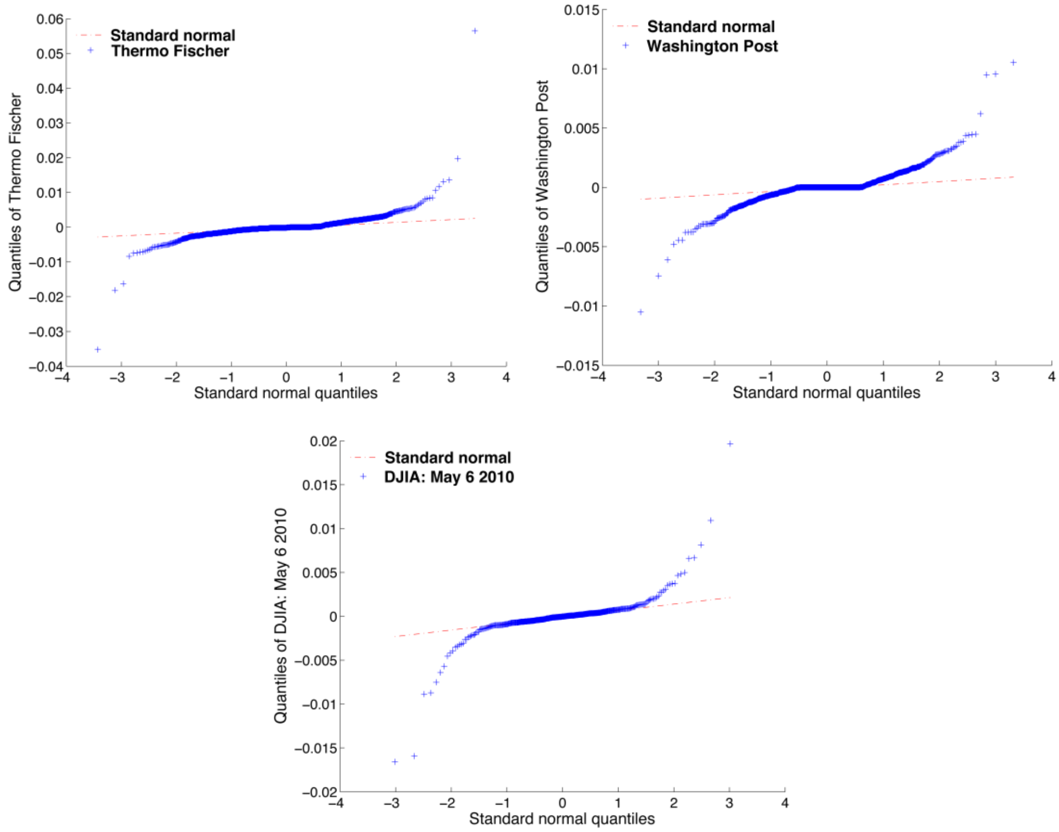


Figure 2. *Q-Q* plots of the stock returns showing departures from Gaussianity.

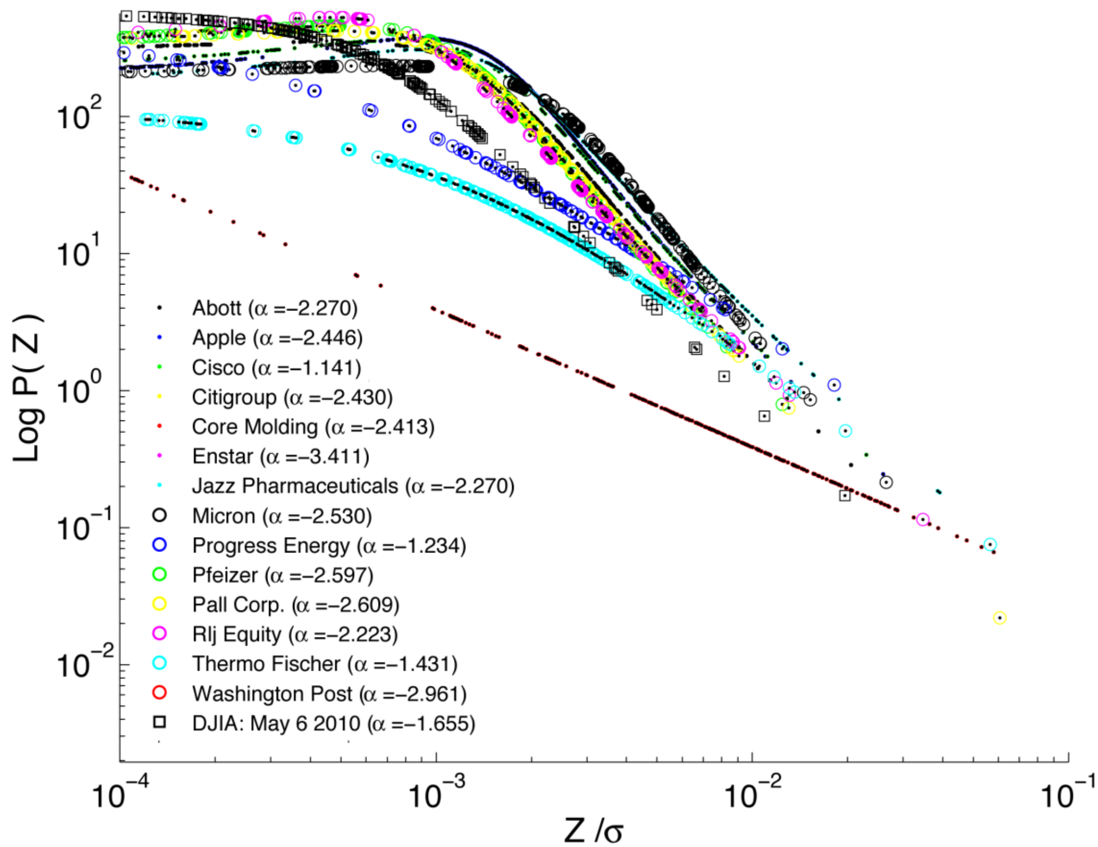


Figure 3. Power law decays for the distribution of normalized stock returns Z/σ .

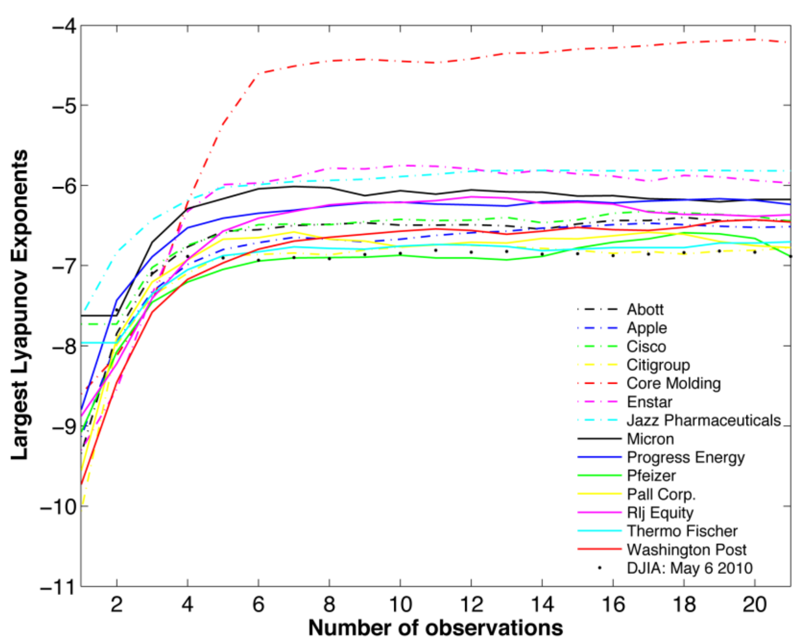
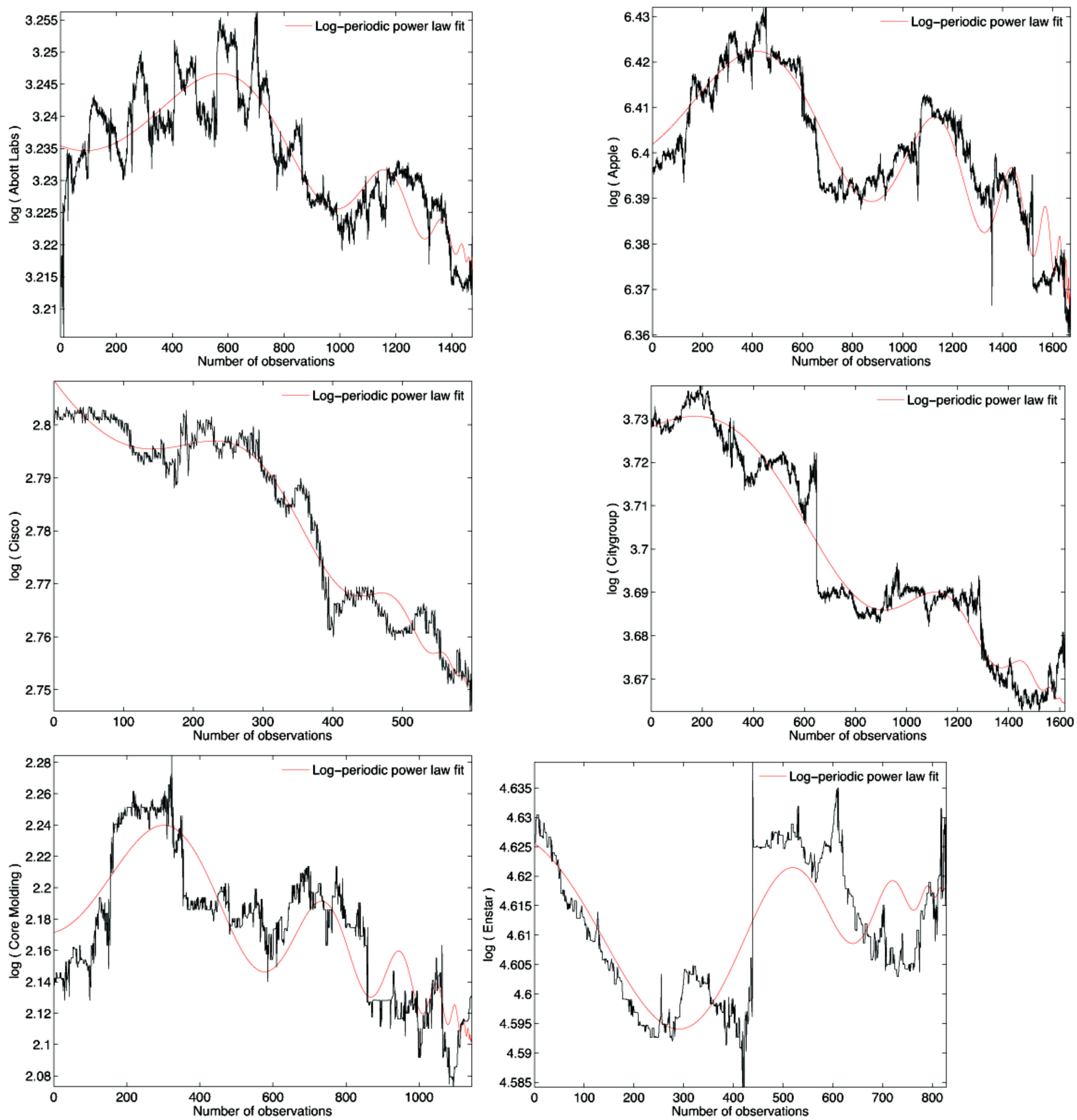
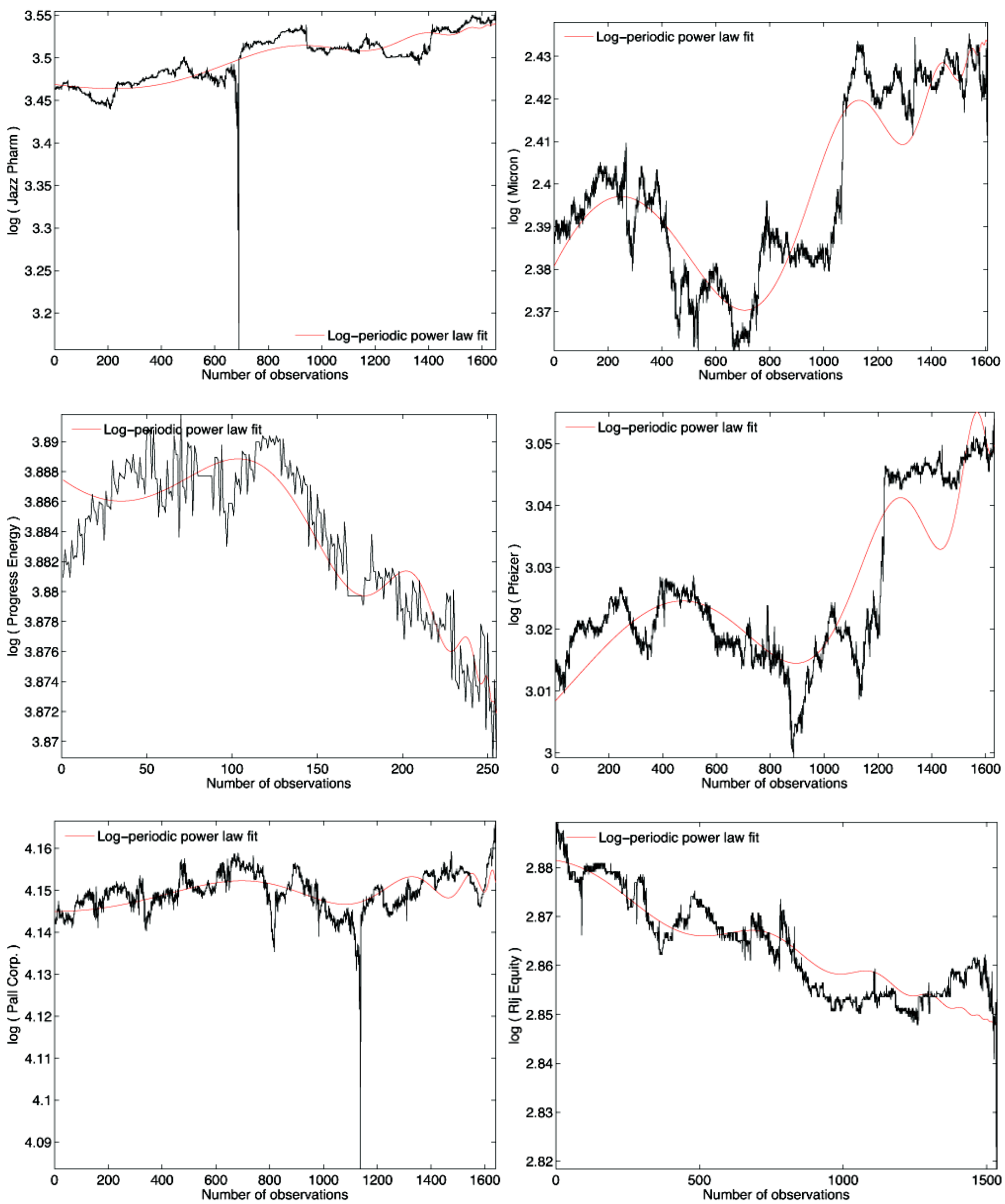


Figure 4. Stable plateaus of the largest Lyapunov exponents for the stock time series.


Log–periodic power law fit
log (Abott Labs)
Number of observations
log (Apple)
log (Cisco)
log (Citygroup)
log (Core Molding)
log (Enstar)


log (Jazz Pharm)
Log−periodic power law fit
Number of observations
log (Micron)
Log−periodic power law fit
Number of observations
log (Progress Energy)
Log−periodic power law fit
Number of observations
log (Pfeizer)
Log−periodic power law fit
Number of observations
log (Pall Corp.)
Log−periodic power law fit
Number of observations
log (Rlj Equity)
Log−periodic power law fit
Number of observations

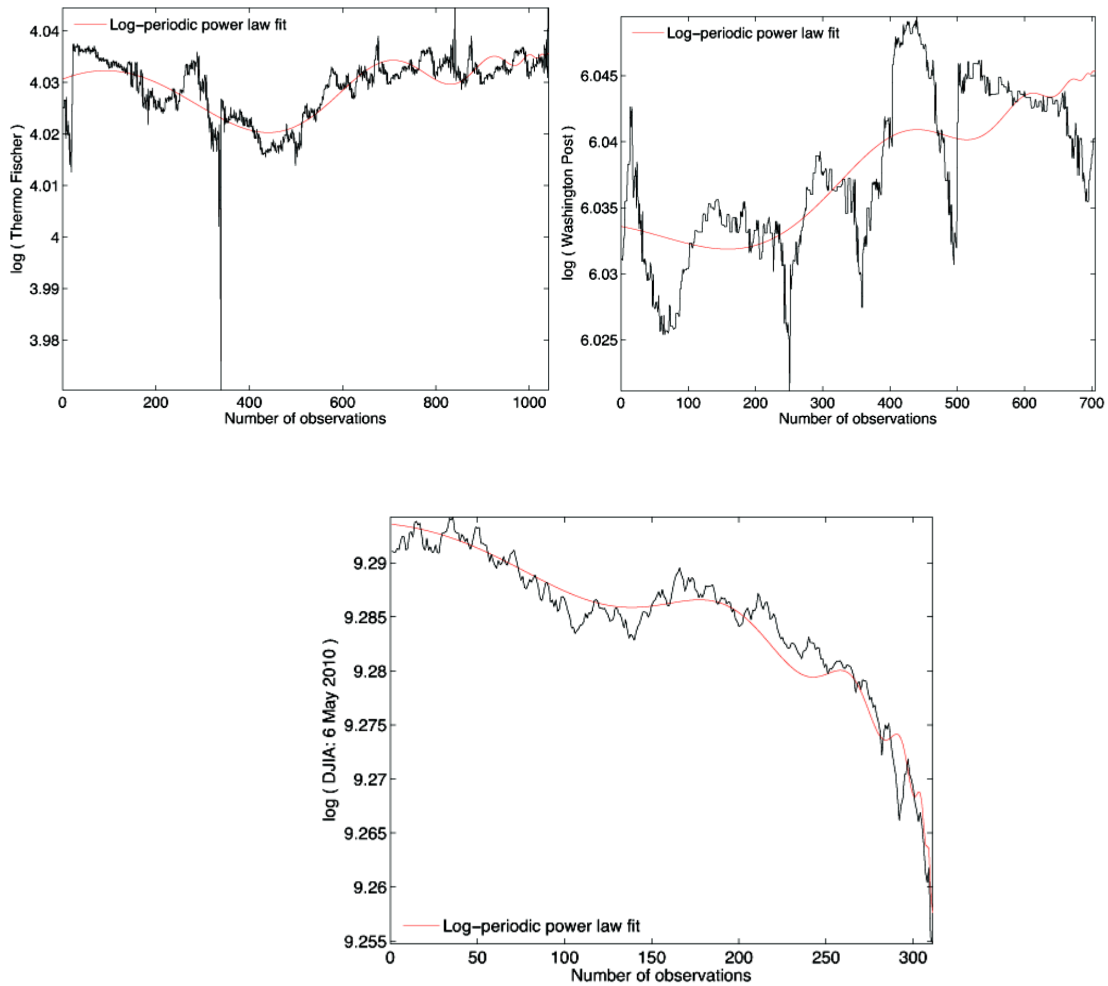


Figure 5. Log periodic power law fits for the stock time series.

Table 1. Description of data

Stock	*Sample size*	*Time period (tic-by-tic from 9:30 a.m. to 4 p.m.)*	*Mean return*	*Variance*
Abott Labs	1672	29 April 2011 – 31 May 2011	6.8648e−07	4.4884e−06
Apple	1671	16 March 2012 – 30 March 2012	−1.4522e−05	5.2526e−06
Cisco Systems	1632	20 July 2011 – 29 July 2011	−1.5533e−05	5.2725e−06
Citigroup	1620	24 June 2011 – 30 June 2011	−3.3082e−05	3.2485e−06
Core Molding	1144	19 August 2011 – 31 August 2011	−1.0227e-05	8.2212e−05
Enstar	828	12 May 2011 – 13 May 2011	−1.2263e-05	7.8117e−06
Jazz Pharmaceuticals	1652	20 April 2011 – 29 April 2011	5.5291e−05	1.4599e−04
Micron	1608	29 April 2011 – 6 May 2011	3.2291e−05	7.6406e−06
Progress Energy	528	30 August 2011 – 07 September 2011	−2.8700e−05	5.5584e−06
Pfeizer	1623	2 May 2011 – 5 May 2011	2.2147e−05	2.4454e−06
Pall Corporation	1640	17 February 2012 – 2 March 2012	1.3602e−05	6.2189e−06
RLJ Equity Partners	1536	11 May 2011 – 12 May 2011	−2.7002e−05	4.6044e−06
Thermo Fischer Co.	1640	17 February 2012 – 2 March 2012	−1.8144e−06	6.5691e−06
Washington Post	1104	14 June 2011 – 20 June 2011	6.9993e−06	1.5832e−06
DJIA index	391	6 May 2010	−7.6953e−05	5.0441e−06

Table 2. Further descriptive statistics of data: Day of crash, skewness, excess kurtosis, and drawdown

Stock	*Day of crash*	*Skewness*	*Excess kurtosis*	*Drawdown*
Abott Labs	2 May 2011	1.1420	15.4381	0.0431
Apple	23 March 2012	0.6084	21.8629	0.0699
Cisco Systems	29 July 2011	0.6777	13.8423	0.0558
Citigroup	29 June 2011	−0.5603	17.3291	0.0724
Core Molding	26 August 2011	0.3821	9.9299	0.1904
Enstar	13 May 2011	4.2109	78.9024	0.0452
Jazz Pharmaceuticals	27 April 2011	11.6698	472.8615	0.2913
Micron	5 August 2011	0.8787	11.5503	0.0476
Progress Energy	27 September 2011	1.2511	11.7679	0.0292
Pfeizer	2 May 2011	0.8677	7.5868	0.0290
Pall Corporation	27 February 2012	6.6160	251.0168	0.0725
RLJ Equity Partners	11 May 2011	0.5516	90.6679	0.0684
Thermo Fischer Co.	27 February 2012	5.1792	173.3840	0.0794
Washington Post	16 June 2011	0.5849	20.4079	0.0213
DJIA index	6 May 2010	−0.0473	33.8842	0.0850

Table 3. Tests of Gaussianity in data

Stock	*Lilliefors*	*Cramer-Von Mises*	*Jarque-Bera*	*Reject normality?*
Abott Labs	0.2071 (< 0.01)	0.0829 (< 0.01)	1.109e5 (< 0.01)	Yes
Apple	0.2058 (< 0.01)	0.0829 (< 0.01)	2.477e5 (< 0.01)	Yes
Cisco Systems	0.1668 (< 0.01)	0.0829 (< 0.01)	8.083e4 (< 0.01)	Yes
Citigroup	0.1843 (< 0.01)	0.0830 (< 0.01)	1.388e5 (< 0.01)	Yes
Core Molding	0.2178 (< 0.01)	0.0816 (< 0.01)	2.301e4 (< 0.01)	Yes
Enstar	0.2517 (0.01)	0.0829 (< 0.01)	1.997e5 (< 0.01)	Yes
Jazz Pharmaceuticals	0.3122 (< 0.01)	0.0825 (< 0.01)	1.517e8 (< 0.01)	Yes
Micron	0.1760 (< 0.01)	0.0828 (< 0.01)	5.081e4 (< 0.01)	Yes
Progress Energy	0.2166 (< 0.01)	0.0829 (< 0.01)	1.804e4 (< 0.01)	Yes
Pfeizer	0.1727 (< 0.01)	0.0830 (< 0.01)	1.627e4 (< 0.01)	Yes
Pall Corporation	0.1868 (< 0.01)	0.0830 (< 0.01)	4.199e7 (< 0.01)	Yes
RLJ Equity Partners	0.2079 (< 0.01)	0.0830 (< 0.01)	4.900e6 (< 0.01)	Yes
Thermo Fischer Co.	0.2106 (< 0.01)	0.0830 (< 0.01)	1.983e7 (< 0.01)	Yes
Washington Post	0.2317 (< 0.01)	0.0831 (< 0.01)	1.391e5 (< 0.01)	Yes
DJIA index	0.2259 (< 0.01)	0.0830 (< 0.01)	1.528e5 (< 0.01)	Yes

Note: *p*-values are in parenthesis. The null of Gaussianity is rejected at the 5 and 10 percent significant levels for all the stocks. And apart from Enstar in the Lilliefors test, Gaussianity is also rejected at the 1 percent level for the other stocks.

Table 4. Tests of stability of the largest Lyapunov exponents

	$\hat{\beta}_1$			
Bootstrap block	*1*	*2*	*3*	*4*
Abott Labs	−0.0133 (−1.40)	−0.0066 (−8.36)	−0.0045 (−1.27)	−0.0040 (−5.96)
Apple	−0.0135 (−1.00)	−0.0069 (−1.69)	−0.0046 (−1.13)	−0.0034 (−7.65)
Cisco Systems	−0.0132 (−3.14)	−0.0066 (−6.06)	−0.0044 (−1.12)	−0.0033 (−3.40)
Citigroup	−0.0145 (−2.19)	−0.0071 (−9.75)	−0.0047 (−1.33)	−0.0043 (−2.00)
Core Molding	−0.0164 (−8.53)	−0.0082 (−9.76)	−0.0056 (−1.65)	−0.0044 (−2.50)
Enstar	−0.0193 (−3.20)	−0.0164 (−1.85)	−0.0108 (−2.62)	−0.0076 (−3.92)
Jazz Pharmaceuticals	−0.0222 (−2.74)	−0.0067 (−2.71)	−0.0050 (−2.03)	−0.0037 (−5.95)
Micron	−0.0118 (−2.68)	−0.0062 (−8.34)	−0.0063 (−2.38)	−0.0039 (−2.35)
Progress Energy	−0.0628 (−2.23)	−0.0323 (−2.66)	−0.0213 (−2.73)	−0.0123 (−3.91)
Pfeizer	−0.0145 (−6.29)	−0.0070 (−8.61)	−0.0047 (−2.10)	−0.0043 (−1.86)
Pall Corporation	−0.0142 (−3.35)	−0.0071 (−1.58)	−0.0046 (−7.10)	−0.0034 (−1.40)
RLJ Equity Partners	−0.0129 (−6.86)	−0.0066 (−6.79)	−0.0044 (−6.31)	−0.0043 (−1.25)
Thermo Fischer Co.	−0.0137 (−2.69)	−0.0070 (−1.96)	−0.0046 (−2.53)	−0.0042 (−2.56)
Washington Post	−0.0231 (−2.14)	−0.0116 (−4.91)	−0.0076 (−5.16)	−0.0063 (−6.76)
DJIA index	−0.0701 (−2.79)	−0.0512 (−1.96)	−0.0200 (−1.18)	−0.0179 (−1.25)

Note: Negative largest Lyapunov exponents for block 1 (1/4 of the time series' size), block 2 (2/4 of the time series' size), and so on. In parentheses: critical values of the empirical distributions of β_i divided by their standard deviations.

Table 5. Log periodic power law fits for the stock time series

Stock	t_c	$\tau = t - t_c$	*A*	*B*	*C*	m^2	ω	π
Abott Labs	78	$\tau > 0$	3.22	5.5e−6	−0.00002	0.9952	3.2827	6.7353
Apple	1500	$0 < \tau < 1600$	6.70	−0.00021	0.00195	0.4016	9.4081	6.0330
Cisco Systems	600	$0 < \tau < 700$	2.75	−0.00564	0.000774	0.4322	6.6690	1.4031
Citigroup	1500	$0 < \tau < 1600$	3.671	0.00443	0.000133	7.5991	9.9002	4.3312
Core Molding	1480	$0 < \tau < 1500$	2.06	−0.00091	0.005421	3.9001	8.4332	5.3321
Enstar	405	$0 < \tau < 800$	4.58	0.00114	0.000675	0.5065	3.5634	3.3093
Jazz Pharmaceuticals	962	$0 < \tau < 100$	3.48	−0.1740	0.00322	−2.7825	2.2054	−0.818
Micron	243	$800 < \tau < 1607$	2.43	−0.00955	0.00001	0.4412	10.233	5.401
Progress Energy	250	$100 < \tau < 250$	3.87	−0.00067	0.00487	0.5441	6.6650	2.332
Pfeizer	1600	$30 < \tau < 1600$	3.05	0.00213	0.06650	0.2320	6.2001	4.880
Pall Corporation	1600	$0 < \tau < 1600$	4.10	−0.00065	−0.0033	0.4322	5.6707	5.551
RLJ Equity Partners.	1510	$0 < \tau < 1510$	2.83	−0.10019	−0.0441	0.6391	8.7710	3.310
Thermo Fischer	1100	$150 < \tau < 1110$	4.03	−0.0800	−0.0392	0.7010	4.2210	2.002
Washington Post	700	$250 < \tau < 700$	6.045	−0.01223	0.00937	0.4804	3.0012	5.901
DJIA index	323	$0 < \tau < 323$	9.255	−0.00142	0.00121	0.7322	6.3223	2.321

Table 6. Shannon entropy and long memory tests

Stock	*Entropy*	*R/S*	*DFA*	*Long memory?*
Abott Labs	0.0468	0.479	0.433	No
Apple	0.0509	0.543	0.675	Yes
Cisco Systems	0.0510	0.531	0.571	Yes
Citigroup	0.0303	0.426	0.445	No
Core Molding	0.3824	0.535	0.563	Yes
Enstar	0.0330	0.473	0.422	No
Jazz Pharmaceuticals	0.4620	0.527	0.223	No
Micron	0.0755	0.575	0.438	No
Progress Energy	0.0178	0.466	0.452	No
Pfeizer	0.0277	0.450	0.388	No
Pall Corporation	0.0520	0.536	0.150	No
RLJ Equity Partners	0.0366	0.405	0.329	No
Thermo Fischer Co.	0.0546	0.617	0.206	No
Washington Post	0.0105	0.508	0.321	No
DJIA index	0.0087	0.398	0.364	No

Note: We consider long memory to be unequivocal when both the *R/S* and *DFA* are greater than 0.5.

Table 7. Shannon entropy and long memory tests

Model	*Entropy*	*R/S*	*DFA*
Threshold GARCH	−0.00025	0.413	0.586
Skewed-*t* GARCH	−0.00038	0.392	0.518

Conflict of Interests

The author declares that there is no conflict of interests.

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