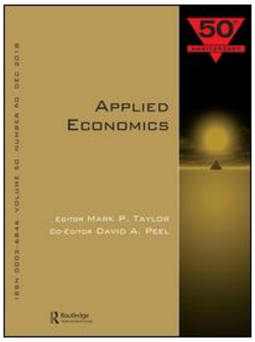

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Optimal trading strategies for Lévy-driven Ornstein–Uhlenbeck processes

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ABSTRACT

This study derives an optimal pairs trading strategy based on a Lévy-driven Ornstein–Uhlenbeck process and applies it to high-frequency data of the S&P 500 constituents from 1998 to 2015. Our model provides optimal entry and exit signals by maximizing the expected return expressed in terms of the first-passage time of the spread process. An explicit representation of the strategy's objective function allows for direct optimization without Monte Carlo methods. Categorizing the data sample into 10 economic sectors, we depict both the performance of each sector and the efficiency of the strategy in general. Results from empirical back-testing show strong support for the profitability of the model with returns after transaction costs ranging from 31.90% p.a. for the sector 'Consumer Staples' to 278.61% p.a. for the sector 'Financials'. We find that the remarkable returns across all economic sectors are strongly driven by model parameters and sector size. Jump intensity decreases over time with strong outliers in times of high market turmoil. The value-add of our Lévy-based model is demonstrated by benchmarking it with quantitative strategies based on Brownian motion-driven processes.

KEYWORDS

Finance; pairs trading; optimal thresholds; Ornstein–Uhlenbeck Lévy process; high-frequency data

JEL CLASSIFICATION

C10; C22; C41; C50; C60

I. Introduction

Pairs trading is a market neutral strategy which has been developed by a group of computer scientists, mathematicians, and physicists at Morgan Stanley in the early to mid-1980s (Vidyamurthy 2004). In its most common form, this strategy finds pairs of related stocks whose prices follow a similar pattern over a historical time period. In case of abnormal divergence, an arbitrageur goes long in the undervalued stock while simultaneously going short in the overvalued stock. The exposure to market risk is substantially reduced for this kind of trading strategy. If history repeats itself, the price relationship converges to the long-term equilibrium and a profit is taken. Gatev, Goetzmann, and Geert Rouwenhorst (2006) expound the first major academic study on statistical arbitrage pairs trading exhibiting excess returns of 11% p.a. for U.S. CRSP securities from 1962 until 2002. This seminal paper characterizes the trigger for an ever-expanding interest in this field of research up to the present. Substantial contributions are supplied by Vidyamurthy (2004), Elliott, van der Hoek, and Malcolm (2005), Avellaneda and Lee (2010), Do and Faff (2012), Huck (2015), Huck and Afawubo (2015), Liu, Chang, and Geman (2017), and Mikkelsen (2018).

The vast majority of the literature determines model-free trading rules via a pre-defined parameter setting, e.g., positions are opened at a two-standard-deviation spread – the criticism of data snooping is omnipresent. Only a small fraction depicts analytic formulae for calculating optimal model-driven statistical arbitrage pairs trading thresholds. This research is confined to Elliott, van der Hoek, and Malcolm (2005), Bertram (2010a, 2010b), Ekström, Lindberg, and Tysk (2011), Govender (2011), Gregory, Ewald, and Knox (2011), Cummins and Bucca (2012), Zeng and Lee (2014), Leung and Xin (2015a, 2015b), Li (2015), Göncü and Akyıldırım (2016a), Bai and Lan (2017), Baviera and Baldi (2017), and Suzuki (2018). These studies model the price spread between two stocks with an Ornstein–Uhlenbeck (OU) process based on Brownian motion.

Given the characteristics of financial data, OU processes based on Brownian motion are not eligible to model high-frequency dynamics – direct consequences are invalid parameter estimates and disregard of stylized facts (Barlow 2002; Carr et al. 2002; Cont and Tankov 2003; Cartea and Figueroa 2005; Meyer-Brandis and Tankov 2008; Jing, Kong, and Liu 2012; Aït-Sahalia and Jacod 2014;

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Jondeau, Lahaye, and Rockinger 2015; Göncü and Akyıldırım 2016b). Surprisingly, only Larsson, Lindberg, and Warfheimer (2013) and Göncü and Akyıldırım (2016b) present academic studies in the context of determining optimal thresholds using Lévy-driven OU processes. It is important to note that both exhibit merely numerical solutions. Larsson, Lindberg, and Warfheimer (2013) generalize the approach of Ekström, Lindberg, and Tysk (2011) by additionally including a jump term. Göncü and Akyıldırım (2016b) give attention to Lévy-driven OU processes with generalized hyperbolic distributed marginals. Together, these two studies provide an initial step in determining optimal trading strategies based on Lévy-driven OU processes.

We enhance the existing research in several aspects. First, our manuscript contributes to the literature by developing an optimal trading framework based on Lévy-driven OU processes, i.e., we derive optimal trade thresholds by maximizing the expected return per unit time. In contrast to the existing literature, Monte Carlo methods are avoided due to a mathematical expression for the expected first-passage time and an explicit representation of the strategy's objective function. Besides mean-reversion, volatility clusters, and drifts, this general and flexible class of stochastic models is able to capture jumps and fat tails. Considering all these effects, our strategy is able to perform both intraday and overnight trading. Most notable, we generalize the Brownian motion-driven OU processes by including the essential jump component. As such, our model is in a position to choose the borderline case in times of full stagnation or quiet behaviour without fat tails in returns at the stock market. Second, we present the first academic survey applying a high-frequency back-testing study to all industry sectors of the S&P 500 over a sample period of 18 years. Therefore, we categorize all companies into 10 economic sectors based on the Global Industry Classification Standard (GICS) and consider all pairs consisting of stocks from the same sector. Thus, we are in a position both to demonstrate the performance of each sector and the efficiency of our strategy in general. For all sectors, we observe statistically and economically significant returns after transaction costs from 1998 to 2015, ranging from 31.90% p.a. for the sector 'Consumer Staples' to 278.61% p.a. for the sector 'Financials'. Third, we analyse common effects across all economic sectors and observe that returns are strongly driven by model parameters and sector size. This finding explains the varying performance between the different branches of trade. Fourth, we demonstrate the value-add of our optimal trading strategy for Lévy-driven OU processes by benchmarking it with well-known quantitative strategies in the same field. We find that our strategy outperforms classic pairs trading approaches over time and obtains almost no loading on systematic sources of risk.

The rest of this paper is organized as follows. Section II describes the data set and software utilized in this study. In Section III, we depict the theoretical framework for determining optimal trading strategies. The model assumptions are evaluated in Section IV. Section V provides the study design for our back-testing application. Results and key findings are reported and discussed in Section VI. Finally, Section VII summarizes the main results of this paper and makes methodological recommendations about further research.

II. Data and software

We run our back-testing framework on minute–by–minute prices of the S&P 500 index constituents from January 1998 until December 2015. Our highly liquid trading universe comprises the 500 leading companies which satisfy strict requirements based on market size, liquidity, and industry grouping. Focusing on large-cap U.S. equities, this subset covers 80% of available market capitalization (S&P 500 Dow Jones Indices 2015). Consequently, this market segment poses a crucial test for any potential stock market inefficiency. Following Stübinger and Endres (2018), we eliminate the survivor bias from our data with the aid of a two-stage process. First, using QuantQuote (2016), we produce a daily constituent list for the S&P 500 stocks from 1998 to 2015. Then, exploiting this information, we create a binary matrix, i.e., for each element of this matrix, we assign a '1′ if the corresponding company is a constituent of the S&P 500 index at the current day, otherwise a '0'. Second, for all companies having ever been part of the S&P 500 index, we download the minute–by–minute stock prices from January 1998 until December 2015 from QuantQuote (2016). An adjustment of the data set is conducted due to stock splits, dividends, and further corporate actions. Applying these two steps, we are in a position to completely replicate the S&P 500 index constituency and the respective prices over time.

III. Methodology

According to Zeng and Lee (2014), the concept of finding an optimal pairs trading strategy follows a three-step process. First, we specify a stochastic model explaining the spread dynamics. In this study, we assume a Lévy-driven Ornstein–Uhlenbeck (OU) process – a highly flexible approach for capturing typical characteristics of financial data, among them jumps, mean-reversion, volatility cluster, and drifts. Most notable, our model also includes the Brownian motion-driven OU process. In the second step, we define the target function of the strategy, e.g., maximize the expected profit or minimize the time until trade termination. Finally, we aim for deriving entry and exit times by optimizing the target function. This section describes the three-step logic outlined above in detail.

Stochastic model

We define the spread for two stocks A and B with prices $S_A(t)$ and $S_B(t)$ as

$$X_t = \ln(S_A(t)/S_A(0)) - \ln(S_B(t)/S_B(0)),\ t \geq 0.$$

For modelling the spread dynamics, we consider the following OU process driven by Lévy noise:

$$dX_t = \theta(\mu_t - X_t)dt + dL_t,\ X_0 = x, \tag{1}$$

with mean-reversion speed $\theta \in \mathbb{R}$, time-dependent mean-reversion level $\mu_t \in \mathbb{R}$, and Lévy process $\{L_t\}_{t\geq 0}$. As a special case, the model covers earlier spread models based on Brownian motion-driven OU processes (Elliott, van der Hoek, and Malcolm 2005; Bertram 2009, 2010a; Ekström, Lindberg, and Tysk 2011; Cummins and Bucca 2012; Bogomolov 2013; Göncü and Akyıldırım 2016a). The solution of Equation (1) is described by

$$X_t = xe^{-\theta t} + \theta \int_0^t \mu(u)e^{-\theta(t-u)}du + \int_0^t e^{-\theta(t-u)}dL_u$$

In line with Kou (2002), Kou and Wang (2003), Sepp (2003), Ramezani and Zeng (2007), Bayraktar and Xing (2011), Cai and Kou (2011), Kou, Cindy, and Zhong (2017), and Su and Bai (2017), we assume a double exponential jump-diffusion representation for the Lévy process $\{L_t\}_{t\geq 0}$, which fits stock data better than a model with normally distributed jump sizes (Ramezani and Zeng 1998; Li, Wells, and Yu 2008; Kou, Cindy, and Zhong 2017).

The sequence of random variables is divided into two parts – a continuous part driven by a Brownian motion and a compound Poisson process possessing a double exponential distribution. Mathematically, we define the Lévy process by

$$L_t = \sigma W_t + \sum_{i=1}^{N_t} \xi_i, \tag{2}$$

with standard Brownian motion $\{W_t\}_{t\geq 0}$, volatility $\sigma \in \mathbb{R}^+$, Poisson process $\{N_t\}_{t\geq 0}$ with rate λ, and jump sizes $\{\xi_1, \xi_2, \ldots\}$. The processes $\{W_t\}_{t\geq 0}$, $\{N_t\}_{t\geq 0}$, and the random variables $\{\xi_1, \xi_2, \ldots\}$ are independent. The common density of ξ is given by

$$f_\xi(x) = p_u \eta_u e^{-\eta_u(x-\delta)} 1_{\{x\geq\delta\}} + p_d \eta_d e^{\eta_d(x+\delta)} 1_{\{x\leq-\delta\}},$$

where $p_u, p_d \in \mathbb{R}_0^+$ with $p_u + p_d = 1$, $\eta_u, \eta_d \in \mathbb{R}^+$ and $\delta \in \mathbb{R}_0^+$. The constants p_u and p_d represent the probabilities of upward (u) and downward (d) jumps. The indicator functions only admit jump sizes greater (smaller) than $\delta\,(-\delta)$. The moment generating function of the jump size ξ is given by

$$E[e^{s\xi}] = p_u \frac{\eta_u}{\eta_u - s} e^{\delta s} + p_d \frac{\eta_d}{\eta_d + s} e^{-\delta s},\ s \in (-\eta_d, \eta_u),$$

from which the moment generating function of the process $\{L_t\}_{t\geq 0}$ is obtained as

$$E[e^{sL_t}] = e^{\lambda t(p_u \frac{\eta_u}{\eta_u - s} e^{\delta s} + p_d \frac{\eta_d}{\eta_d + s} e^{-\delta s} - 1)} e^{\sigma^2 s^2 t/2}. \tag{3}$$

In the double exponential jump–diffusion model, the upward and downward jumps are both generated by a single Poisson process with fixed intensity λ. Ramezani and Zeng (1998) propose a similar model, in which upward and downward jumps are caused by two independent Poisson processes $\left\{N_t^j(\lambda^j)\right\}_{t\geq 0}$ with intensity parameters λ^j $(j \in \{u, d\})$. The corresponding Lévy process is defined by

$$L_t = \sigma W_t + \sum_{j\in\{u,d\}} \sum_{i=1}^{N_t^j(\lambda^j)} \xi_i^j. \tag{4}$$

The density functions for jump magnitudes are $f_{\xi^u}(x) = \eta_u e^{-\eta_u(x-\delta)} 1_{\{x\geq\delta\}}$ and $f_{\xi^d}(x) = \eta_d e^{\eta_d(x+\delta)} 1_{\{x\leq-\delta\}}$. Besides economic reasons, the distinction between upward and downward variations leads to better analytical tractability. Following Ramezani and Zeng (2007), we transform the parameters of (2) and (4) into each other by setting $\lambda = \lambda_u + \lambda_d$ and $p_u = \frac{\lambda_u}{\lambda}$. Furthermore, we set $\eta_u = \eta_d = \eta$ and $p_u = 1/2$ (Carr and Liuren 2004; Tsay 2005; Ramezani and Zeng 2007) – an assumption that has been commonly invoked.

Objective function

As a natural target of any rational investor, we maximize the expected return per unit time in our trading strategy. We define the entry and exit signals x and b $(x, b \in \mathbb{R})$ and assume $x < b$ without loss of generality. The time over which the return takes place is the first-passage time $\tau_{b,x}$:

$$\tau_{b,x} = \inf\{t \geq 0 | X_t \geq b\}$$

for $X_0 = x$. Figure 1 illustrates the setting for simulated spread dynamics and thresholds x and b. Positions are entered and exited when the spread crosses the thresholds. The squares representing the respective times are not exactly on the dashed lines because the spread exhibits jumps and additionally, trading is discrete. The investor enters at $t_0 = \inf\{t \geq 0 | X_t \leq x\}$ and exits at $\tau_{b,x} = \inf\{t \geq 0 | X_t \geq b\}$ for the first time.

The return r is a function of the thresholds x and b and transaction costs c $(c \in \mathbb{R}_0^+)$. This results in the optimization problem

$$\max_{b,x} \frac{r(b,x,c)}{E[\tau_{b,x}]}, \tag{5}$$

which is actually a problem of determining or approximating the distribution of the first-passage time $\tau_{b,x}$. In contrast to the Brownian motion-driven OU process, the first-passage time problem for general Lévy processes has not been solved analytically. Nevertheless, the construction of the Lévy process in Equations (2) and (4) offers a rare case allowing for analytical solutions for the first passage times, derived in Borovkov and Novikov (2008).

Optimal variables

In this subsection, we determine the optimal deviation thresholds to generate trade signals and execute pairs trading. Therefore, we decompose

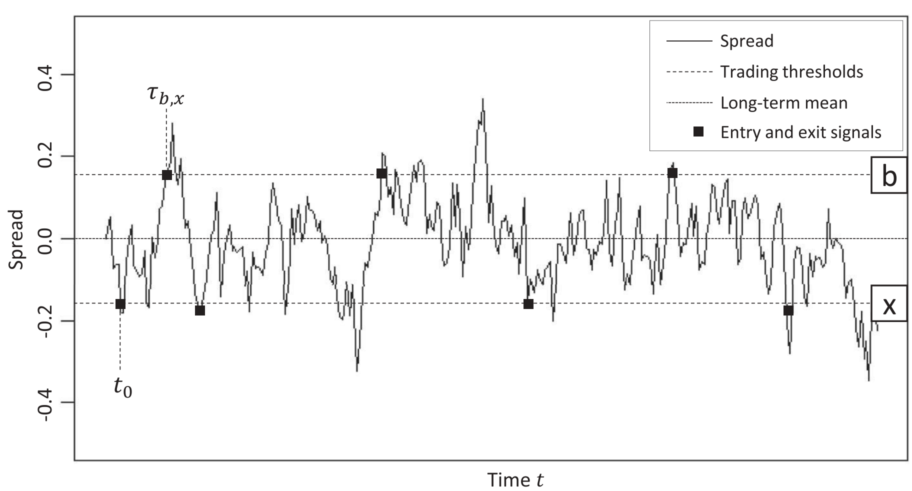

Figure 1. Trading strategy for simulated spread dynamics (black curve) and thresholds x and b (dashed lines). Squares label the times of opening and closing positions.

the Lévy process $\{L_t\}_{t\geq 0}$ into two parts – $\{Q_t\}_{t\geq 0}$, representing the downward jumps and the Brownian motion, and $\{R_t\}_{t\geq 0}$, representing the upward jumps. Both jump sizes follow exponential distributions. Due to the memoryless property of the exponential distribution, it holds that

$$E(\tau_{b,x}) = \frac{1}{\theta}\int_0^K (e^{ub} - e^{ux}(1 - u/K))$$

$$(1 - u/K)^{\lambda/(2\theta)-1} e^{-\Delta(u)} u^{-1} du \qquad (6)$$

for

$$\Delta(u) = \frac{1}{\theta}\int_0^u \frac{\log(Ee^{vQ_1})}{v} dv \qquad (7)$$

and

$$K = \sup\{u \geq 0 | Ee^{uL_1} < \infty\} \qquad (8)$$

with $X_0 = x$ (Borovkov and Novikov 2008). The moment generating functions in Equations (7) and (8) are given by

$$E[e^{sQ_t}] = e^{\lambda_d t\left(\frac{\eta}{\eta+s}e^{-\delta s}-1\right)} e^{\sigma^2 s^2 t/2}$$

and

$$E[e^{sL_t}] = m_{Q_t}(s) e^{\lambda_u t\left(\frac{\eta}{\eta-s}e^{\delta s}-1\right)}.$$

Due to the explicit representation of $E(\tau_{b,x})$, we are in a position to directly solve the problem (5). Typically, such optimization problems are solved by a box-constrained optimization routine using the Port library (Gay 1990; Fox 1997). We obtain thresholds that maximize the expected return per unit time in the respective trading strategies.

IV. Model evaluation

From a total of 85 studies in the pairs trading context based on stochastic differential equations, the vast majority explains the spread by a Brownian motion-driven OU model, e.g., Elliott, van der Hoek, and Malcolm (2005), Bertram (2010a), and Zeng and Lee (2014). We use a model that is even more flexible: the Lévy-driven OU model that includes the classic OU model as a special case. The latter model is used by Larsson, Lindberg, and Warfheimer (2013), Göncü and Akyıldırım (2016b), and Endres and Stübinger (2018). This section conducts two analyses in order to justify the assumptions of our Lévy-driven OU model. Therefore, we divide our data set from 1998 to 2015 into disjoint subperiods with a length of 30 days. For each subperiod, we consider the constituent stocks of the S&P 500 index and identify all possible pair combinations within each economic sector. Per pair combination, we determine the corresponding spread process X_t.

In the first analysis, we test whether it is reasonable to assume that X_t is Lévy-driven, i.e., we follow Abdelrazeq, Ivanoff, and Kulik (2014) and Abdelrazeq (2015) and test the hypothesis of uncorrelated increments. Therefore, from each spread X_t, observed at discrete times $0, 1/M, 2/M, \ldots, N$, the unobserved driving process L_t is approximated by estimating the increments $\Delta_1\hat{L}^{(M)}$. Then, the authors define a test statistic

$$W_{\Delta_1\hat{L}^{(M)}}(k) = \sqrt{N}\frac{\hat{\gamma}_{\Delta_1\hat{L}^{(M)}}(k)}{\hat{\eta}^2}$$

for $\hat{\eta}^2$ and $\hat{\gamma}_{\Delta_1\hat{L}^{(M)}}(k)$ the sample variance and covariance, respectively, of $\Delta_1\hat{L}^{(M)}$, and lag k, $k = 1, \ldots, N$. The null hypothesis of uncorrelated increments is rejected for large absolute values of the statistic $W_{\Delta_1\hat{L}^{(M)}}(k)$, i.e., at significance level α the hypothesis is rejected if

$$\left|W_{\Delta_1\hat{L}^{(M)}}(k)\right| > z_{1-\alpha/2}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$ quantile of the standard normal distribution. Table 1 presents the test results for all 10 economic sectors over a period from 1998 to 2015, for lag $k = 1$, $M = 500$ and $\alpha = 5\%$, chosen according to Abdelrazeq, Ivanoff, and Kulik (2014). The mean of the test statistics ranges from 0.56 for CD and HC sector to 1.32 for the UT sector – values clearly below the critical value $z_{1-\alpha/2} = 1.96$. This picture does not change considering the p-values, which are clearly above the 5% significance level. Consequently, and across all sectors, the rejection quote, i.e., the percentage of rejecting the null hypothesis, is at low levels, e.g. the average percentage of rejections varies between 8% for CD and HC, and 29% for UT. As such, the Lévy assumption does not have to be

Table 1. Model evaluation of Lévy-driven OU process for the 10 economic sectors of the S&P 500 from 1998 to 2015 for disjoint study periods of 30 days length for $\alpha = 5\%$, $k = 1$ and $M = 500$.

	CD	CS	EN	FI	HC	IN	IT	MA	TS	UT
Mean of test statistic	0.56	0.64	0.84	0.80	0.56	0.57	0.66	0.67	1.06	1.32
Median of test statistic	0.57	0.63	0.83	0.80	0.54	0.56	0.67	0.65	1.06	1.26
Mean of *p*-value	0.45	0.43	0.39	0.40	0.45	0.44	0.43	0.43	0.34	0.29
Median of *p*-value	0.46	0.43	0.40	0.41	0.46	0.45	0.44	0.43	0.35	0.29
Mean of rejection quote	0.08	0.10	0.15	0.14	0.08	0.09	0.11	0.12	0.21	0.29
Median of rejection quote	0.07	0.09	0.12	0.13	0.07	0.08	0.10	0.11	0.19	0.25

rejected in the majority of cases, which justifies the selection of our Lévy-driven model.

In the second analysis, we follow Abdelrazeq, Ivanoff, and Kulik (2018) and perform a goodness-of-fit test to evaluate whether a simple Brownian motion-driven OU process would have been sufficient for modelling the spread dynamics. We prove the value-add of additional flexibility of the Lévy-driven model compared to the Brownian motion-driven model especially in times of high market turmoils. Abdelrazeq, Ivanoff, and Kulik (2018) define a statistic following the usual Kolmogorov-Smirnov test statistic, with the difference that the increments of L_t are not observed directly. Table 2 reports test results for the 10 economic sectors for the period from 1998 to 2015. The null hypothesis normality is rejected in about 30% of the cases across all sectors at a 5% significance level. The value-add of the additional flexibility beyond the Gaussian framework becomes more apparent when considering times of crisis. We find that the highest percentages of normality rejection occur in 2001 and 2011, after the September 11 attacks and the global financial crisis. Consequently, our Lévy-driven model is of particular importance in times of high market turmoils because heavy tails cannot be attributed to a Gaussian framework (see, e.g., Meyer-Brandis and Tankov 2008; Bollerslev, Todorov, and Li 2013; Jondeau, Lahaye, and Rockinger 2015; Göncü and Akyıldırım 2016b).

In summary, we prove that the assumed Lévy-driven OU model is suited for our high-frequency data and that the flexibility of the Lévy-framework pays off – the value-add is particularly strong in times of high market volatility.

V. Study design

For our back-testing framework, we follow Jegadeesh and Titman (1993) and Gatev, Goetzmann, and Geert Rouwenhorst (1999, 2006) and divide our high-frequency data set from January 1998 to December 2015 into 4484 overlapping study periods (Figure 2). Each study period is shifted by one day and consists of a 30-day formation period and an out-of-sample 5-day trading period . We set the length of the formation and trading period consistent with Knoll, Stübinger, and Grottke (2018), Liu, Chang, and Geman (2017), and Stübinger and Endres (2018).

Formation period

In the 30-day formation period, we fit Ornstein–Uhlenbeck (OU) processes driven by Lévy noise as specified in Equation (1) to all possible combinations of pairs. Following Liu, Chang, and Geman (2017), we receive the mean-reversion level μ_t by a step function obtained from the average of the last two available data points out of the daily

Table 2. Goodness-of-fit test for Brownian motion for all economic sectors from 1998 to 2015 for disjoint study periods of 30 days length and $\alpha = 5\%$.

	CD	CS	EN	FI	HC	IN	IT	MA	TS	UT
Mean of test statistic	0.79	0.79	0.77	0.78	0.79	0.78	0.80	0.78	0.79	0.78
Median of test statistic	0.79	0.79	0.76	0.78	0.79	0.78	0.79	0.78	0.77	0.77
Mean of *p*-value	0.26	0.26	0.28	0.27	0.26	0.27	0.25	0.27	0.26	0.27
Median of *p*-value	0.26	0.26	0.28	0.27	0.26	0.27	0.25	0.27	0.26	0.28
Mean of rejection quote	0.33	0.34	0.30	0.32	0.34	0.32	0.34	0.32	0.33	0.31
Median of rejection quote	0.33	0.33	0.29	0.31	0.34	0.31	0.35	0.31	0.33	0.30
Maximum of rejection quote	0.50	0.47	0.52	0.50	0.49	0.58	0.50	0.62	0.81	0.60
Month of maximum of rejection quote	Sep 01	Feb 10	Apr 02	Jul 11	Jul 06	Sep 01	Sep 01	Sep 01	Jul 11	Sep 01

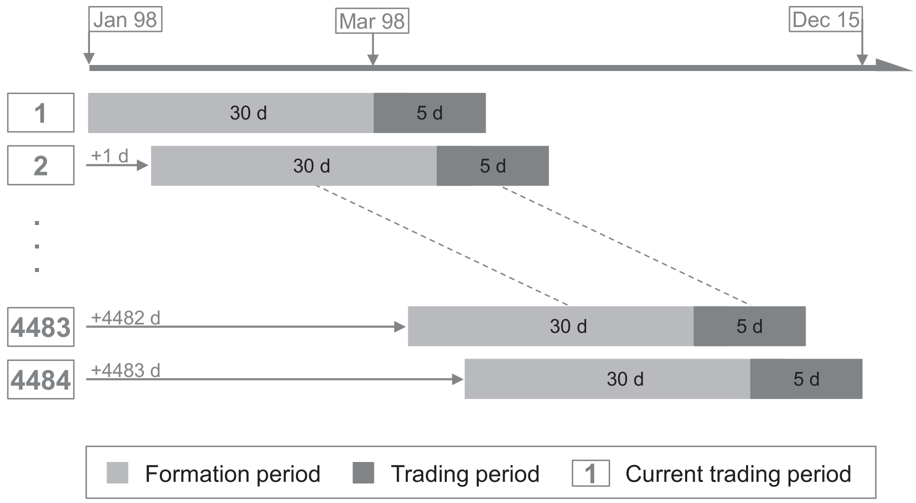

Figure 2. The back-testing application deals with 4484 overlapping study periods from January 1998 to December 2015. Each study period consists of a 30-day formation and a 5-day out-of-sample trading period.

opening and closing values. According to Mai (2012, 2014), there is a maximum likelihood estimation for the mean-reversion speed θ available. Given $X_{t_1}, \ldots, X_{t_n}$, the estimator is obtained from the discretization of the stochastic differential equation for $\Delta_i X = X_{t_{i+1}} - X_{t_i}$ and $\Delta_n = \max_{1 \leq i \leq n-1} \{|t_{i+1} - t_i|\}$ under the assumption that the jumps are of finite activity:

$$\hat{\theta}_n = \frac{\sum_{i=0}^{n-1} (\mu_{t_i} - X_{t_i}) \Delta_i X 1_{\{|\Delta_i X| \leq v_n\}}}{\sum_{i=0}^{n-1} (\mu_{t_i} - X_{t_i})^2 (t_{i+1} - t_i)}.$$

Increments larger than the threshold $v_n = \Delta_n^{\beta}$, $\beta \in (0, 1/2)$ are deleted. This is due to the fact that increments of the continuous part over an interval length of Δ_n are, with high probability, smaller than $\Delta_n^{0.5}$ (Mancini 2009; Mai 2012; Aït-Sahalia and Jacod 2014). The time interval Δ_n in our context is $\Delta_n = \frac{1}{250 \cdot 391}$ (Cont and Mancini 2011; Liu, Chang, and Geman 2017). We set the most restrictive variant $\beta = 0.4999$ to specify the threshold exponent, a value well in line with Cont and Mancini (2011), who use $\beta = 0.999/2$ in an application to the S&P 500 index. It should be stressed that the estimator does not detect any jumps in the special case of a Brownian motion-driven OU process (Mai 2012). The procedure in which the jump process is distinguished from the continuous part enables to separately estimate the parameter set of the jumps (λ, η) and the continuous part (θ, σ, μ_t). In accordance with Cartea and Figueroa (2005), the volatility σ is estimated by the sample standard deviation. The remaining jump parameters λ and η are calibrated by their maximum likelihood estimators.

We transfer the top 10 pairs (Miao 2014; Stübinger, Mangold, and Krauss 2018) exhibiting both a high mean-reversion speed θ and a high volatility σ to the trading period. A larger θ leads to a higher trading frequency, and a larger σ leads to a bigger fluctuation of the process, both resulting in a higher profit in each trade (Zeng and Lee 2014). This selection criterion is in line with Liu, Chang, and Geman (2017), who select pairs with low long-term and high short-term variance, both functions of θ and σ, and Stübinger and Endres (2018), who select pairs with high mean-reversion speed θ and high jump intensity λ.

Trading period

In line with Bertram (2010a), Cummins and Bucca (2012), and Zeng and Lee (2014), we construct trading bands $\mu_t \pm b$ symmetric about the mean level μ_t of the process. Following Bertram (2010a), the return per trade is defined by the range between the upper and lower trading threshold, adjusted for transaction costs c $(c \in \mathbb{R}_0^+)$. Hence, we receive a return per trade of $r(b, -b, c) = 2b -$

c and require $b \geq \frac{c}{2}$ for any profitable strategy. The optimal value for b is achieved as a result of the optimization routine outlined before.

From the symmetric construction and the relationship $\tau_{b,-b} = \tau_{-b,b}$, we are able to use both thresholds as entry and exit signals at the same time. From Equation (5), we obtain

$$\max_{b,x} \frac{r(b,x,c)}{E[\tau_{b,x}]} = \max_{b} \frac{2b-c}{E[\tau_{b,-b}]}, \tag{9}$$

with $X_0 = x = -b$. We define the following trading rules:

- Go short in stock A and go long in stock B, if $X_t \geq \mu_t + b$, i.e., stock A is overvalued and stock B undervalued.
- Go long in stock A and go short in stock B, if $X_t \leq \mu_t - b$, i.e., stock A is undervalued and stock B overvalued.
- Do not execute any trade, if $\mu_t - b < X_t < \mu_t + b$, i.e., the spread is in its 'normal' region.

We make a trade immediately upon every entry signal, buying 1 dollar worth of the undervalued stock and shorting 1 dollar worth of the overvalued stock. Allowing for only one active position per pair, we neglect any additional entry signals until the position is closed. Trades are held until the spread crosses the opposite trading band, the trading period ends, or one of the stocks of the respective pairs is delisted.

We follow Gatev, Goetzmann, and Geert Rouwenhorst (2006) for return computation. The sum of daily pay-offs across all pairs is related to the sum of invested capital at the beginning of the respective days. We show both the return on committed capital (scale pay-offs by the number of considered pairs) and the return on employed capital (scale pay-offs by the number of active pairs). A pair is called active if it possesses at least one round-trip trade during the corresponding trading period.

For our back-testing application, we decide on all S&P 500 constituents from January 1998 to December 2015. According to the Global Industry Classification Standard (GICS), all companies are categorized into the following 10 economic sectors (valuation date: 2015/12/31): Consumer Discretionary (CD), Consumer Staples (CS), Energy (EN), Financials (FI), Health Care (HC), Industrials (IN), Information Technology (IT), Materials (MA), Telecommunications Services (TS), and Utilities (UT). We generate all pairs consisting of stocks from the same sector. By this procedure, we are in a position both to analyse the performance of each sector and to examine the efficiency of our strategy in general.

VI. Results

In the following section, we run a full-fledged performance evaluation after transaction costs for the top 10 stocks of each sector from March 1998 to December 2015, compared to a naive buy–and–hold strategy of the S&P 500 index (MKT). Therefore, we examine the risk–return characteristics and trading statistics (and evaluate the strategy performance across the sectors . Furthermore, we investigate the influence of jumps to financial and statistical factors and demonstrate the value-add of our optimal trading strategy based on Lévy-driven Ornstein–Uhlenbeck (OU) processes to less complex approaches built on regular OU processes . In line with the vast majority of the literature, returns are calculated based on committed capital. We follow Avellaneda and Lee (2010) and Liu, Chang, and Geman (2017) and depict transaction costs of 5 bps per share per half-turn.

Risk–return characteristics and trading statistics

Table 3 reports daily risk–return characteristics after transaction costs for the top 10 pairs per sector from March 1998 until December 2015. Most of the presented performance metrics are examined by Bacon (2008). Across all branches of trade, we observe statistically significant returns with Newey–West t-statistics between 5.84 for CS and 16.00 for EN. The economic point of view confirms this finding – daily returns after transaction costs range from 0.12% for CS to 0.54% for FI, compared to 0.01% for the S&P 500 benchmark. In most instances, the return distribution of the sectors possesses right skewness – a desired property for any investor (Cont 2001). Kurtosis well above 3 indicates leptokurtic distribution for all variants.

Table 3. Daily return characteristics and risk metrics after transaction costs for the top 10 pairs of the 10 economic sectors, compared to an S&P 500 long-only benchmark (MKT) from March 1998 until December 2015. NW denotes Newey–West standard errors with five-lag correction and CVaR the Conditional Value at Risk.

	CD	CS	EN	FI	HC	IN	IT	MA	TS	UT	MKT
Mean return	0.0023	0.0012	0.0040	0.0054	0.0024	0.0028	0.0040	0.0034	0.0023	0.0032	0.0001
Standard error (NW)	0.0003	0.0002	0.0002	0.0004	0.0003	0.0002	0.0004	0.0003	0.0003	0.0002	0.0002
t-Statistic (NW)	7.6116	5.8389	15.9977	14.7785	9.1559	11.9127	11.0501	12.3458	7.2136	13.9580	0.8889
Minimum	−0.1603	−0.1354	−0.1406	−0.1340	−0.2814	−0.1267	−0.1303	−0.0954	−0.2042	−0.1498	−0.0947
Quartile 1	−0.0048	−0.0037	−0.0025	−0.0016	−0.0045	−0.0036	−0.0053	−0.0036	−0.0056	−0.0017	−0.0056
Median	0.0022	0.0009	0.0033	0.0035	0.0024	0.0025	0.0034	0.0027	0.0017	0.0018	0.0005
Quartile 3	0.0095	0.0056	0.0100	0.0101	0.0095	0.0090	0.0129	0.0098	0.0097	0.0074	0.0061
Maximum	0.1836	0.1580	0.1552	0.1742	0.1020	0.1577	0.1711	0.1390	0.1641	0.1083	0.1096
Standard deviation	0.0173	0.0110	0.0125	0.0166	0.0154	0.0135	0.0201	0.0135	0.0184	0.0120	0.0126
Skewness	0.2388	0.9116	0.2152	1.2468	−1.8829	0.4082	0.4566	0.5782	−0.0652	−0.7257	−0.1983
Kurtosis	14.9316	23.2571	13.3305	15.4023	34.0079	10.3881	7.5709	7.0291	13.1650	21.4208	7.5250
Historical VaR 1%	−0.0458	−0.0270	−0.0259	−0.0344	−0.0400	−0.0337	−0.0528	−0.0332	−0.0451	−0.0262	−0.0350
Historical CVaR 1%	−0.0709	−0.0386	−0.0410	−0.0587	−0.0652	−0.0473	−0.0712	−0.0432	−0.0683	−0.0494	−0.0506
Historical VaR 5%	−0.0219	−0.0145	−0.0136	−0.0120	−0.0186	−0.0170	−0.0253	−0.0164	−0.0234	−0.0113	−0.0197
Historical CVaR 5%	−0.0382	−0.0229	−0.0224	−0.0268	−0.0337	−0.0277	−0.0416	−0.0266	−0.0388	−0.0234	−0.0302
Maximum drawdown	0.4322	0.7575	0.6629	0.3267	0.4838	0.2315	0.6944	0.7142	0.5174	0.3369	0.6433
Share with return >0	0.5883	0.5624	0.6512	0.6802	0.5892	0.6115	0.6079	0.6200	0.5662	0.6363	0.5306

Following Mina and Xiao (2001), we depict historical Value at Risk (VaR) measures, typically used to gauge the amount of assets required to cover potential losses. Tail risks of all sectors are approximately at the same level as the market, e.g., the historical VaR 5% vary from −2.53 percent for IT to −1.13 percent for UT, compared to −1.97 percent for a naive buy–and–hold strategy. Most notable, the maximum drawdown level varies widely within the different branches of trade from 0.23 for IN to 0.76 for CS. Across all sectors, the hit rate, i.e., the percentage of days with positive returns, exceeds clearly the market (53.06 percent) with a top value of 68.02 percent for FI.

Table 4 contains statistics on trading frequency, which show a similar picture for all sectors. At least 9.98 of the 10 regarded pairs possess trading activity during the 5-day trading period. This high number is explained by the second part of our pair selection algorithm – high volatility of the spreads creates superior trading opportunities . The average time pairs are open ranges between 0.74 days for IT and 1.27 days for UT, 8 out of 10 sectors exhibit intraday trade durations. This short-term horizon of our strategy is based on the first part of our selection – high mean-reversion speed leads to fast reinvestment of capital and diminishes the risk of losing financial resources caused by a divergent pair.

In Table 5, annualized risk–return measures for all sectors are depicted. We observe annualized mean returns after transaction costs ranging from 31.90% for CS to 278.61% for FI – the naive buy–and–hold strategy is clearly outperformed (1.76%). This picture barely changes considering the Sharpe ratio, i.e., the excess return per unit of deviation, since all sectors attain approximately the standard deviation of the market. Most sectors generate downside deviations below the market – a favourable effect for investors because volatility is largely driven by upside deviations. The lower partial moment risk results in Sortino ratios above 2.94, compared to 0.12 for the general market. The results based on committed and employed capital show only marginal deviations – this fact is not astonishing since the top pairs trade in almost all cases (Table 4). Summarizing, we may carefully conjecture that our optimal pairs trading strategy based on Lévy -driven OU processes outlined in Section V is meaningful.

Table 4. Trading statistics for the top 10 pairs of the different sectors per 5-day trading period.

	CD	CS	EN	FI	HC	IN	IT	MA	TS	UT
Avg. traded pairs per 5-days period	10.0000	9.9996	10.0000	9.9989	10.0000	10.0000	10.0000	10.0000	9.9853	9.9770
Avg. round-trip trades per pair	7.6069	5.6809	7.0411	7.1301	7.3590	6.9598	8.6773	7.0300	7.4031	5.4587
St. dev. round-trip trades per pair	3.5827	2.9196	3.2275	4.0860	3.3319	3.2904	4.3661	3.3780	4.1741	3.3399
Avg. time pairs are open in days	0.8209	1.1200	0.8772	0.9459	0.8334	0.8951	0.7394	0.8973	0.9242	1.2670
St. dev. of time open, per pair, in days	0.4737	0.6892	0.5076	0.6417	0.4589	0.5137	0.4456	0.5424	0.6670	0.9070

Table 5. Annualized risk–return measures after transaction costs for the top 10 pairs and the different sectors, compared to an S&P 500 long-only benchmark (MKT) from March 1998 until December 2015.

	CD	CS	EN	FI	HC	IN	IT	MA	TS	UT	MKT
Mean return	0.7176	0.3190	1.6729	2.7861	0.7647	0.9931	1.6247	1.2712	0.7293	1.1717	0.0176
Mean excess return	0.6835	0.2928	1.6200	2.7112	0.7297	0.9536	1.5727	1.2263	0.6950	1.1287	−0.0027
Standard deviation	0.2741	0.1745	0.1990	0.2627	0.2448	0.2146	0.3190	0.2147	0.2916	0.1908	0.2005
Downside deviation	0.1766	0.1086	0.1073	0.1272	0.1671	0.1271	0.1883	0.1201	0.1846	0.1137	0.1441
Sharpe ratio	2.4939	1.6772	8.1403	10.3186	2.9807	4.4444	4.9297	5.7121	2.3835	5.9154	−0.0136
Sortino ratio	4.0629	2.9365	15.5841	21.9019	4.5778	7.8124	8.6290	10.5877	3.9507	10.3048	0.1218
Employed capital											
Mean return	0.7176	0.3190	1.6729	2.7861	0.7647	0.9931	1.6247	1.2712	0.7287	1.1718	0.0176
Sharpe ratio	2.4939	1.6772	8.1403	10.3186	2.9807	4.4444	4.9297	5.7121	2.3802	5.9149	−0.0136

Given the remarkable returns of our strategies, we compare them with 200 bootstraps of random trading to check on robustness. Similar to Gatev, Goetzmann, and Geert Rouwenhorst (2006), we combine the original entry and exit signals of the top pairs with two randomly chosen securities of the S&P 500 at that time. As expected, the average daily returns before transaction costs of the random trading are close to zero at −0.01% per day – a value well in line with Gatev, Goetzmann, and Geert Rouwenhorst (2006). In contrast, our strategies produce daily returns between 0.12% and 0.54% across all sectors even after transaction costs (Table 3) – results are far superior to the returns of random bootstrap trading. Thus, our strategy identifies temporal variations and exploits market inefficiencies.

Strategy performance across the sectors

Motivated by the varying performance across the branches of trade, we directly compare the results from the different sectors considering model parameters and the number of stocks. In this way, we aim to identify driving sources of positive returns across all sectors.

Our trading strategy relies on the mean-reversion of spread processes (Hameed and Mian 2015; Leung and Xin 2015a; Lubnaua and Todorova 2015), combined with a high degree of variation (Zeng and Lee 2014; Liu, Chang, and Geman 2017). Strong exposure to mean-reversion provides a high process' predictability and thus, spreads generate profits from trading (Leung and Xin 2015b; Stübinger and Endres 2018). Therefore, we rank the top pairs by mean-reversion speed θ in ascending order and record the ranking r_θ. Besides fast convergence to the equilibrium level, high fluctuation enables a high trading frequency. Volatile periods of a process are identified by high volatility σ, while sudden, large movements in the spread process are created by a high jump intensity λ. Thus, pairs are ranked by σ and λ, both in ascending order, and rankings r_σ and r_λ are recorded. All three rankings constitute $r_{m'} = r_\theta + r_\sigma + r_\lambda$, which is again ranked ascending to create the joint model parameter ranking r_m. We identify r_m as one potential source that drives returns in a positive way.

Besides the influence of model parameters on our strategies' performance, we analyse the effect of different sector sizes. The average sector size ranges approximately between 10 stocks for TS and 80 stocks for FI. Since with increasing number of stocks the number of potential pairs increases exponentially, greater sectors offer wider selection possibilities. This motivates us to create the second ranking r_s, chosen ascending with an increasing number of average included stocks.

Figure 3 presents the distribution of the branches of trade with respect to r_m and r_s in a three-dimensional bubble chart. The bubble sizes represent the magnitudes of annualized mean returns for the different sectors, compared to the general market (MKT) (see Table 5). First of all, the outstanding performance across all risk–return measures of FI is strongly driven by both a large sector size and a high parameter rank, which vindicates our trading strategy. EN and IT with high annualized returns of 167.29% and 162.47% also range in the right chart area with pleasant parameter ranking. As expected, the sector CS, which possesses the worst rank concerning model parameters, achieves the smallest annualized return of 31.90%. Surprisingly, the smallest sector TS exhibits a very high rank r_m. Concluding, we find that the dissimilarity between the sector returns is mostly driven by the corresponding mean-reversion speed, volatility, jump intensity, and sector size.

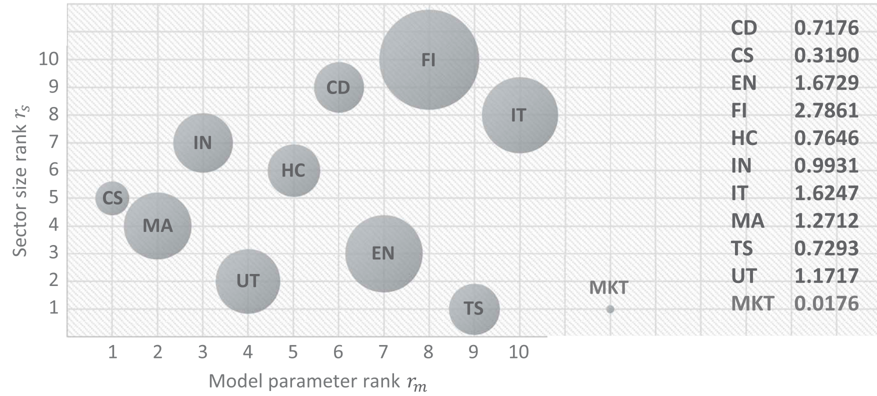

Figure 3. Performance results after transaction costs for the period from 1998 until 2015 and the different sectors in a three-dimensional bubble chart. The first two dimensions visualized as coordinates are model parameter rank r_m and sector size rank r_s. The magnitudes of annualized mean returns are represented by bubble size and depicted on the right side.

Jump analysis

Johannes, Kumar, and Polson (1999), Eraker (2004), and Kou, Yu, and Zhong (2017) report significant changes in jump rates in equity index returns over time. Motivated by the literature, we investigate the extent of jump activity in our data.

We specify a very flexible model to explain spread dynamics. The Lévy process is defined by

$$L_t = \sigma W_t + \sum_{i=1}^{N_t} \xi_i, \tag{10}$$

with standard Brownian motion $\{W_t\}_{t\geq 0}$, volatility $\sigma \in \mathbb{R}^+$, Poisson process $\{N_t\}_{t\geq 0}$ with rate λ, and jump sizes $\{\xi_1, \xi_2, \ldots\}$. As a special case for Poisson intensity $\lambda = 0$, the model covers the Brownian motion-driven OU process without any jumps. As such, our model is feasible for processes both with and without jumps. In line with this, the estimation procedure only detects jumps if variations cannot be explained by simple Gaussian shocks.

Figure 4 shows the number of detected jumps for the different sectors over time. In a 5-day trading period, there are 1955 data points, thus 1954 time intervals and potential jumps. First of all, we observe a substantial number of detected jumps across all sectors and over the whole sample period – including jumps pays off. We confirm the finding of Eraker (2004) and observe many jumps during the first years of our data set. Especially, IT exhibits a comparatively high jump intensity during this phase, mainly affected by the dot–com bubble. The jump rates increase strongly in times of the global financial crisis from 2007 to 2009 (see Kou, Yu, and Zhong 2017). As expected, FI contributes significantly to the high jump activity, with clearly lower jump frequencies before and after the crisis. In present days, the intensity of jumps decreases across all sectors and a major part of these jumps are caused by EN – this

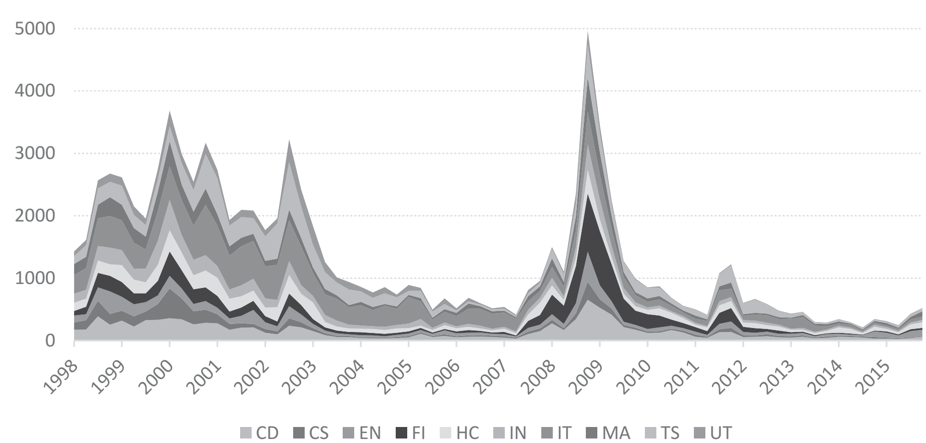

Figure 4. Average number of jumps during 5-day trading periods for the different sectors over time.

seems reasonable taking into account the sharp drop of oil prices beginning in the mid of 2014 and lasting until the end of 2015. We may conclude that in recent years, when jump rates are at much lower levels, trading frequencies decrease and taking advantage of temporary mispricing becomes harder. Summarizing and in line with Kou, Yu, and Zhong (2017), we observe a declining trend of jump numbers across all sectors with strong outliers in times of high market turmoils.

Assessment of the developed strategy

To demonstrate the additional benefit of our optimal trading strategy for Lévy-driven OU models (OLM), we compare it with well-established quantitative strategies in the field of pairs trading based on stochastic differential equations. Following Bertram (2010a) and Cummins and Bucca (2012), we implement an optimal trading strategy for Brownian motion-driven OU models (OBM). Specifically, an analytic solution maximizing the expected return is derived. The second benchmark is given by Zeng and Lee (2014) and represents a classic asymmetric strategy based on Brownian motion-driven OU models (CBM). The strategy takes positions at two-standard deviations and clears positions when the spread reverts back to the mean (Bollinger 1992, 2001; Avellaneda and Lee 2010; Clegg and Krauss 2018; Stübinger, Mangold, and Krauss 2018). In stark contrast to the optimal trading rules of OLM and OBM, the entry and exit signals in CBM are model-free. Identical to OLM, we select pairs based on highest mean-reversion speed and highest variance in both benchmarks . Following Kanamura, Rachev, and Fabozzi (2010), Cummins and Bucca (2012), and Liu, Chang, and Geman (2017), our comparison study focuses on the energy sector and depicts the performance and risk profile for OLM, OBM, and CBM in light of strategy performance over time and systematic sources of risk .

Performance over time

Figure 5 displays the development of an investment of 1 USD after transaction costs for OLM, OBM, and CBM over four sub-periods, compared to the cumulative returns of the S&P 500 (MKT) and the crude oil price of West Texas Intermediate (WTI), the most often referenced grade in oil pricing.

The first sub-period ranges from 1998 to 2003 and describes the growth and collapse of the dot–com bubble, the September 11 attacks and the start of the Iraq war. Simultaneously, there is neither an invention of methods or algorithms used in this paper nor the awareness level of high-frequency trading. As such, it is not surprising that the pairs trading strategies show strong and consistent outperformance. Optimal thresholds produce superior results compared to the model-free 2-σ rule, leading to annualized returns of 584.55% and 481.01% for OLM and OBM, and 156.81% for CBM.

The second sub-period ranges from 2004 to 2006 and characterizes the time of moderation and rising costs of oil in consequence of several events, e.g., the destruction caused by hurricane Katrina and the reducing strength of the U.S. dollar. We observe increasing developments for all time series with OLM showing particularly high performance. Specifically, our strategy outlined in Section V generates 143.16% p.a.

The third sub-period ranges from 2007 to 2012 and corresponds with the global financial crisis and its aftermath. The S&P 500 index, as well as the crude oil price, are strongly affected by this crash – the S&P 500 loses approximately 50% of its value and the oil price decreases fourfold in this time. In stark contrast, OLM, OBM, and CBM generate strongly positive returns, mostly driven by the long–short portfolios we are constructing.

The fourth sub-period ranges from 2013 to 2015 and specifies the period of comebacks and 2010s oil glut. The pairs trading benchmarks OBM and CBM show declining trends in comparison to the general market caused by increasing public availability of these methods. Most notable, OLM exhibits a clear growth in value up to 2.5 during the years 2013 to 2015, after transaction costs.

Common risk factors

Finally, Table 6 investigates the exposure of OLM, OBM, and CBM to common systematic sources of risk. We follow Krauss and Stübinger (2017) and apply (i) the Fama–French 3-factor model (FF3) introduced by Fama and French (1996), (ii) the Fama–French 3+2-factor model

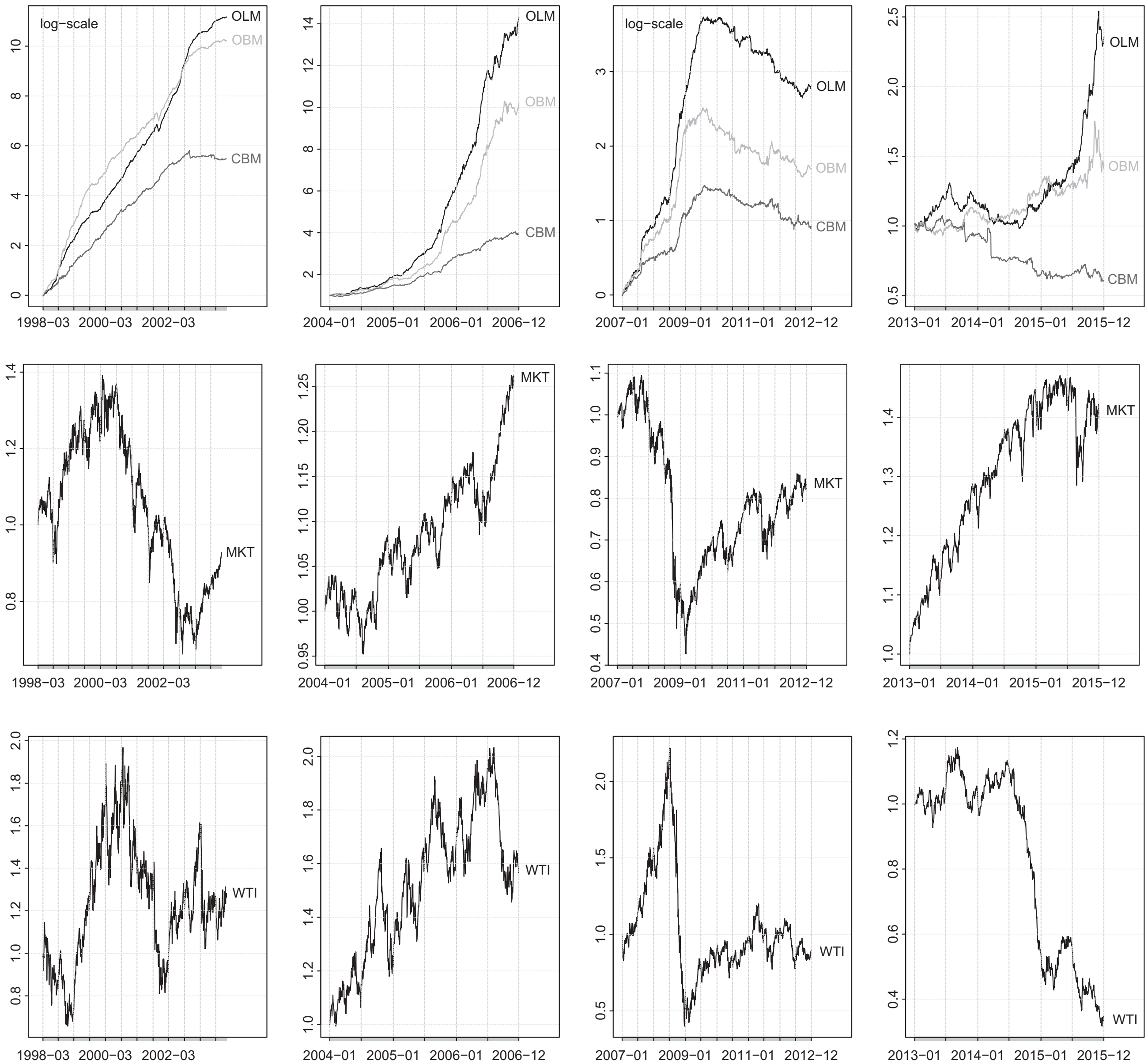

Figure 5. Development of an investment of 1 USD after transaction costs for the top 10 pairs of OLM, OBM, and CBM in the first row, compared to the S&P 500 index (MKT) in the second row and the crude oil price (WTI) in the third row. The time period from 1998 until 2015 is split into four sub-periods (1998–2003, 2004–2006, 2007–2012, 2013–2015). In the top, left corner of each plot log-scales is indicated where applied.

(FF3+2) outlined in Gatev, Goetzmann, and Geert Rouwenhorst (2006), and (iii) the Fama–French 5-factor model (FF5) discussed by Fama and French (2015). FF3 measures systematic risk exposure to the general market, small minus big capitalization stocks (SMB), and high minus low book–to–market stocks (HML). FF3+2 augments the first model by a momentum factor and a short-term reversal factor. FF5 appends two additional factors to the FF3, namely portfolios of stocks with robust minus weak profitability (RMW) and conservative minus aggressive investment behaviour (CMA). All data related to these models are downloaded from Kenneth R. French's website.[1]

Irrespective of the model employed, we observe that returns depict statistically and economically significant daily alphas of 0.39% for OLM, 0.33%

[1]We thank Kenneth R. French for all relevant data for these models on his website.

Table 6. Exposure to systematic sources of risk for the daily returns of the top 10 pairs of OLM, OBM, and CBM after transaction costs from March 1998 until December 2015. Standard errors are depicted in parentheses.

	OLM			OBM			CBM		
	FF3	FF3+2	FF5	FF3	FF3+2	FF5	FF3	FF3+2	FF5
(Intercept)	0.0039***	0.0039***	0.0039***	0.0033***	0.0032***	0.0033***	0.0016***	0.0016***	0.0016***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Market	0.0026	− 0.0242	− 0.0027	− 0.0125	− 0.0457**	− 0.0188	0.0071	− 0.0120	0.0096
	(0.0148)	(0.0164)	(0.0171)	(0.0158)	(0.0175)	(0.0183)	(0.0130)	(0.0144)	(0.0151)
SMB	− 0.0353	− 0.0282		− 0.0674*	− 0.0619		− 0.0711**	− 0.0731**	
	(0.0298)	(0.0299)		(0.0319)	(0.0320)		(0.0262)	(0.0262)	
HML	− 0.0058	− 0.0148		− 0.0453	− 0.0413		− 0.0358	− 0.0101	
	(0.0280)	(0.0300)		(0.0299)	(0.0321)		(0.0246)	(0.0263)	
Momentum		− 0.0385			− 0.0229			0.0252	
		(0.0209)			(0.0223)			(0.0183)	
Reversal		0.0668**			0.1032***			0.0913***	
		(0.0210)			(0.0225)			(0.0184)	
SMB5			− 0.0400			− 0.0733*			− 0.0521
			(0.0322)			(0.0344)			(0.0283)
HML5			0.0094			− 0.0149			− 0.0134
			(0.0318)			(0.0340)			(0.0279)
RMW5			− 0.0172			− 0.0020			0.0439
			(0.0415)			(0.0444)			(0.0365)
CMA5			− 0.0185			− 0.0506			− 0.0415
			(0.0509)			(0.0544)			(0.0447)
R^2	0.0003	0.0035	0.0004	0.0015	0.0066	0.0019	0.0019	0.0076	0.002
Adj. R^2	−0.0004	0.0024	−0.0007	0.0008	0.0055	0.0008	0.0012	0.0064	0.0009
Num. obs.	4484	4484	4484	4484	4484	4484	4484	4484	4484
RMSE	0.0125	0.0125	0.0125	0.0134	0.0134	0.0134	0.011	0.011	0.011

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$.

for OBM, and 0.16% for CBM. Across all strategies, loading on the reversal factor is well expressed and highly significant, confirming the effect of mean-reversion that our strategy relies on. Additionally, we observe small and insignificant loadings on the remaining factors for OLM. In stark contrast, OBM and CBM capture systematic risk by possessing significant and negative loadings on the market and SMB. Overall, OLM generates statistically significant and economically considerable daily intercept alphas of 0.39%, does not load on any common sources of systematic risk, and outperforms the benchmark approaches – complexity pays off.

VII. Conclusion

In this paper, we introduce an optimal pairs trading framework based on Lévy-driven Ornstein–Uhlenbeck (OU) processes and apply it to high-frequency data of the S&P 500 constituents from January 1998 to December 2015. In respect thereof, we make four main contributions to the existing literature.

The first contribution bears on the developed optimal trading strategy for Lévy-driven OU processes. Concerning this matter, we present the first manuscript deriving an explicit representation of the expected return per unit time which serves as a target function in our jump-based strategy. This takes place without the necessity of Monte Carlo approaches or numerical methods. To be more specific, the expected first-passage time is formulated by means of a mathematical expression, allowing for direct optimization of the objective function. In this way, we obtain the desired trade thresholds. By generalizing the classic Gaussian-driven OU model, we are in a position to capture additionally jumps and fat tails – both depict typical characteristics of financial data.

The second contribution refers to our high-frequency back-testing study of the S&P 500 constituents from 1998 to 2015. We observe statistically and economically significant returns after transaction costs for the top 10 pairs across all sectors demonstrating the efficiency of our strategy. Specifically, annualized returns range between 31.90% for the sector 'Consumer Staples' and 278.61% for the sector 'Financials', compared to 1.76% for a naive buy–and–hold strategy of the S&P 500. We find that the profits of all branches of trade are not being arbitraged away in the recent past.

The third contribution focuses on the varying performance results between the branches of trade. Our results reveal that there are common effects influencing returns across all economic sectors. We find that the dissimilarity is mostly driven by the respective mean-reversion speed, volatility, jump intensity, and sector size.

The fourth contribution relies on the value-add of our optimal trading strategy for Lévy-driven OU models in comparison to well-established Brownian motion-driven approaches in this area of research. We find that the jump-based pairs trading framework outperforms classic strategies in light of performance over time and the exposure to systematic sources of risk.

For future research in this field, time-changed Lévy processes may be explored to capture non-normal return innovations, stochastic volatility, and leverage effects. However, with increasing model complexity, the first-passage time problem becomes more challenging. Second, simultaneous trading of more than two stocks which co-move in some pattern could be executed. Therefore, a multivariate model that accounts for common interactions has to be estimated. Third, interdependencies and dissimilarities among the sector returns should be elaborated and explained by additional influencing factors.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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