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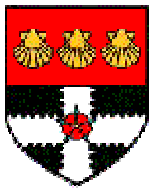

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The Cointegration Alpha: Enhanced Index Tracking and Long-Short Equity Market Neutral Strategies

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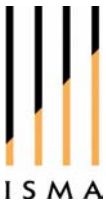

Abstract

This paper presents several applications of cointegration based trading strategies: a classic index tracking strategy, a long-short equity market neutral strategy and a number of strategies combining index tracking and long-short market neutral. As opposed to other traditional index tracking or long-short equity strategies, the portfolio optimisation is based on cointegration rather than correlation. The first strategy aims to replicate a benchmark accurately in terms of returns and volatility, while the other seeks to minimise volatility and generate steady returns under all market circumstances. The combinations of index tracking and long-short market neutral are designed as to enhance the properties of the basic strategies.

To validate the applicability of the cointegration technique to asset allocation, pioneered by Lucas (1997) and Alexander (1999), and explain how and why it works, we have employed a panel data on DJIA and its constituent stocks. When applied to constructing trading strategies in the DJIA, the cointegration technique produced encouraging results. For example, between January 1995 and December 2001 the most successful self-financing statistical arbitrage strategies returned (net of transaction and repo costs) approximately 10% with roughly 2% annual volatility and negligible correlation with the market.

The comprehensive set of back-test results reported is meant to offer a detailed picture of the cointegration mechanism, and to emphasise its practical implementation issues. Its key characteristics, i.e. mean reverting tracking error, enhanced weights stability and better use of the information contained in stock prices, allow a flexible design of various funded and self-financing trading strategies, from index and enhanced index tracking, to long-short market neutral and alpha transfer techniques. Further enhancement of the strategy should target first, the identification of successful stock selection rules to supplement the simple cointegration results and second, the investigation of the potential benefits of applying optimal rebalancing rules.

Key words: cointegration, enhanced index tracking, long-short equity, market neutral, hedge fund, alpha strategy
JEL classification: C32, C51, G11, G23

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1. Introduction

The search for appropriate quantitative techniques to construct long-short equity strategies is not a last moment development in the financial markets. Newcomers in this game are constantly joining the traditional players and, currently, the most fervent searchers in quantitative strategies are the hedge funds involved in equity trading. Their operational flexibility and lack of constraints are ideally suited to allow them to benefit from the application of these types of trading strategies. Irrespective of the actual players, the characteristics of a successful long-short equity strategy are usually recognised to be steady pattern in returns, low volatility and market neutrality.

Alpha, market neutrality and traditional long-short equity strategies

When addressing the returns issue, 'alpha' is the most frequently used term. Derived from statistics, alpha is used in finance in connection with an assumed linear relationship between the returns to a particular asset or portfolio and the returns to some factors or a benchmark (Schneeweis, 1999). In the hedge fund industry, alpha is a proxy for excess return to active management, adjusted for risk (Jensen, 1969). Its two main sources are usually credited to be a successful stock selection and market timing.

Traditional sources of alpha through stock selection, 'long-short' equity strategies are often seen as being 'market-neutral'[1] by construction. However, unless they are specifically designed to have zero-beta, long-short strategies are not necessarily market neutral. In a recent paper, Brooks and Kat (2001) find evidence of significant correlation of classic long-short equity hedge funds indexes with equity market indexes such as S&P500, DJIA, Russell 2000 and NASDAQ, correlation which may still be underestimated due to the auto-correlation of returns. Moreover, some hedge fund indexes' returns possess statistical properties which, by being far from normality, limit the straightforward applicability of traditional performance measures (e.g. Sharpe ratio) or portfolio allocation techniques (i.e. mean variance analysis) based on normality assumptions.

Generally, long-short strategies are designed to exploit market inefficiencies, generating alpha through both stock selection and market timing. Many such self-financing strategies consist in buying undervalued and/or selling overvalued assets. However, usually, no stable relationship between the two groups of stocks is hypothesised when setting up a long-short strategy. The undervalued stocks are expected to grow more or decrease less than the overvalued stocks, and consequently, the price differential between them is expected to get lower. However, this does not imply by any means market neutrality, as there is no proven relation between the two separate equity groups to ensure that this will be eventually the case.

As opposed to simple long-short strategies, market neutral strategies involve only equities or securities with proved interdependencies. Such interdependencies, sometimes taking the form of convergence, ensure that, over a given time horizon, the equities will reach an assumed pricing relation. Examples of market neutral strategies are convertible securities arbitrage, futures/index arbitrage, fixed income, currency and options arbitrage, merger arbitrage or corporate structure arbitrage[2].

According to Barra RogersCassey Research (2000), the advantages of market neutral long-short equities investing are perceived to be independence of the market direction, more efficient use of

[1] Generally, a strategy is said to be market neutral if it generates returns which are independent of the relevant market returns. Market neutral funds actively seek to avoid major risk factors, but take bets on relative price movements (Fund and Hsieh, 1999)

[2] Usually exploit mismatches in the pricing of 'equivalent' instruments, such as: convertibles and their underlying securities, index futures and baskets replicating the index, fixed income instruments generating similar cash-flows, interest rates differentials between currencies, options and other derivatives on the same underlying. Merger arbitrage trades on the convergence of the stock prices of two companies involved in a merger.

information as compared to long only strategies, double alpha and also its potential portability through derivatives.

The independence of market direction, or put in other words, low correlation between strategy and market returns, is the effect of netting beta between the long and the short parts of the portfolio. In some market neutral strategies, the portfolio is optimised under the explicit constraint that beta is zero. In others, beta is only controlled for not exceeding some limits.

Apart from the above, market neutral strategies remain exposed to other sources of risk, such as cross hedging errors (in case a given position is hedged with an imperfect replica) or mismatches between the investor's time horizon and the timing of positions convergence.

The fact that long-short equity strategies ensure a more efficient use of information than long only strategies is the result of not restricting the weights of the undervalued assets to zero. By allowing portfolio returns to be borne by both the short set under-performing the market and the long set over-performing the market, the strategy generates 'double alpha'.

Another important feature of long-short market neutral strategies is that, once generated, alpha may be easily transported to other markets through the use of derivatives. Jacobs and Levy (1999) have demonstrated the portability of alpha between asset classes through derivatives. As the long-short strategy is self-financing, its alpha can be transported to virtually any index through the use of a futures contract, for example. The concept is based on separating first beta from alpha and then re-associating them in the portfolio construction (Ineichen, 2000).

Still, there are several issues that have limited the more extensive use of long-short investing. Among them we note that double transaction costs will usually correspond to double alpha opportunities, and that low volatility and low correlation with market returns in normal circumstances may disappear in extreme market events. Some studies (e.g. Barra RogersCassey Research) have also mentioned market narrowness (lack of liquidity) as an impediment to a wider use of long-short market neutral equity strategies.

Among market neutral strategies based on convergence assumptions, pairs trading, index and enhanced index tracking are of particular interest to us, as belonging to the same category of strategies as our cointegration-based strategies. Traditionally, all three of them are based on correlation assumptions. However, correlation assumptions have a number of shortcomings, amongst which instability is the most hazardous.

Cointegration and correlation in long-short strategies

In the last decade, the concept of cointegration has been widely applied in financial econometrics in connection with time series analysis and macroeconomics. It has evolved as an extremely powerful statistical technique because it allows the application of simple estimation methods (such as least square regression and maximum likelihood) to non-stationary variables. Still, its relevance to investment analysis has been rather limited so far, mainly due to the fact that the standard in portfolio management and risk measurement is the correlation analysis of asset returns.

However, correlation analysis is valid only for stationary variables. This requires prior de-trending of prices and other level financial variables, which are usually found to be integrated of order one or higher. Taking the first difference in log prices is the standard procedure for ensuring stationarity and leads all further inference to be based on returns. However, this procedure has the disadvantage of loosing valuable information. In particular, de-trending the variables before the analysis removes any possibility to detect common trends in prices. Moreover, when the variables in a system are integrated of different orders, and therefore require different numbers of differentiations to become

stationary, the interpretation of the results becomes difficult. By contrast, the aim of the cointegration analysis is to detect any stochastic trend in the price data and use these common trends for a dynamic analysis of correlation in returns (Alexander, 2001).

The fundamental remark justifying the application of the cointegration concept to, for example, stock prices analysis, is that a system of non-stationary stock prices in level can share common stochastic trends (Stock and Watson, 1991). According to Beveridge and Nelson (1981), a variable has a stochastic trend if its difference has a stationary invertible ARMA(p,q) representation plus a deterministic component. Since ARIMA(p,1,q) models seem to characterise many financial variables, it follows that the growth in these variables can be described by stochastic trends.

The main advantage of cointegration analysis, as compared to the classical but rather limited concept of correlation, is that it enables the use of the entire information set comprised in level financial variables. Moreover, a cointegration relationship is able to explain the long run behaviour of cointegrated series, while correlation, as a measure of co-dependency, usually lacks stability, being only a short run measure. While the amount of history that may be used to support the cointegration relationship may be large, the attempt to use the same sample to estimate correlation may face many obstacles such as outliers in the data sample and volatility clustering. The enhanced stability of a cointegration relationship generates a number of significant advantages for a trading strategy as, for instance, reducing the amount of rebalancing trades in a hedging strategy and, consequently, the associated transaction costs.

Separately, the use of cointegration analysis for long-run inferences does not impede in any way the use of correlation as a short-term guide. For example, short-run correlation may be used as a stock selection technique, which is followed by a portfolio optimisation based on cointegration.

When applied to stock prices and stock market indexes, which are usually found to be integrated of order one, cointegration exists when there exists at least one stationary linear combination of them. Such stationary linear combination of stock prices/market indexes can be interpreted as mean reversion in price spreads. The finding that the spread in a system of prices is mean reverting does not provide any information for forecasting the individual prices in the system, or the position of the system at some point in the future, but it provides the valuable information that, irrespective to its position, the prices in the system will stay together on a long-run basis.

The literature on cointegrated time series is huge and still rapidly expanding. New methods have been developed for testing the presence of cointegrating relationships (Engle and Granger (1987); Engle and Yoo (1987); Johansen (1988); Park (1992); Balke and Fomby (1997)) and much research concerns the distributional properties of the different estimation and inference procedures (Stock (1987); Phillips and Oularis (1990); Johansen (1991); MacKinnon (1991)).

Numerous empirical studies have examined the nature of cointegrating relationships in different systems of variables. In macroeconomics, cointegration techniques have been applied to modeling exchange rates (Baillie and Bollerslev (1989 and 1994); Diebold, Gardeazabal and Yilmaz (1994)), purchasing power parity and international capital mobility (Corbae and Ouliaris (1988); Enders (1988); Taylor (1988); Fisher and Park (1991)), money demand and monetary dynamics (Johansen and Juselius (1990); Hafer and Jansen (1991); Miller (1991)), treasury bill yields (Hall, Anderson and Granger, 1992), productivity, aggregate investments, savings, inflation, unemployment, government spending and international trade (Clarida, 1994).

In the area of equity markets, cointegration analysis has frequently targeted two objectives: to estimate the degree of co-movement in stocks within a given market index (Hersom, Sutti and Szego, 1973) and to identify the economic fundamentals generating this type of behaviour. Generally, co-movements in stock prices are seen as being the effect of common underlying economic factors, such

as macroeconomic conditions (both domestic fundamentals and international economic developments), investors' expectations and behaviour (Cerchi and Havenner (1988); Bossaerts (1988)).

Cointegration techniques have also been applied to examine price linkages and information transmission mechanisms (Harris, McInish, Shoesmith and Wood (1995)), the relationship between spot and forward prices (Brenner and Kronner (1995); Ackert and Racine (1998)), the degree of integration between stock exchanges (Taylor and Tonks (1989)), to test for the presence of asset prices bubbles (Hamilton and Whiteman (1985); Diba and Grossman (1988)) or for rational expectations present value models (e.g. in term structures and stock prices, Campbell and Shiller, 1987).

One application of cointegration analysis to asset management that is particularly relevant to our line of research was performed by Lucas (1997). His paper deals with the optimal asset allocation in the presence of possibly cointegrated time series, and produces encouraging results. Using a stylised asset allocation problem with a risk adverse investment manager, Lucas shows that cointegrating combinations of time series reveal less long-term variability and therefore, less long-term risk. From a short term or tactical asset allocation perspective, cointegration implies that the price series have an error-correcting behaviour, allowing the anticipation of future developments. According to Lucas' results, the presence of cointegration relations has important consequences for the short-term predictability of time series, the coherency displayed by the simulated series over time and the range of possible scenarios on time series (e.g. asset prices).

Outline of our long-short equity strategies

Considering the important comparative advantages of cointegration analysis in modelling integrated series, one straightforward application would be to exploit, if found, the cointegration relationship between stock prices and indexes and construct trading strategies.

This paper presents several applications of cointegration based trading strategies:

- a classic index tracking strategy;
- a long-short equity market neutral strategy; and
- a number of strategies combining index tracking and long-short market neutral.

The first strategy aims to replicate a benchmark in terms of returns and volatility, while the other seeks to minimise volatility and generate steady returns under all market circumstances. The combinations of index tracking and long-short market neutral are designed to enhance the properties of the basic strategies.

As opposed to other traditional index tracking or long-short equity strategies, portfolio optimisation is based on cointegration rather than correlation. This allows us to make use of the full information contained in stock prices and base our portfolio weights on the long-run behaviour of stocks.

The first target of our portfolio construction analysis is index tracking. Through the means of cointegration we will construct portfolios replicating the index. Such portfolios are expected to have *similar returns, similar volatility and high correlation with the index*.

Special attention will be devoted to the analysis of the tracking error[3], i.e. the difference between the tracking portfolio returns and market returns. Ideally, the tracking error will prove to be a white noise

[3] Please note that we use the term tracking error to denote the excess returns of the tracking portfolio over the market index and not the standard deviation of this excess, which is the case for other authors.

process, with zero mean and low variance. This would ensure that the tracking portfolios do not have consistent or large deviations from the benchmark. Another important property of the tracking error would be its low correlation with market returns. This is a necessary (but not sufficient) condition for the market neutrality of the long-short strategies.

The second step of our analysis is the long-short equity market neutral strategy. This is also based on the tracking ability of cointegrated portfolios, but now cointegrated portfolio prices are proven to have a long-run equilibrium relationship with an enhanced index. That is, cointegration is used to replicate 'plus' and 'minus' benchmarks (i.e. enhanced index tracking), and then a self-financing strategy is constructed by being short on the portfolio tracking the 'minus' benchmark and long on the portfolio tracking the 'plus' benchmark.

The following observations indicate why the long-short strategy will generate *double alpha with low volatility and low correlation with the market*. First, provided that each tracking portfolio in the strategy is a suitable replica of its 'plus' or 'minus' benchmark, the long-short market neutral equity strategy should generate returns according to the spread between the 'plus' and the 'minus' benchmarks. Secondly, the volatility of the strategy returns depends on the volatilities of the 'plus' and 'minus' portfolios' returns and on the correlation between them. If the volatilities of the 'plus' and 'minus' portfolios equal the volatility of the market index, and they are highly correlated with each other, as they are individually correlated with the market index, then the volatility of the long-short strategy returns and its correlation with the index will be very low.

The final part of our analysis concerns the enhancement of the simple index tracking and long-short market neutral strategies by combining them to accommodate different types of investment profiles. Three main alternatives are investigated: a 'fund of funds' approach, which, by investing in a number of long-short strategies should reduce the returns volatility; a combination of the excess return from index tracking with the long-short strategies aiming to improve the performance of the statistical arbitrage during market declines; and a strategy transferring the alpha gained in a market neutral framework to the market index through the use of a tracking portfolio.

For testing the performance of the cointegration-based trading strategies we have used a panel of data on DJIA and its constituent stocks. Our main results indicate that:

- The cointegration-based tracking strategy generates *accurate replicas* of the market index, provided that a minimum number of stocks is included in the tracking portfolio and an appropriate calibration period is used;

- Special attention should be given to the *stock selection method*, especially to the amount of trades required to rebalance the portfolio, as the transaction costs may erode the returns of the tracking portfolios;

- The results of the long-short strategies are highly dependent on the stock selection method used and on the spread between the 'plus' and 'minus' benchmarks tracked. Selected strategies generate *returns according to the spread between the benchmarks tracked*, and display *no significant correlation* with the market returns. However, as the spread between the benchmarks tracked increases, the cointegration relationship begins to break down and, consequently, the results of the long-short market neutral strategy become more volatile. The most *consistent* positive returns, with low volatility and no significant correlation with the market are generated by strategies tracking *narrow spreads* between the 'plus' and the 'minus' benchmarks.

- In terms of returns, *similar performance* to hedge funds indexes can be obtained by adding *leverage* to our long-short strategies. The returns, even if more volatile than the index, have a significantly *lower correlation* with the market returns.

- The characteristics of the individual index tracking and long-short market neutral strategies can be significantly improved by combining them to create market neutral or enhanced index tracking strategies.

Further enhancement of the strategy should target first, the identification of successful stock selection rules to supplement the simple cointegration results and second, the investigation of the potential benefits of applying optimal rebalancing rules.

The remainder of this paper is organised as follows: section 2 gives a description of the data, section 3 presents the results of simple index tracking strategies, section 4 analyses different long-short strategies, section 5 explores a number of strategies combining index tracking and long-short market neutral, and section 6 concludes.

2. Data

In order to construct and back-test several cointegration-based strategies, we have used the daily prices of the stocks included in the Dow Jones Industrial Average index as of 31-Dec-01. These stocks, their ticker symbols and their weights in the index on 31-Dec-01 are given in Appendix 1. For the historical cointegration analysis of price equilibrium we required the DJIA daily historical series over the period 1-Jan-90 to 31-Dec-01. An artificial 'reconstructed' DJIA historical series was computed from daily close prices of the stocks currently included in DJIA basket, using the last available value of the DJIA divisor as of 31-Dec-01 (i.e. 0.14452124). The value of the reconstructed index for one particular day in our sample was computed as an equally weighted sum of all stock prices divided by the constant divisor value.

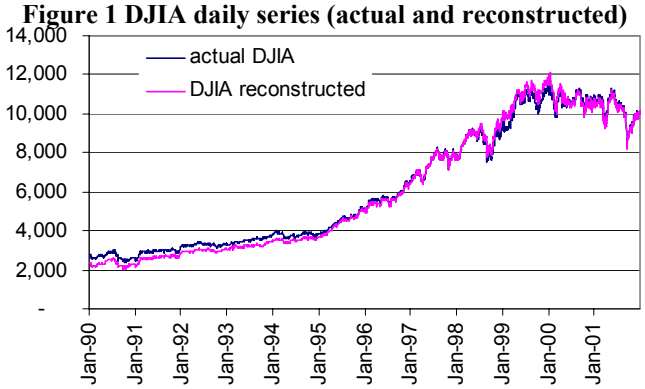
There are two differences between the actual index and the reconstructed one: the value of the divisor and the constituent stocks (both of which change periodically in the actual index but not in the reconstructed index).

$$\text{actual\_DJIA}_t = \text{divisor}_t \sum_{k=1}^{30} \text{stock\_price}_{k,t} \tag{1}$$

$$\text{reconstructed\_DJIA}_t = \text{divisor}_T \sum_{k=1}^{30} \text{stock\_price}_{k,t} \tag{2}$$

The use of a reconstructed index instead of the actual one is justified by our interest in the current structure of the index: that is, we compare the performance of portfolios comprising the stocks currently included in DJIA with a market index constructed from the same stocks. Additionally, the use of the reconstructed index ensures consistency in the treatment of dividends and stock splits.

Figure 1 DJIA daily series (actual and reconstructed)



The most significant changes in the constituents of the DJIA occurred in 1999, when 4 new stocks (out of which 3 were technology stocks) were introduced, replacing more traditional stocks. These changes are the main cause of the difference between the actual and the reconstructed DJIA returns series: Figure 1 shows that the reconstructed index, which has always included the technology stocks, under-performed the actual index at the beginning of the '90s and started to over-perform the actual index at the end of 1998. The difference disappeared at the end of 1999, with the inclusion of the technology stocks in the actual DJIA.

The daily stock closing prices were downloaded directly from yahoo-financial.com. Missing observations were replaced by the last close price available for that particular stock. The statistical properties of the log price series are reported in Appendix 2. Based on ADF test[4] results, all series, including the market index, prove to be integrated of order one.

3. Index tracking

The rationale for constructing the tracking portfolio based on a cointegration relationship with the index, instead of simple correlation, rests on the following features of cointegrated systems:

- Tracking error is, by construction, mean reverting;
- Stability of stock weights in the portfolio, and consequently reduced amount of rebalancing trades; and
- Better use of information, in particular the information contained in stock prices.

An index tracking process entails two, equally important stages: first, selecting the stocks to be included in the tracking portfolio and then, determining the portfolio holdings in each stock based on a cointegration optimisation technique.

The first stage, stock selection, can be the result of proprietary selection models, technical analysis or just stock picking skills of a portfolio manager. The degree of cointegration and, consequently, the tracking performance will depend very much on the selection process. However critical, the selection process does not have special features in a cointegration based tracking technique and identifying the most successful selection technique does not constitute the focus of this paper. But it is important to emphasise that the tracking results will depend highly on the stock selection and this constitutes in practice a control variable in identifying the most appropriate tracking portfolio.

The second stage of index tracking concerns determining the portfolio holdings in each of the stocks selected in the previous stage. The stocks weights in each portfolio are estimated based on the ordinary least square (OLS) coefficients of the cointegration equation that regresses the index log price on the portfolio stocks log prices over a given calibration period prior to the portfolio's construction moment.

$$\log(\text{index}_t) = c_1 + \sum_{k=1}^{n} c_{k+1} * \log(P_{k,t}) + \varepsilon_t \qquad (3)$$

The log transformation is applied to produce more homogenous series, provided that, if the level variables are cointegrated, so will be their logarithms. As shown in Hendry and Juselius (2000), 'if a variable had a unit root in its original units of measurement, it would become essentially deterministic over time if it had a constant error variance. Thus, absolute levels have heteroscedastic errors to make sense; but if so, there is not a sensible place to start modelling.'

[4] ADF tests are based on the null hypothesis of unit root: if the test is found to be statistically significant, the null hypothesis is rejected and we conclude that the series is stationary. We have tested for stationarity each series in levels and first differences.

We note that the application of OLS to non-stationary dependent variables such as log(index) is only valid in the special case of a cointegration relationship. The residuals in (3) are stationary if, and only if, the log(index) and the tracking portfolio $\sum_{k=1}^{n} c_{k+1} * \log(P_{k,t})$ are cointegrated. Unless the residuals from the above regression are found to be stationary, the OLS coefficients will be inconsistent and further inference based on them will be invalid. Therefore, testing for cointegration is an essential step in constructing cointegration-based tracking portfolios.

Further to estimation, the OLS coefficients are normalised to sum up to one, thus providing the composition of the tracking portfolio. For the sake of simplicity, in the remainder of the paper we will refer to the normalised regression coefficients as 'stock weights'.

The procedure described above will provide a unique portfolio solution, in the case of cointegration, for each given selection of stocks and each fixed calibration period. If different stock selection methods and/or calibration periods are used, there will be multiple portfolio solutions.

3.1. Back-test procedure

For the purpose of our analysis, we have used the simplest stock selection criterion available, i.e. the price ranking of the stocks in the index at the moment of the portfolio construction. We have set up portfolios comprising the first 10, 15, 20 and 25 stocks, ordered descendingly according to their weights in the index. The composition of a given portfolio is not constant through time, as the stock ranking is based on prices that are changing. Additionally, we have constructed tracking portfolios using *all* 30 stocks in the index.

As calibration periods we have used 1 to 5 years of data prior to the moment of portfolio construction. The first cointegration based tracking portfolios were constructed on 1-Jan-95 and the last were constructed on 12-Dec-01.

All portfolios were rebalanced every 10 trading days, based on the new ranking and the new OLS coefficients of the cointegration regression estimated over the rolling calibration period.

In order to assess the performance of each strategy, we have considered several criteria:

a. Engle-Granger cointegration test

The residuals of each cointegration regression were tested for stationarity following the Engle-Granger methodology for cointegration testing. This method was particularly appealing to us for its intuitive and straightforward implementation. Moreover, its well-known limitations (small sample problems, asymmetry in treating the variables, at most one cointegration vector) are not effective in our case: the estimation sample ranges from 250 to 1250 observations, there is a strong economic background to treat the market index as the dependent variable, and identifying only one cointegration vector is sufficient for our purposes.

The cointegrating ADF regression estimated on the residuals of the cointegration regressions is:

$$\Delta\hat{\varepsilon}_t = \gamma\hat{\varepsilon}_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta\hat{\varepsilon}_{t-i} + u_t \qquad (4)$$

The null hypothesis tested is of no cointegration, i.e. $\gamma = 0$, against the alternative of $\gamma < 0$. The critical values for the t-statistic of γ have been obtained using the response surfaces provided by MacKinnon (1991).

b. Returns on the tracking strategy

Further to ensuring that the portfolios were constructed on a cointegration relationship, we have estimated the daily prices for each of the portfolios monitored. Between consecutive re-estimations of the cointegration equation, the number of shares in each portfolio was kept constant.

Ideally, tracking the index based on the cointegration coefficients would imply keeping constant the weights and not the number of stocks in the portfolio. This means daily (or even intra-day) rebalancing of the portfolio to account for the effect of price changes on the stock weights. This type of active strategy is likely to reduce the tracking errors but may generate huge transaction costs. As the practical relevance of this very active strategy is limited, we have based the tracking error on no trading within the 10-days period between consecutive re-estimations of the cointegration coefficients and have kept the number of stocks (instead of the weights) constant.

Assuming that the portfolio weights $w_{k,T}$ are estimated at time T, the price of the portfolio at time $T+x$, $x <= 10$, can be computed based on the prices $P_{k,T}$ and $P_{k,T+x}$ of the n stocks in the portfolio as follows:

$$\pi_{T+x} = \pi_{T-1} \sum_{k=1}^{n} \frac{w_{k,T}}{P_{k,T}} P_{k,T+x} \tag{5}$$

The portfolio returns were further estimated as the first difference in log prices of the portfolio.

c. Transaction costs

Additionally, to account for the impact of the price spread and the brokerage fees on the portfolio returns, we have assumed a fixed amount of 20 basis points transaction costs on each trade value. However arbitrary, choosing an amount of 20 basis points as transaction costs is in line with previous studies on the transaction costs incurred on NYSE (NYSE research report (2001); Chalmers, Edelen and Kadlec (1999)). The stocks in DJIA are known to be very liquid, and their trading generates low transaction costs. Moreover, the impact on our results of choosing any fixed level of transaction costs is rather limited, as we are interested in their comparative effect on different strategies rather than in stating the overall profitability of the strategies.

In the framework of our strategy, the transaction costs were incurred on each portfolio re-balancing, i.e. every 10 trading days. However, in order to avoid creating artificial jumps in the returns series, the transaction costs were equally distributed to all the daily returns during the non-trading period. If the portfolio weights $w_{k,T}$ are estimated at time T, the transaction costs at time T can be computed as follows:

$$TC_T = 0.002 \sum_{k=1}^{n} abs(w_{k,T} - w_{k,T-10}) P_{k,T} \tag{6}$$

d. Volatility of strategy's returns

For each of the tracking portfolios constructed, we have computed the annual volatility of the excess returns, using the 250 days per annum convention. As estimation methods we have used an equally weighted approach to compute the unconditional volatility over the entire data sample and an exponentially weighting method to analyse the short-term volatility behaviour.

e. Correlation of the tracking portfolio returns with the index returns

For each tracking portfolio, we have computed and reported:

- The correlation of its returns with the market returns; and
- The correlation of the excess returns (i.e. tracking error) with the market returns.

As estimation methods we have used again an equally weighted approach to compute the unconditional correlations over the entire data sample and an exponentially weighting method to analyse the short-term correlation.

f. Skewness and excess kurtosis of tracking portfolio excess returns

To complete the characterisation of the tracking error distribution, we have computed and reported for each of the tracking portfolios constructed the skewness and excess kurtosis[5].

g. Sharpe ratios and information ratios

As a summary statistic useful in the classical framework of mean-variance analysis, we have computed the Sharpe ratios[6] for each tracking portfolio and compared them with the Sharpe ratio of the benchmark. To this end we have used the average interest rate of the 3-months T-bills over our sample period, which was of 5.26% p.a. In addition to the Sharpe ratios, we have reported the information ratios[7], as a purely statistical measure which does not assume a particular investment behaviour (computing the Sharpe ratios assumes that the risk free rate for any investment is the US T-bill rate, which may not necessarily be true, for example, in case of a Japanese investor).

3.2. Back-test results

a. Engle-Granger cointegration test

In order to ensure that the tracking portfolios were validly constructed, we have tested the residuals of each OLS regression estimated for stationarity, using the Engle-Granger methodology for testing cointegration relationships.

Based on the Engle-Granger tests results (Figure 2 and Appendix 3), a number of portfolios proved not to be sufficiently cointegrated with the market index. For instance, the null hypothesis of no cointegration could not be rejected in approximately 80% of cases for tracking portfolios containing only 10 stocks, even for a calibration period of 5-years. For tracking portfolios comprising 15-stocks, the null hypothesis of no cointegration could not be rejected in more than 30% of cases, even for large calibration periods. Large proportions of non-significance cases were also obtained for small calibration periods.

[5]The skewness and excess kurtosis were computed as:

$$sk = \frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{TE_i - \overline{TE}}{\sigma_{TE}}\right)^3$$

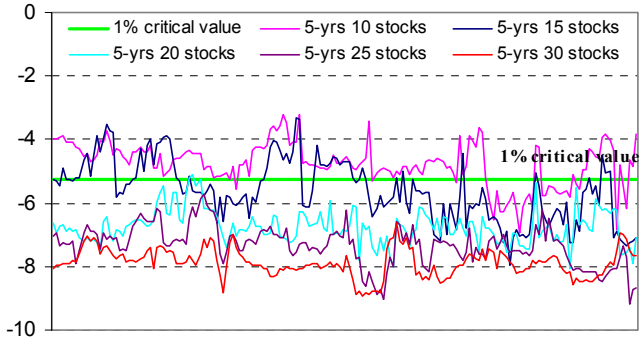
$$excesskurt = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^{n} \left(\frac{TE_i - \overline{TE}}{\sigma_{TE}}\right)^4 - 3\frac{(n-1)^2}{(n-2)(n-3)}$$

[6] The Sharpe ratio was computed as the average annual excess return of an investment strategy over the risk free rate divided by the annualised standard deviation of returns.

[7] The information ratio is simply the average annual return of an investment strategy divided by its annualised standard deviation.

As expected, the degree of cointegration increases with the number of stocks in the tracking portfolio and with the calibration period. This result is rather intuitive, as one would expect the degree of cointegration between the market index and part of its stocks to increase with the number of stocks in the tracking portfolio. Also, since the cointegration aims to identify long-run equilibrium relationships, it requires for a good specification a rather long calibration period.

Figure 2 ADF test statistics for tracking portfolios with a calibration period of 5 years



Based on these results, it can be concluded that a number of only 10 or 15 stocks is too small to allow the construction of a portfolio cointegrated with the index. Also, using only 1 or 2-years as calibration periods does not provide sufficient ground for strong cointegration. Therefore, in the following results we shall use only 20, 25 and 30-stocks portfolios for tracking the index, with calibration periods from 3 to 5 years.

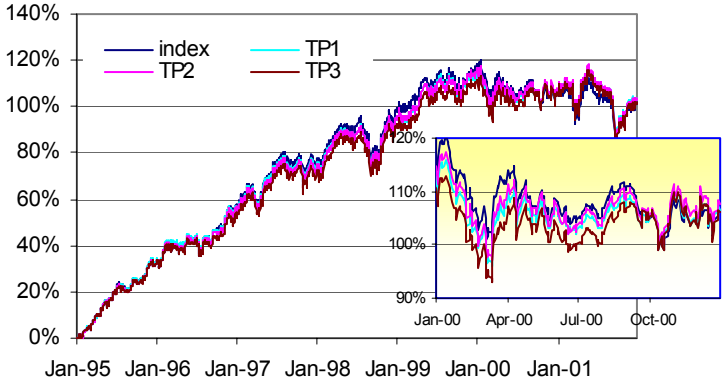
The average ADF statistics over the entire data period are reported for each tracking portfolio in Appendix 3.

b. Returns of the tracking portfolios

The summary results of the tracking portfolio returns, as well as other statistics, are reported in Appendix 3, before and after considering the transaction costs. They are quoted as excess returns of the tracking portfolio over the market index, which we define as the tracking error.

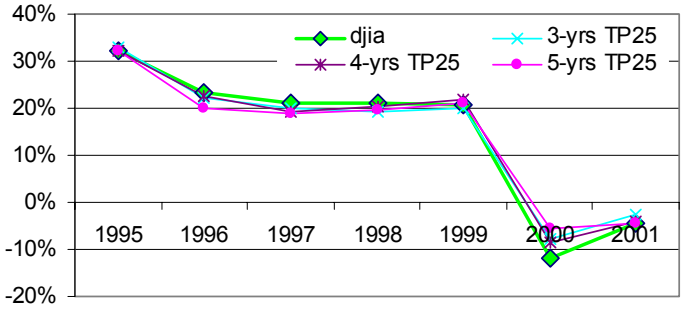
The first observation is that all tracking portfolios produce *results fairly close to the market index* before accounting for transaction costs. The 20-stocks tracking portfolios tend to under-perform the index, with an annual average of 2%. The tracking portfolios comprising 25 stocks produce the closest to the index average return, while the 30-stocks tracking portfolios over-perform the index in average by one percent annually. Please refer to Figure 3 for a plot of the cumulative returns on the 25 stocks tracking portfolio with different calibration periods as compared to the cumulative returns on DJIA. Also, the cumulative returns on the 30-stocks portfolios are reported in Figure 8.

Figure 3 Cumulative returns on DJIA and tracking portfolios based on 25 stocks



Regarding the impact of the calibration period, for a given number of stocks, the returns tend to stay in the same range irrespective of the amount of historical data (over 3 years) used to estimate the cointegration coefficients. This may lead to the conclusion that once the minimum calibration period for ensuring cointegration is used, increasing it does not necessarily improve the cointegration results.

Figure 4 Annual returns on DJIA and the tracking portfolios comprising 25 stocks



When examining the cumulative returns of the tracking portfolios as compared to the index returns, it appears that the difference between them is not uniformly accumulated. If we review the comparative performance of the tracking portfolios and the index on a year-by-year basis, it will become clear that only a small number of years is responsible for generating the largest part of the overall tracking error. In case of 20-stocks portfolios, year 1999 has generated the largest tracking error, while for 25 and 30-stocks tracking portfolios, the worst[8] years were 2000 and 2001. Figure 4 illustrates a year-by-year plot of returns on the 25-stocks strategies.

c. Transaction costs

The analysis of the transaction costs turned out to be very revealing in respect of the characteristics of the cointegration-based tracking strategies and critical to their understanding. The key result is that the transaction costs for rebalancing the tracking portfolios are highly dependent on the number of stocks in the portfolio and the length of the calibration period. They tend to act as a proxy for the degree of cointegration of the portfolio: *the stronger the cointegration, the steadier the stock weights in the portfolio and the smaller the transaction costs* incurred in connection with rebalancing the portfolio.

The overall transaction costs, computed at 0.2% of each trade value over the period 1-Jan-95 to 27-Dec-01, range from 8.8% for a 20-stocks tracking portfolio with a 3-years calibration period to 2.7% for a 30-stocks tracking portfolio with a 5-year calibration period. Put in other words, transaction costs of 2.7% over 7 years are equivalent to trading 100% of the portfolio almost twice per year, while 14.8% are equivalent to turning over the entire portfolio more then six times per year.

The impact of the transaction costs on the tracking portfolio returns is reported in Appendix 3. The transaction costs decrease significantly with the number of stocks in the portfolio and also, but less obviously, with the number of years in the calibration period.

As the main drive of the transaction costs is the stability of the weights, the inspection of weights would be useful. When analysing the stock weights in each portfolio (given in Appendix 4), except for 30-stocks tracking portfolios, they appear to be quite unstable through time, despite, for example, the relatively low transaction costs for 25-stocks portfolios. This type of instability, which appears only in portfolios employing part of the stocks in the index, might have been induced by the portfolio

[8] We qualify these years as 'worst' from the tracking error minimising perspective. Apart from this, they have actually generated excess return for the tracking portfolios over the market index returns, which turns out to be a key feature of the cointegration based tracking processes.

selection method. To investigate this issue, we have also analysed alternative stock selection methods. Their results are summarised in the final part of the back-test section.

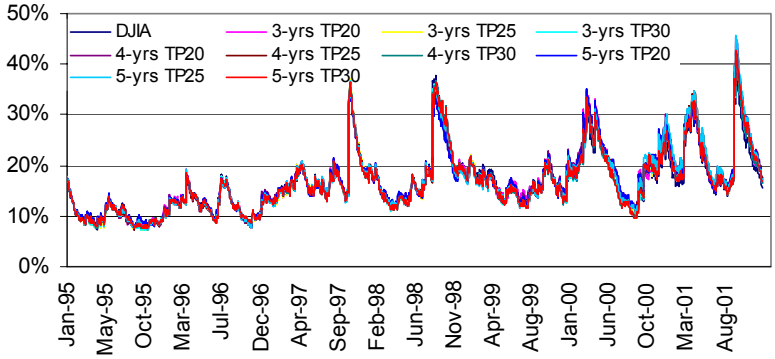
d. Volatility of tracking portfolio returns

In terms of volatility, all tracking portfolios display the same pattern as the market index. The annualised unconditional volatility of the tracking portfolios ranges from 17% to 19%. The tracking portfolios with smaller number of stocks appear to be slightly more volatile than the market, but the difference in the annualised unconditional volatility is very low.

The statistics in Appendix 3 report the annualised volatility of the tracking error. Again, smaller number of stocks portfolios display higher volatility of the excess returns. For instance, the tracking error of the 30-stocks portfolios is associated with an annualised volatility of approximately 2.5%.

Accounting for transaction costs does not add anything to the unconditional volatility figures, since the daily transaction costs display a steady pattern and are very low as compared to the daily returns. Also, the calibration period appears not to have a big impact on the volatility of the tracking portfolio returns.

Figure 5 EWMA volatilities of the tracking portfolios based on RD stock selection method (lambda 0.94)



The similarity in volatility behaviour between the market index and the tracking portfolios is also present when we move from unconditional volatility to exponentially weighted moving average volatility (Figure 5). Each significant spike in the market index volatility, e.g. the Asian and Russian crises or September 11th, is experienced also by the tracking portfolios, at comparable levels. The smoothing parameter used for computing the exponentially weighted moving average volatilities was of 0.94.

e. Correlation of tracking portfolios returns with market returns

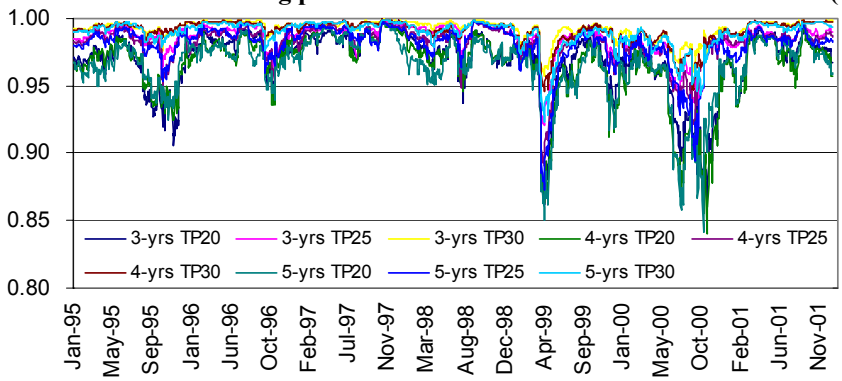
As reported in the statistics of the tracking portfolios (Appendix 3), the unconditional correlation of the tracking portfolio returns with the market returns is close to one, for all numbers of stocks and calibration periods used. Again, tracking portfolios with 20 and 25-stocks display a slightly lower correlation with the market returns, when compared to the tracking portfolio constructed from all 30 stocks of the index.

Moreover, also the exponentially weighted moving average correlation plotted in Figure 6 remains high during the entire back-testing period, ranging from 0.85 to 1. The tracking portfolios displaying the lowest correlation (which is still satisfactory high) are the ones constructed from only 20-stocks.

As noted previously, the cointegration tracking portfolio appears to have some 'bad periods', which account for the largest part of the overall deviation of the tracking portfolio returns from the

benchmark. Therefore, it does not come as a surprise the fact that the same periods (i.e. year 1999 for the 20-stocks portfolios and years 2000 and 2001 for 25 and 30-stocks tracking portfolios) are also characterised by declines in correlation with the market returns.

Figure 6 EWMA correlations of the tracking portfolios based on RD stock selection method (lambda 0.94)



Another important issue for the cointegration strategy is the correlation between the tracking error and the benchmark. The results in Appendix 5 show that *the tracking error is not correlated with the benchmark returns*, and this feature will play an important role in the success of the cointegration strategy when implemented in a market neutral long-short framework in the next section.

f. Skewness and excess kurtosis

To complete the analysis of the statistical properties of the daily tracking error distribution for our strategy, we have reported in Appendix 3 the skewness and excess kurtosis.

Generally, all tracking errors display small positive skewness, in the range of 0.01 for 20-stocks portfolios to 0.3 for 30-stocks portfolios. For 30-stocks portfolios, the positive skewness of the excess returns over the market index should be interpreted as an enhancement of the tracking portfolio's chances to consistently over-perform the benchmark.

Regarding the excess kurtosis, all portfolios' tracking errors appear to have leptokurtic distributions. Depending mainly on the number of stocks in the tracking portfolio and on the selection method used, the excess kurtosis ranges from 3.04 (20-stocks tracking portfolio) to 4.8 (30-stocks tracking portfolio).

To conclude, when analysing higher distribution moments, the tracking errors generated by different portfolios appear to have different degrees of non-normality, but generally they have small positive skewness and excess kurtosis.

g. Sharpe ratios

The Sharpe ratio computed for the benchmark was of 0.54. Provided that our tracking portfolios have generated average returns very close to market index returns with similar volatilities, the Sharpe ratios generally stay in the same range (please refer to Appendix 6).

The lowest Sharpe ratios (i.e. at 0.33) are displayed by 20-stocks strategies, after accounting for transaction costs. This comes as no surprise, as this strategy generated the lowest returns, with the highest volatility, being additionally penalised by the highest transaction costs. By contrast, the highest Sharpe ratio (0.57) is provided by 30-stocks tracking portfolios. The latter exceeds also the Sharpe ratio of the benchmark, even when accounting for transaction costs.

Alternative stock selection methods

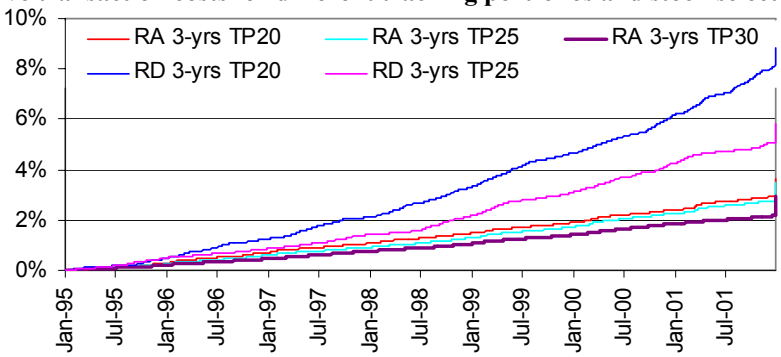
To investigate whether the stock selection method is responsible for the weights instability, we have employed alternative stock selection methods, still based on price ranking criteria. First, to reduce the effect of changes in the stocks ranking (and implicitly in the groups on which the cointegration regressions are estimated), each group was maintained constant for 6-months and respectively 1-year. The initial strategy, which was based on daily re-ranking of stocks, will be referred to as RD, while the semi-annually and annually re-ranking strategies will be denoted by RSA and RA.

Additionally, instead of using the stocks ranking based on the prices observed at one point in time, which may not be sufficiently stable or relevant, we have based the ranking on an indicator function counting the number of times in the previous period (for the purposes of our analysis 1, 3 and 5 years) when a particular stock was in the first n-group. The strategies based on this kind of frequency ranking will be denoted by F1, F3 and F5.

The statistics for all the alternative strategies are presented in Appendix 3.

The main features of the tracking portfolios identified for the daily re-ranking stock selection method in respect of cointegration tests, returns before transaction costs, volatility, correlation, skewness and kurtosis, are also displayed by the alternative stock selection methods. The important difference between the daily re-ranking stock selection method and the alternative ones concerns the transaction costs, and affects implicitly the returns after transaction costs and Sharpe ratios.

Figure 7 Cumulative transaction costs for different tracking portfolios and stock selection methods



As expected, the transaction costs decline significantly when moving from daily re-ranking to semi-annually or annually re-ranking, or to the strategies based on frequency. The lowest transaction costs, which occur when the cointegration is the strongest, are displayed by the annual re-ranking strategy and the frequency strategy based on the indicator function observed over 3-years. Furthermore, the plots of the stock weights in each tracking portfolio (Appendix 7) sustain the idea of improved stability in the cointegration relationship as we move from daily re-ranking to semi-annual, annual and frequency-based stock selection.

From the plot of the cumulative transaction costs for daily re-ranking and annual re-ranking with different numbers of stocks (Figure 7), the decreasing pattern in transaction costs will become more evident as the number of stocks in the portfolio is increased and we move from daily re-ranking to annual re-ranking. Moreover, when coupling the relative under-performance of the 20-stocks with higher transaction costs, the difference between 30-stocks tracking portfolio returns and 20-stocks or 25-stocks tracking portfolios will increase. This effect will be more evident in the case of the daily re-ranking strategy, which has the largest variability in transaction costs between tracking portfolios with different number of stocks.

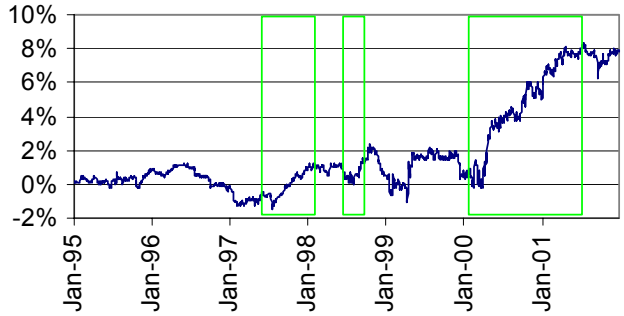
The impact of changing the stock selection method is the most evident for 20-stocks strategies, on the net returns (after transaction costs) and Sharpe ratios.

Considering the returns, before and after accounting for the transaction costs and depending on the number of stocks in the portfolio and the calibration period, we see that different methods have generated the closest, the highest and the lowest returns as compared to the benchmark. Tables with these results are presented in Appendix 8.

3.3. Some stories about cointegration

One of the advantages of estimating the portfolio weights based on cointegration coefficients is their enhanced stability. Being constructed on a rather long history of prices, they tend to ignore short-term movements in stock prices, such as bubbles or just noise, and focus on the long-run behaviour of the prices. As already shown, the fact that the tracking error is, by construction, mean reverting ensures that the tracking portfolio will stay 'tied together' with the index in the long-run, irrespective to short-term movements in individual stock prices. There can, however, be short-term de-correlations between the tracking portfolio and the index. In fact, this is a potential source of 'alpha', i.e. excess return, in the tracking portfolios.

Figure 8 Cumulative excess return of the 30-stocks tracking portfolios over the DJIA



Excess return occurs when the tracking portfolio is over-weighted in respect of a particular stock as compared to the benchmark, and that stock price increases, or when the tracking portfolio is under-weighted on some stock and that stock prices decreases. As shown by the plot of the cumulative tracking error of the 30-stocks tracking portfolios (Figure 8), the periods during which most of this excess return was accumulated coincide with the main market crises within our data sample: the Asian crisis, the Russian crisis and the TMT crash. In particular, during the bust of the bubble in the year 2000, the tracking portfolio produced significant alpha[9].

Some specific examples are detailed in the Appendix 9, together with the graphs of some stock weights and their prices evolution, to illustrate this discussion.

3.4. Final remarks on index tracking

To summarise, the cointegration index tracking strategy has the following features:

- To ensure cointegration, a minimum number of stocks in the portfolio (in our case 20 stocks, 0.67% of the index basket) and a minimum calibration period (in our case of 3 years) are required;

[9] A thorough analysis of cointegration relationships during periods of market decline will be presented in another paper, forthcoming in the ISMA Discussion Papers in Finance series.

- The tracking portfolios have similar returns and volatility with the market index, and are highly correlated with the latter;

- The excess returns from the index tracking, i.e. tracking errors, are uncorrelated with the market, have low volatility and slightly leptokurtic distributions with positive skewness;

- The periods of significant market decline, such as the Asian and Russian crises and the burst of the technology bubble are generating the largest part of the excess returns on the tracking portfolios; and

- The overall performance of the tracking strategy is dependent on the portfolio selection method used, the number of stocks and calibration period. Special attention should be given to the stock selection method, especially to the amount of trades required to rebalance the portfolio, as the transaction costs may erode the returns of the tracking portfolios.

From the analysis of tracking portfolios with different number of stocks, we have found that most of the performance measures have favoured the 30-stocks tracking portfolio. However, the slight under-performance of the 20-stocks portfolios as compared to the market index will be of further use when designing the short part of the market neutral strategy. Of the stock selection methods considered, the annual re-ranking and frequency based re-ranking with an indicator function estimated over 3-years appear to provide the best results in terms of returns and consistency.

4. Long-short market neutral strategy

Having constructed the simple tracking strategy, a natural extension for exploiting the tracking potential of the cointegrated portfolios would be to replicate 'plus' and 'minus' benchmarks. Then, self-financing long-short strategies can be set up with portfolios tracking different 'plus' and 'minus' benchmarks. This type of long-short strategy is expected to generate returns according to the 'plus'/'minus' spread with a fairly low volatility. Moreover, even if there is no explicit constraint in our model to ensure zero correlation of the long-short strategy returns with market returns, since the 'plus' and 'minus' portfolios are both highly correlated with the original benchmark, a low correlation of their difference with the benchmark results, provided that each tracking error is individually not correlated with the market.

'Plus' and 'minus' benchmarks can be constructed by adding to/subtracting from the benchmark returns an annual excess return of x%, uniformly distributed to daily returns. We expect to become more and more difficult to construct cointegrated portfolios as the magnitude of x increases. The cointegration relationship between the market index and its component stocks has a solid rationale, but this is not necessarily the case for portfolios tracking artificial benchmarks, which, for example, may be chosen to over-perform the market index by 50%. The difficulty in finding an appropriate cointegration relationship leads to an increased instability of the stock weights, higher transaction costs and higher volatility of returns. To avoid this, it is essential to ensure that all the portfolios tracking 'plus' or 'minus' benchmarks pass the cointegration test.

The new cointegration regressions can be written as:

$$\log(\text{index\_plus}_t) = a_1 + \sum_{k=1}^{n} a_{k+1} * \log(P_{k,t}) + u_t \tag{7}$$

$$\log(\text{index\_minus}_t) = b_1 + \sum_{k=1}^{n} b_{k+1} * \log(P_{k,t}) + u_t \tag{8}$$

We note that the stock weights are not restricted to be positive in the tracking portfolios above; in fact it is likely that we shall take some short positions in the portfolios tracking both 'plus' and 'minus' benchmarks.

Further to estimating the stock weights in the individual 'plus'/'minus' tracking portfolios, the self-financing strategy will consist of a long position on the 'plus' portfolio and a short position on the 'minus' portfolio.

The stock holdings in the long-short strategy will be obtained by netting their individual weights in the 'plus' and 'minus' portfolios. Therefore, the transaction costs on the long-short strategy will be less than the sum of the costs incurred by trading the 'plus' and 'minus' portfolios individually. Specifically, the transaction costs will be determined as follows:

$$TC_T = 0.002\sum_{k=1}^{n} abs((wplus_{k,T} - wminus_{k,T}) - (wplus_{k,T-10} - wminus_{k,T-10}))P_{k,T} \quad (9)$$

In order to examine the correlation of the long-short strategy returns with the market returns, we can write the returns on the 'plus' and 'minus' portfolios separately as functions of benchmarks' returns:

$$R_{+,t} = \rho_+ \frac{\sigma_+}{\sigma_{B+}} R_{B+,t} + \varepsilon_{+,t} \quad (10)$$

$$R_{-,t} = \rho_- \frac{\sigma_-}{\sigma_{B-}} R_{B-,t} + \varepsilon_{-,t} \quad (11)$$

where $\rho_{+/-}$ are the correlation coefficients between the 'plus' and respectively 'minus' portfolio returns with the 'plus'/'minus' benchmark returns, σ_+ / σ_{B+} and σ_- / σ_{B-} are the relative volatilities of the 'plus' respectively 'minus' portfolio returns to the 'plus'/'minus' benchmark returns, and ε_+ and ε_- are the tracking errors of the 'plus'/'minus' portfolios.

The 'plus'/'minus' benchmarks' returns can be written in terms of market returns +/- the fixed annual spreads, denoted by μ_+ and μ_-, while the return on the long-short strategy will be the difference between the returns on the individual 'plus' and 'minus' portfolios:

$$R_{+/-,t} = R_{+,t} - R_{-,t} = R_{M,t}(\rho_+ \frac{\sigma_+}{\sigma_{B+}} - \rho_- \frac{\sigma_-}{\sigma_{B-}}) + \mu_+\rho_+ \frac{\sigma_+}{\sigma_{B+}} + \mu_-\rho_- \frac{\sigma_-}{\sigma_{B-}} + \varepsilon_{+,t} - \varepsilon_{+,t} \quad (12)$$

As pointed out in section 3.2, the tracking errors for simple tracking strategies were not correlated with the market. Consequently, the condition for market neutrality (i.e. zero correlation between the long-short strategy and the market returns), is that the second term in the equation (12), the difference between the 'plus' market beta and 'minus' market beta, is zero.

This will happen if the 'plus'/'minus' portfolios are suitable replicas of their benchmark, i.e. have the correlation coefficients close to 1, as well as unit relative volatilities. Under such circumstances, the difference between the 'plus' market beta and the 'minus' market beta will be zero, and the strategy will be market neutral.

4.1. Back-test results

For testing the long-short market neutral strategy outlined above, we have used the same principles as for the straight index tracking strategy. As basis for tracking, we have created 6 'plus'/'minus' benchmarks by adding/subtracting annual returns of 5%, 10% and, respectively, 15% from the

reconstructed DJIA returns. Similar to the simple tracking strategy, we have set up portfolios comprising 20, 25 and 30 stocks. As calibration periods we have used again 3, 4 and 5 years.

In respect of the stock selection strategy recall that, based on the simple tracking results, we could not distinguish a single best stock selection strategy, independently of the number of stocks in the portfolio and calibration period chosen. Consequently, we have employed the most representative three stock selection methods from the ones used in the simple tracking, i.e. daily re-ranking (RD), annual re-ranking (RA) and frequency based re-ranking with an indicator function estimated over the previous 3 years (F3).

As long-short strategies, we have examined all possible 'plus'/'minus' combinations for a particular stock selection strategy, calibration period and number of stocks in the portfolios. For example, with annual re-ranking, 20-stocks portfolios and calibration period of 3 years, we have set up 16 portfolios from all possible combinations of +/- 0, 5%, 10% and 15%. Additionally, we have examined, for each selection strategy and calibration period, all possible combinations with 30-stocks in the 'plus' portfolio. The overall number of long-short strategies monitored was 720.

The same re-balancing procedure was followed for the long-short strategies: the cointegration coefficients were re-estimated every 10 trading days and the number of stocks kept constant between consecutive re-balances.

In order to analyse the performance of the long-short strategies we have considered the following:

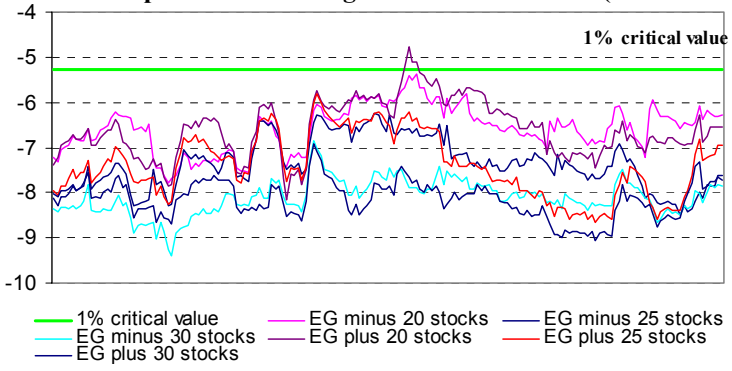
a. Engle-Granger cointegration tests;
b. returns on the long-short strategy;
c. volatility of strategy's returns;
d. correlation of the tracking portfolio returns with the index returns;
e. skewness and excess kurtosis of tracking portfolio returns; and
f. Sharpe ratios.

a. Engle-Granger cointegration tests

In order to ensure that each of the 'plus' and 'minus' portfolios is adequately cointegrated with the 'plus' respectively 'minus' benchmark, the cointegration regression residuals have been tested for stationarity.

The results of section 3 showed that the portfolios tracking the market index with at least 20-stocks and a calibration period of minimum 3 years proved to be sufficiently cointegrated with the benchmark. Our concern was whether the degree of cointegration remains the same as we move further from the market index by creating artificial 'plus' and 'minus' benchmarks. The results plotted in Figure 9 for tracking 'plus' and 'minus' 15% benchmarks, the most likely to break the cointegration tests, show that the portfolios remain fairly cointegrated with the benchmarks tracked, even if the latter diverge significantly from the actual market index.

Figure 9 ADF test statistics for portfolios tracking +/-15% benchmarks (calibration period 5 years)



b. Returns of the long-short market neutral equity strategies (with transaction costs[10])

The cumulative returns of the long-short strategies estimated over the entire back-test period (1995 to 2001) are reported in Appendix 10.

All long-short strategies based on the **daily re-ranking** stock selection method produced negative results after accounting for the transaction costs with one exception, i.e. the strategies with the 'plus' portfolio comprising 30-stocks (which, of course, are not affected by the stock selection method employed).

The main responsibility for the strategy's failure to generate returns according to the spread between the 'plus' and 'minus' benchmarks stays with the transaction costs. As displayed also by simple tracking strategy results, the stock weights tend to be quite unstable for the daily re-ranking selection method, which generates high transaction costs. Additionally, when tracking large 'plus' or 'minus' spreads, it is expected that the degree of cointegration will decrease; this will also contribute to the stock weights instability. The highest cumulative amount of transaction costs for a simple index tracking strategy was approximately 9%, but when moving to a long-short strategy the highest transactions costs reached almost 35% (for +/- 15% strategy with 30 stocks in the 'plus' portfolio and 20 in the 'minus').

The total returns, net of transactions costs, that were generated by the **annual re-ranking** and **frequency based re-ranking** stock selection methods tend to be positive, with only few exceptions (for the shortest calibration period, i.e. 3 years). The impact of the calibration period is rather mixed in case of the annual re-ranking, while for the frequency based re-ranking the returns are consistently higher for longer calibration periods.

Significantly higher and more consistent results are produced by all stock selection methods for strategies comprising less than 30-stocks in the 'minus' portfolio. For instance, the strategies with 20 stocks in the portfolio tracking 'minus' 15% and 30 stocks in the portfolio tracking at least 'plus' 5% generated returns over 49% over the whole back-testing period in case of the frequency based re-ranking. This should come as no surprise, considering the tendency of 30-stocks tracking portfolios to over-perform the market, and the tendency of 20-stocks tracking portfolios to under-perform the market, creating by itself a spread arbitrage opportunity.

Additional insight on the individual 'plus' and 'minus' performance can be obtained by examining the combinations 'plus' α-'minus' zero and 'plus' zero-'minus' α. By construction, the 'plus' α and 'minus' α tracking portfolios should be symmetrical to the benchmark and the 'plus' α-'minus' zero and 'plus' zero - 'minus' α strategies should be equivalent. However, in case of the daily re-ranking stock selection method, there are significant differences: the under-performance of 'plus' α-'minus' zero strategies is always smaller than the under-performance of 'plus' zero-'minus' α strategies. These results appear to indicate that it is more difficult to construct 'minus' tracking portfolios than 'plus' tracking portfolios in case of less than 30-stocks portfolios with the daily re-ranking stock selection method. In the case of the annual re-ranking and frequency based re-ranking, it appears that, as opposed to daily re-ranking, the 'minus' tracking portfolios are responsible for the largest part of 'alpha' in the long-short strategy (there are, however, exceptions).

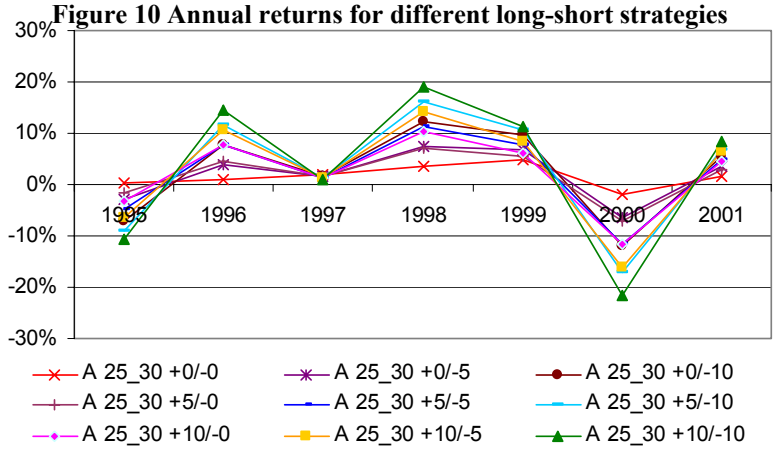
To summarise, for the daily re-ranking stock selection method, the long-short strategy failed to produce profits due to the instability of the stock weights in the tracking portfolios and the associated transaction costs. The other two stock selection methods have produced mainly positive results,

[10] In addition to the transaction costs, we have examined the impact of the repo costs for the short equity positions in the long-short strategies. Please refer to Appendix 13 for these results.

highly dependent on the number of stocks in the 'plus'/'minus' portfolios, the benchmark tracked and the calibration period.

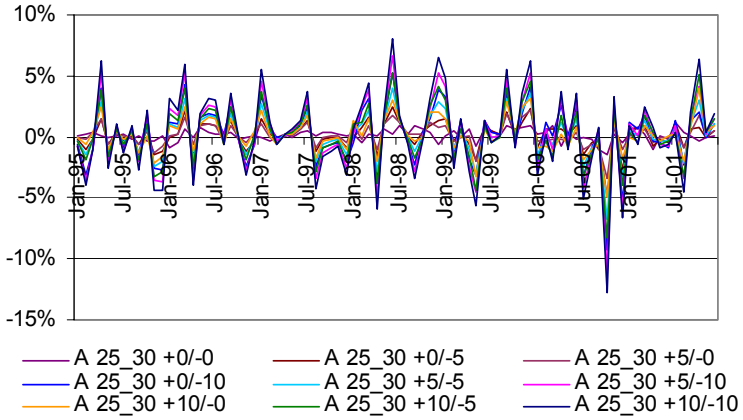
As pointed out in the simple tracking, the strategy's performance was not uniform during the back-testing period. Therefore, it would be worth investigating the annual performance of the long-short strategies. To this end, we have selected some of the best combinations identified in the previous sections, i.e. 'plus' 30/'minus' 25-stocks portfolios with a calibration period of 3 years based on annual re-ranking stock selection method.

Figure 10 Annual returns for different long-short strategies



The annual returns for different combinations available with the above strategy are plotted in Figure 10. As the results indicate, the long-short strategies have generated negative results in only two of the seven years under examination, i.e. 1995 and 2000. Year 2000 was pointed out also by the simple tracking strategy as being a bad year for the cointegration. However, during that year the market index lost 11.73%, which is more than the loss generated by four out of the nine long-short strategies plotted below.

Figure 11 Monthly returns for different long-short strategies



Another remark suggested by the graph above is that the long-short strategies have performances consistent with the 'plus'/'minus' spread for which they were designed. The highest returns/losses are generated constantly by the +/-10% long-short strategy, while the lowest returns/losses are always generated by the +/-0, which is in fact the difference between the 25 and 30-stocks portfolios tracking the index. The same is true for the monthly returns, shown in Figure 11.

To summarise, from the long-short strategies analysed, the ones producing the most consistent positive results are the ones tracking small spreads, even if the magnitude of these returns is reduced.

By contrast, the strategies tracking large spreads are generating less consistent and less frequent positive returns, which also have a higher magnitude.

c. Volatility of the strategy returns

The annualised volatility for all long-short strategies monitored is reported in Appendix 10. Generally, the volatility turned out to be highly dependent on the spread between the benchmarks tracked in the long-short strategies.

One consistent pattern, which can be identified for all stock selection methods employed, is that the volatility of the combinations increases with the spread between the 'plus' and 'minus' benchmarks tracked. The annualised volatility ranges from 3% for +5%/-0 or +0/-5% strategies to over 30% for +/-15%. Another common feature is that the long-short strategies tracking very small spreads, i.e. benchmarks close to the actual DJIA, exhibit very low volatility, less than 5% p.a.

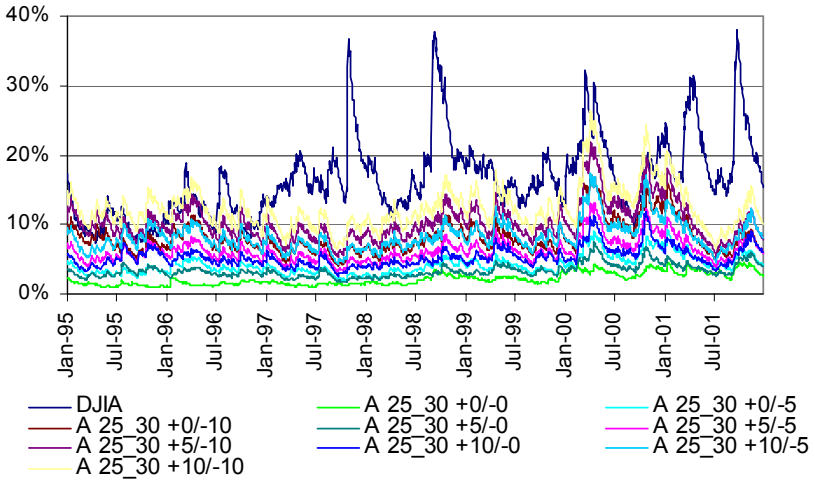
When separating the impact of tracking a 'plus' and a 'minus' benchmark, we find evidence that the 'minus' part of the long-short strategy generates more volatility than the 'plus' part. The cause of this excess volatility should be related to the increased difficulty of constructing 'minus' portfolios as compared to 'plus' portfolios.

The impact of the number of stocks in the portfolios and the calibration period are rather diffuse. Slight and inconsistent increases in the level of volatility can be observed when increasing the calibration period, as well as when reducing the number of stocks in the portfolio.

When compared to daily re-ranking results, the volatility of the annual re-ranking tends to be slightly lower, but generally in the same range. Separately, F3 appears to penalise less in terms of volatility the returns of the combinations tracking benchmarks with the largest spreads, i.e. the difference in volatility between strategies tracking narrow spreads and strategies tracking wider spreads is less evident than in the case of annual re-ranking stock selection method.

The short-term volatilities during the back-test period have been estimated using an exponentially weighted moving average (EWMA) with a smoothing parameter of 0.94. Short-term volatilities, for the same combinations as previously (i.e. 'plus' 30/'minus' 25-stocks portfolios with a calibration period of 3 years based on annual re-ranking stock selection method and different choices for alpha), are plotted in Figure 12.

Figure 12 EWMA volatilities of different long-short strategies (lambda 0.94)



As the graph indicates (and as suggested by the long-term volatility figures in Appendix 10), with few exceptions mainly during the year 1995, the volatilities of the strategies considered are far lower than the volatility of the market index. Also, the effect of increasing the spread between the benchmarks tracked is very obvious: consistently, the lowest volatility is displayed by the +/-0 strategy, and the highest by the +/-10% strategy. If we consider only the strategies tracking up to +/-5%, they all display stable short-term volatilities that never rise above 10% per annum, and the difference between the market index volatility and the volatilities of the long-short strategies is substantial.

Another important feature of the long-short strategies' volatilities is that they did not experience a significant number of spikes present in the market index volatility. *The large spikes of the market volatility from November 1997, September 1998, and April 2001 were not reflected in the volatility of the long-short strategies*. The only exception is the year 2000, in which the volatility of the long-short strategies behaved in a very similar manner to the volatility of the market.

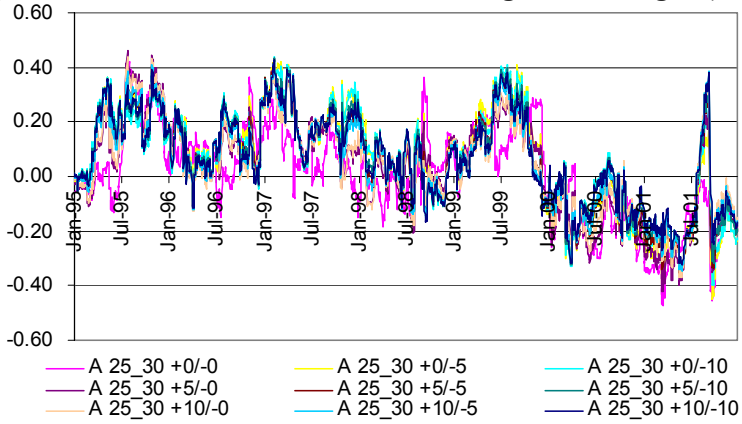
Considering the results of the long-short strategies in respect of volatility, a reasonable conclusion would be that some of the most aggressive strategies, tracking benchmarks that are quite far from the market index, display high volatility. But this is almost never greater than the market volatility and the more conservative strategies have much lower volatility than the market.

d. Correlation of the long-short strategies' returns with the market returns

As shown in the outline of the strategy, unless the 'plus' and 'minus' portfolios are suitable replicas for the 'plus' and 'minus' benchmarks, the long-short strategy will not necessarily be market neutral. In the simple tracking strategy back-tests, the correlation coefficients of all cointegration-based portfolios were very close to one, but occasionally, the 'minus' portfolios was more volatile than the 'plus' portfolios, for the same spread. In case there is a difference between the relative volatilities of the 'plus'/'minus' portfolios, the market neutrality of the strategy may be affected.

The unconditional correlation coefficients reported in Appendix 10 are all very low, generally positive for strategies employing a calibration period of 3 years and negative for strategies based on 4 or 5 years of calibration. The stock selection method and the spread between the benchmarks tracked are influencing the level of correlation, but the relationship is not straightforward. Also, there is a slight decrease in correlations as the number of stocks in the tracking portfolios increases.

Figure 13 EWMA correlations for different long-short strategies (lambda 0.97)



From the perspective of the unconditional correlation coefficients, *the long-short strategies are very close to market neutrality*. As a comparison, Brooks and Kat (2001) found in their study much higher correlations between the returns of market neutral hedge funds and the returns on the market indexes, up to 0.54.

Short-term correlations are, typically, very unstable and an analysis of the exponentially weighted moving average correlation allows a more thorough check on market neutrality. The exponentially weighted moving average correlation with a smoothing parameter of 0.97 is reported in Figure 13 for the same long-short combinations selected previously.

The EWMA correlations range from 0.4 to –0.4. The episodes with significantly higher correlation between the market returns and long-short returns are year 1995, the beginning of 1997, mid of 1999 and end of 2001. Significant negative correlation is displayed during the years 2000-2001.

Year 1995 was previously identified as being one of the worst for the long-short strategies: negative returns, higher volatility than the market, and now also significant positive correlation with the benchmark. As observed also for the volatility, the effects of the Asian and Russian crises are not experienced by the long-short strategies, as their correlation with the market during that time was fairly low. The same thing happened during the burst of the technology bubble, i.e. the end of year 2000. Even if the returns of the long-short strategies were also quite volatile, they were not correlated with the market. We will revert to the behaviour of the long-short strategy during the end of the technology bubble burst when we will analyse the stock weights in the long-short portfolios.

e. Skewness and kurtosis of the long-short strategy returns

An important point to investigate in connection with the long-short strategy is the normality of its returns. The skewness and excess kurtosis figures for all strategies are reported in Appendix 10.

When analysing the skewness of the returns, common patterns can be identified: for most strategies, the skewness tends to decrease with the spread between the benchmarks tracked; the lowest skewness is displayed by the 30-stocks strategy with a calibration period of 3 years (around – 0.38) but this is hardly significant; slightly significant positive skewness is generated by strategies based on combinations with 30-stocks in the 'plus' portfolio and less in the 'minus' portfolio and a calibration period of 5 years.

Most of the strategies exhibit excess kurtosis between 1 and 2, which is also hardly significant. As a reference, the kurtosis of the market index returns is of 4.5. Generally, the excess kurtosis increases with the spread between the benchmarks tracked, with some exceptions. Also, on average, the excess kurtosis generated by annual re-ranking strategies is lower than the excess kurtosis of the daily re-ranking based strategies. Apparently, a straightforward relationship between calibration period and the excess kurtosis cannot be identified for the daily and annual re-ranking stock selection methods. A significant reduction in the excess kurtosis can be observed when moving to frequency based re-ranking stock selection method. All these strategies display excess kurtosis between 0.7 and 2 with few exceptions.

To sum up the results on skewness and excess kurtosis, *the daily returns to the long-short strategies appear to have only a small degree of non-normality. This is far lower than the non-normality that is usually identified for hedge fund returns*. Even at a monthly frequency, many hedge fund returns display highly significant kurtosis (see Brooks and Kat, 2001). Also, when compared with the market index (or with the simple tracking strategy outlined in the previous section), the statistical properties of the long-short strategies returns are closer to the characteristics of a normal distribution.

f. Sharpe ratios

In order to compute the Sharpe ratios, we have assumed that, as the 'plus'/'minus' portfolios are financing each other, the corresponding amount is invested in 3-months T-bills. This assumption was

made in order to ensure comparability between this self-financing strategy and other strategies requiring financing, such as simple index tracking or equity long-only.

The Sharpe ratios[11] for all long-short strategies are reported in Appendix 11. As a common feature for all the stock selection methods employed, the Sharpe ratios tend to increase with the number of stocks in the portfolios. The best strategies in terms of Sharpe ratios remain the combinations 'plus' 30/'minus' 20-stocks and 'plus' 30/'minus' 25 stocks.

An important observation concerning the annual re-ranking and frequency based re-ranking strategies is that the increase in the spread between the benchmarks tracked is done at the expense of reducing their Sharpe ratios. The highest Sharpe ratios, in the range of 0.66 to 0.78 are obtained as the difference between the returns of the 30 and 25-stocks portfolios tracking the simple index. However, this may be interpreted as being the result of very low levels of volatility for these strategies (around 2-3% annualised volatility), rather then the result of exceptionally high returns.

The Sharpe ratios produced by the annual re-ranking and frequency-based re-ranking are considerably larger than the ones displayed by the daily re-ranking strategies. In the frequency based re-ranking method, for calibration periods of 3 and 4 years, the 20 and 25-stocks strategies generate negative Sharpe ratios, which are successfully used in the 30-stocks 'plus'/20 or 25-stocks 'minus' combinations.

4.2. Application of a simple strategy selection rule

We have tested a very simple strategy selection rule based on monitoring the returns on the 9 selected long-short strategies: during each 10-days trading period, we have invested in the strategy that had the best performance during the previous 10-days.

In terms of returns before transaction costs and volatility, the selection rule provided a better Sharpe ratio (i.e. 0.88) than any of the 9 individual strategies. However, after accounting for the transaction costs, the strategy ends up in a loss position. The transaction costs generated by switching every 10-days from the weights produced by one cointegration regression to the weights produced by another are huge. This experiment shows how sensitive are the cointegration weights to the selection of the strategy parameters, i.e. the benchmarks to be tracked and the number of stocks in each tracking portfolio.

4.3. Study case – long-short portfolio weights during year 2000

As shown by the annual returns during year 2000, most of the long-short strategies had a better performance than the market index, even if still negative. Therefore, it would be worth investigating the source of this relative excess return as compared to the benchmark. The graphs with the evolution of the stock weights in some long-short strategies during the year 2000 are plotted in Appendix 14.

As a first comment, the graphs are based on the annual re-ranking strategy and at the end of February the stocks were re-ranked, and the composition of the first 20 and 25-stock groups changed. The significant displacements displayed by the stock weights at this point are a direct result of this re-ranking. Apart from this, when analysing the graphs based on strategies tracking different benchmarks, we observe that the patterns of certain stock weights are similar for all strategies, and the main differences consist in the magnitude of these weights. The most 'aggressive' strategies are the ones tracking large spreads between the benchmarks.

[11] In addition to the Sharpe ratios, we have also computed the information ratios and reported them in Appendix 12.

The stocks on which the strategies are significantly long are: JNJ, MMM, XOM, WMT, and HD. By contrast, the strategies are significantly short on IP, KO, CAT, HWP and IBM. The common feature of the stocks in the long group is that they can all be defined as traditional, while from the short group, two are leading technology stocks. Thus the success of the strategy should come from the stocks with long positions increasing and the stocks with short positions falling.

Johnson & Johnson (JNJ) was one of the year 2000 winners in the Dow Jones, as its price increased by almost 13% when the market index lost 6%. The company, having an individual sound performance during the year 2000, was also favoured by being part of a growth industry sector, despite the general downside of the market. Therefore, the significantly long position in JNJ proved to be beneficial for the long-short strategy.

Another winner of the year 2000 with a significant long position in the long-short strategy was Minnesota Mining (MMM), with 23.12% annual return. Its performance was generated by company's specific strategy on research and development and by a successful merger and acquisition policy, which explain its positive evolution on the background of a bear market.

Exxon Mobil (XOM) also had a general good performance in terms of stock price growth during year 2000, yielding 7.92% as annual return. The favourable conjunction of the oil prices and a successful merger are the main reasons for the company's performance during the year 2000. But year 2000 followed a number of years in which XOM has over-performed the market, which explains its significant long weight in the long-short strategy. Again, this proved to be a good choice for the cointegration based strategy.

Wal-Mart Stores (WMT), by contrast, had a very bad year in 2000, loosing 23% of its stock price. However, this followed a series of years in which this stock has highly over-performed the market, which explains its significant long weight in the long-short strategy. Its long position in the cointegration strategy generated losses further to the price decline.

The same story underlies Home Depot (HD) weight in the long-short strategy. Home Depot had a very bad year 2000, which however followed a number of years of significant over-performance of the index. Therefore, the cointegration strategy lost during the year 2000 from its long bet on HD.

From the short positions, the weights of Coca-Cola (KO) and International Paper (IP) are the result of a general under-performance, which lasted for a number of years preceding year 2000. Such significant short positions have generated excess returns for the long-short strategy only in case of International Paper, which continued to significantly under-perform the market. For Coca-Cola, the short position generated relative loses for the long-short strategy, as its price increased during the year 2000.

The cointegration based long-short strategy proved to be very successful in dealing with the burst of the technology bubble during year 2000. Technology stocks like IBM and Hewlett-Packard (HWP) had significant short positions in the long-short portfolios in the same year in which they lost more than 20% of their stock prices.

To conclude, the stock weights in the long-short strategy are determined as the difference between the stock weights in the long and short portfolio. To the extent that a particular stock has over-performed the market index and was cointegrated with the 'plus' benchmark during the calibration period, it will have a long position in the long-short portfolio. Alternatively, if a certain stock has under-performed the market index and was cointegrated with the 'minus' benchmark during the calibration period, it will have a short position in the long-short portfolio. The success of the strategy rests on the stability of the cointegration relationship over the trading horizon and in the mean reversion of the stock prices, which has already been discussed in some detail in section 3. The above case of the

technology bubble is a good example of mean reversion in prices, which was successfully exploited by the cointegration based long-short strategy.

4.4. Final remarks on the long-short strategies

Originally, we have described the features of a successful long-short market neutral equity strategy as being:

- returns according to the spread between the benchmarks tracked;
- lower volatility than the market; and
- low correlation with the benchmark.

The results obtained from back-testing prove that, when setting up a long-short market neutral strategy based on cointegration, the following parameters have a significant impact on the strategy's success:

1. *Stock selection method* – in terms of returns, the stock selection method is critical to the success of the long-short strategy. As shown by the negative results of the daily re-balancing stock selection method, a high variability of the stock weights may generate huge transaction costs affecting the strategy's potential to generate returns. Based on the same grounds, the other two stock selection methods employed (annual and frequency based re-ranking) have proved to be equally appropriate, considering that they were generating similarly low transaction costs in the simple tracking strategies.

2. *Benchmarks to be tracked* by the 'plus'/'minus' portfolios – as shown by the back-test results, the spread between the benchmarks tracked in the long-short strategy cannot be increased without a corresponding increase in the volatility and kurtosis of the returns, and also a potential reduction of the strategy skewness. The Sharpe ratios, as a measure of the trade-off between returns and volatility, favour low spreads between the benchmarks tracked. In addition to an increase in volatility, the cointegration relationship will weaken as the reconstructed benchmarks diverge from the market index. Therefore, it is essential to test each portfolio for cointegration with its underlying benchmark.

3. *Number of stocks* in each portfolio – as displayed by the residuals' stationarity tests, in order to identify a cointegration relationship, a minimum number of stocks is required in the tracking portfolio. Apart from the minimum number of stocks, the long-short strategies appear to provide best results with close to the maximum number of stocks in each 'plus'/'minus' portfolio. A particularly successful strategy appeared to be the combination of 30-stocks in the 'plus' portfolio with 20 or 25-stocks in the 'minus' portfolio, a combination that is able to exploit the difference in the cointegration regressions intercepts.

4. *Calibration period* – as showed by the cointegration tests and implied by the theory, a minimum number of years is required to construct a cointegration relationship. Beyond this number, the effect of the calibration period on the returns of different stock selection strategies is not uniform. Still, the best results were obtained for longer calibration periods.

Considering these results, it becomes obvious that the practical implementation of the cointegration-based trading strategies should be designed as to allow at each step the selection of the most appropriate long-short strategy, provided the set of solutions available for different parameters. To this end, the impact of using parameters such as calibration period, number of stocks, benchmarks to be tracked and stock selection method should be considered independently for both 'plus' and 'minus' portfolios.

5. Alternative strategies combining index tracking and long-short market neutral

The index tracking and long-short market neutral strategies introduced in the previous sections do not need by any means to be implemented individually. Their combination may target either the enhancement of some properties of the long-short market neutral strategies, or the transport of the alpha gained in the long-short market neutral strategy to the market index.

a. 'Fund of funds' approach to long-short market neutral strategies

The first enhancement of the long-short market neutral strategies is a 'fund of funds' approach, which, by investing in a number of similar strategies, should reduce the volatility of its returns. This is similar to constructing an index, which is known to have more attractive statistical properties for investors than the individual returns series from which it is constructed.

Such an approach also allows a basic comparison of our strategy returns with some hedge funds indexes. To this end, we have used publicly available data on hedge funds provided by Hennesse Group (www.hennessegroup.com) and Zurich Capital Markets (www.marhedge.com). The two monthly returns indexes employed by us are Hennesse Market Neutral index and Zurich Long-Short Market Neutral Median index.

The 'fund of funds' returns series was constructed from the same long-short strategies selected previously, i.e. annual re-ranking stock selection method, calibration period 3 years, 30 stocks in the 'plus' portfolio and 25 stocks in the 'minus' portfolio, tracking spreads up to 10% away from the market index. The 'fund of funds' was assumed to equally invest in the nine selected strategies.

When interpreting the results of such comparison, there are certain issues to consider. First, the index is constructed by averaging the returns of a much larger number of funds than our 'fund of funds' approach that is based only on nine individual strategies. Consequently, the index should have, by means of construction, a lower volatility than the returns on our 'fund of funds' approach. Second, the returns reported by individual funds are obtained through different levels of leverage, while our strategies' returns are not leveraged.

The cumulative returns on the cointegration-based 'fund of funds' and the hedge fund indexes are plotted in Figure 14. The first observation is that the cointegration based 'fund of funds' cumulative returns are considerably more volatile than the indexes returns but this can be an artefact of the indexes construction method which averages the returns over hundreds of funds. Then, there is a significant difference in the cumulative returns generated by our strategy and the ones of the indexes, which may be the effect of the leverage.

Figure 14 Cumulative returns (no leverage)

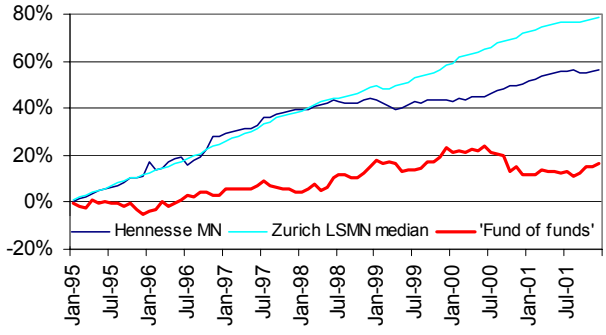
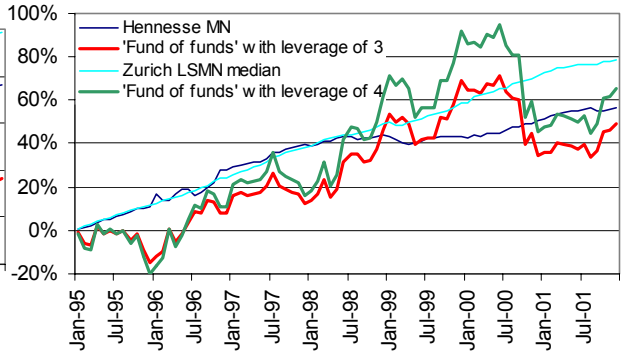


Figure 15 Cumulative returns (with leverage)



To account for this second issue, from the cumulative returns series, we have backed out the leverage, which applied to the 'fund of funds' cumulative returns would make them similar to the indexes cumulative returns. The levels of leverage implied by regressing the indexes cumulative returns on

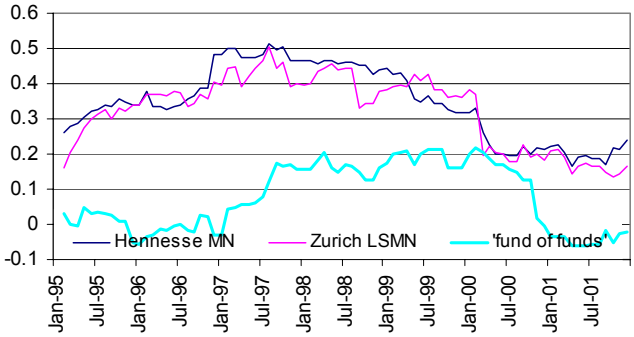
the 'fund of funds' cumulative returns (without intercept) are 2.9 for Hennesse MN and 3.7 for Zurich LSMN. Figure 15 is plotting the cumulative returns of the 'fund of funds' against the returns of the indexes, by using a leverage of 3 and, respectively, 4.

We may conclude that, when adding leverage to our strategy, its returns become similar to relevant hedge funds indexes, even if more volatile. But this is to be expected, given that the cointegration is a single strategy and not an index.

Regarding the strategy's performance in terms of market neutrality compared with the hedge funds indexes, we have computed the exponentially weighted correlation with the market index (DJIA) by using a smoothing parameter of 0.97 for our strategy monthly returns, as well as for the indexes.

As Figure 16 shows, with few exceptions, the correlation of our long-short strategy returns is significantly lower than the correlation of the hedge fund indexes with the market returns.

Figure 16 EWMA correlations with the market index for monthly returns (lambda=0.97)



To conclude, in the presence of leverage, the 'fund of funds' approach to long-short market neutral strategies generates similar, even if more volatile, returns with the hedge fund indexes, which are, however, significantly closer to market neutrality than the hedge fund indexes.

b. Combination of the index tracking excess return with long-short market neutral strategies

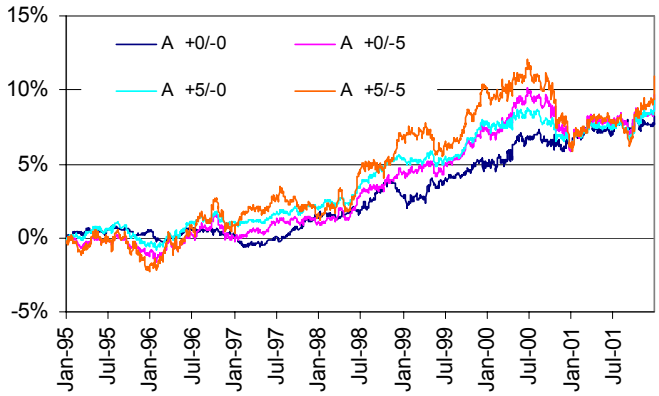
Another enhancement of the long-short market neutral strategy concerns mainly its performance during significant market downturns. In such cases, the market index under-performs the artificial 'minus' benchmark, and therefore, it is more profitable to be short on the market index instead of the 'minus' benchmark.

To account for this type of event, we have set up a strategy that combines the long short market neutral strategy with the excess returns generated through index tracking. Specifically, we have examined the performance of our selected long-short strategies when combined with another market neutral strategy which is long on the portfolio tracking the index with all 30-stocks and short on the index (the portfolio composition should be determined as stock weights deviation from the market index, rather than being individually long on the tracking portfolio and short on the market index). The cumulative returns and the statistics of this strategy are reported in figure 17. Additionally, we have reported in Appendix 13 the Sharpe ratios for this strategy after accounting for repo costs.

As shown previously, the excess return from index tracking has low volatility, zero correlation with the market, and is close to normality, representing by itself an alpha generating market neutral strategy. When combined with the long-short market neutral strategies, it manages to improve consistently their Sharpe ratios. The absolute returns on the combined strategies decline as compared with the corresponding long-short only strategies, because of the reduced magnitude of the excess returns of the 30-stocks tracking portfolio over the index. However, the associated decrease in volatility is larger, which improves the Sharpe ratios of the strategies.

Moreover, the combination remains market neutral, with a certain degree of non-normality, which is however less than the one usually found in respect of hedge funds returns.

Figure 17 Cumulative returns on strategies combining index tracking excess return with long-short market neutral selected strategies



LSMN	-\+	0%	5%	10%
total return	0	8.25%	9.72%	11.67%
annual volatility		1.63%	2.02%	3.82%
correlation		-0.03	-0.01	0.01
skewness		0.02	0.01	0.02
excess kurtosis		2.53	2.71	1.82
Sharpe ratio		0.72	0.69	0.44
total return	5%	9.47%	10.95%	12.89%
annual volatility		1.62%	2.84%	4.85%
correlation		-0.02	0.01	0.02
skewness		-0.02	-0.08	-0.06
excess kurtosis		2.43	1.51	1.26
Sharpe ratio		0.84	0.55	0.38
total return	10%	10.46%	11.93%	13.88%
annual volatility		2.40%	3.83%	5.87%
correlation		0.00	0.01	0.02
skewness		-0.14	-0.15	-0.12
excess kurtosis		1.72	1.25	1.11
Sharpe ratio		0.62	0.44	0.34

** Period 1-Jan-1995 to 27-Dec-2001*

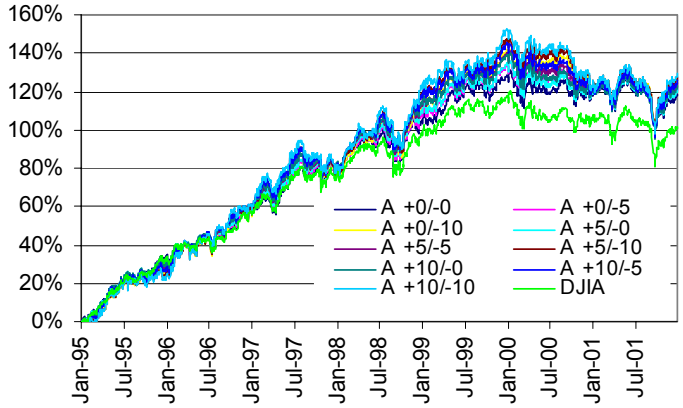
An important observation is that, also in this case, tracking wider spreads does not improve the performance of the long-short strategy, mainly due to an increase in volatility that is penalised by the Sharpe ratios.

c. Transport of alpha to the market index

The previous two combinations of strategies were market neutral. In case market neutrality is not a requirement and an exposure to a market index is desired, another possibility, mentioned in the preamble of our paper, would be to transport the alpha gained in the long-short market neutral framework to an index, through the use of derivatives (e.g. index futures). Or, instead of derivatives, the enhanced cointegration-based index tracking procedure can be implemented.

To illustrate this, we have examined the returns of a strategy combining the simple index tracking (where also, instead of index tracking, a 'plus' benchmark can be tracked) with the long-short market neutral strategy.

Figure 18 Cumulative returns on the alpha transport strategy



LSMN	-\+	0%	5%	10%
total return	0	118.48%	121.42%	125.31%
annual volatility		17.72%	17.97%	19.21%
correlation		0.98	0.97	0.92
skewness		-0.41	-0.37	-0.28
excess kurtosis		4.39	3.85	2.74
Sharpe ratio		0.66	0.67	0.66
total return	5%	120.92%	123.87%	127.76%
annual volatility		17.78%	18.46%	20.18%
correlation		0.98	0.95	0.88
skewness		-0.39	-0.32	-0.22
excess kurtosis		4.04	3.20	2.09
Sharpe ratio		0.68	0.67	0.64
total return	10%	122.89%	125.84%	129.73%
annual volatility		18.18%	19.21%	21.27%
correlation		0.96	0.92	0.83
skewness		-0.34	-0.26	-0.18
excess kurtosis		3.46	2.56	1.61
Sharpe ratio		0.68	0.66	0.62

** Period 1-Jan-1995 to 27-Dec-2001*

The cumulative returns and statistical characteristics of the alpha transport strategy are reported in Figure 18. As expected, the strategy returns have the proper correlation with the market index, while gaining alpha from two sources: excess return in index tracking and the double alpha from the long-short strategies. The returns of these strategies are slightly more volatile than the market index, but have significantly better Sharpe ratios (around 0.65 as compared with 0.54). Again, tracking wider spreads in the long-short strategies does not improve their overall performance. The Sharpe ratios for this strategy after accounting for repo costs are reported in Appendix 13.

This was only an example of how alpha gained in the long-short market neutral strategies may be transported to an index in the context of cointegration-based trading strategies, but the range of possible destinations is much wider.

5. Conclusions

The main point of our analysis was to show that, when applied to constructing trading strategies, the cointegration technique produces encouraging results. Its key characteristics, i.e. mean reverting tracking error, enhanced weights stability and better use of the information contained in stock prices, allow a flexible design of various trading strategies, from index tracking to long-short market neutral.

Our main results, while demonstrating how cointegration works and validating its applicability to the construction of trading strategies, can be summarised as follows:

A. Index tracking

- Appropriate replicas can be constructed for the market index, as well as for artificial benchmarks linearly over-performing and under-performing the market index, provided that a minimum number of stocks is included in the tracking portfolio and an appropriate calibration period is used;

- During sharp market declines, such as the Russian crisis and the burst of the technology bubble, the cointegration-based tracking strategies have significantly over-performed the market index;

B. Long-short market neutral strategies

- Long-short market neutral strategies can be set up to track different spreads. Most of them displayed much lower volatility than the market, no correlation with the latter, normal returns and sometimes better Sharpe ratios than the market index;

- The spread between the benchmarks tracked can only be increased at the expense of increasing the volatility and eventually reducing the Sharpe ratios. The most consistent positive returns, with low volatility and no significant correlation with the market are generated by strategies tracking narrow spreads between the 'plus' and the 'minus' benchmarks;

- Special attention should be given to the amount of trades necessary to rebalance the portfolios. If not controlled for, the transaction costs may erode the performance of the long-short strategies;

- In terms of returns, similar performance to hedge funds indexes can be obtained by adding a leverage of between 2 and 4 to our long-short strategies; the cointegration long-short equity returns, even if more volatile than the index, have a very low correlation with the market returns. In fact, this correlation is significantly lower than the market correlations of the major market neutral hedge fund indexes;

C. Combinations of index tracking and long-short market neutral strategies

- We have shown, using 'fund of fund' and 'portable alpha' techniques, how the characteristics of the individual index tracking and long-short market neutral strategies can be significantly improved by combining them to create market neutral or enhanced index tracking strategies.

Still, our results are based on rather rudimentary stock selection and rebalancing rules, and conservative transaction and repo costs. The refinement of the strategy in these respects is likely to produce more attractive outcomes.

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Appendix 1
Stocks comprised in DJIA as of 31-Dec-01 (ordered ascendingly according to their weight in DJIA)

No.	Company	Ticker	Weight in DJIA
1	International Business Machines Corp.	IBM	8.35%
2	Minnesota Mining & Manufacturing Co.	MMM	8.16%
3	Procter & Gamble Co.	PG	5.46%
4	Microsoft Corp.	MSFT	4.57%
5	United Technologies Corp.	UTX	4.46%
6	Johnson & Johnson	JNJ	4.08%
7	Merck & Co. Inc.	MRK	4.06%
8	Wal-Mart Stores Inc.	WMT	3.97%
9	Caterpillar Inc.	CAT	3.61%
10	Home Depot Inc.	HD	3.52%
11	Citigroup Inc.	C	3.49%
12	General Motors Corp.	GM	3.36%
13	Coca-Cola Co.	KO	3.26%
14	Philip Morris Cos.	MO	3.17%
15	E.I. DuPont de Nemours & Co.	DD	2.94%
16	International Paper Co.	IP	2.79%
17	General Electric Co.	GE	2.77%
18	Exxon Mobil Corp.	XOM	2.71%
19	SBC Communications Inc.	SBC	2.70%
20	Boeing Co.	BA	2.68%
21	J.P. Morgan Chase & Co.	JPM	2.51%
22	American Express Co.	AXP	2.46%
23	Alcoa Inc.	AA	2.45%
24	Honeywell International Inc.	HON	2.34%
25	Intel Corp.	INTC	2.17%
26	Eastman Kodak Co.	EK	2.03%
27	McDonald's Corp.	MCD	1.83%
28	Walt Disney Co.	DIS	1.43%
29	Hewlett-Packard Co.	HWP	1.42%
30	AT&T Corp.	T	1.25%

Appendix 2
Statistical features of the log prices series

Log Returns	**DJIA**	**IBM**	**MMM**	**PG**	**MSFT**	**UTX**	**JNJ**	**MRK**	**WMT**	**CAT**	**HD**	**C**	**GM**	**KO**	**MO**	**DD**
Mean	0.00048	0.00053	0.00047	0.00050	0.00132	0.00060	0.00075	0.00058	0.00078	0.00050	0.00110	0.00101	0.00024	0.00057	0.00056	0.00036
Median	0.00063	0.00000	0.00000	0.00000	0.00029	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Maximum	0.05014	0.12366	0.10500	0.09097	0.17869	0.08291	0.07591	0.09165	0.08736	0.10296	0.12128	0.16838	0.07411	0.09360	0.14842	0.08376
Minimum	-0.07284	-0.16892	-0.10078	-0.36005	-0.16958	-0.30290	-0.10189	-0.09860	-0.10260	-0.12968	-0.33878	-0.11522	-0.14540	-0.11072	-0.26139	-0.10951
Std. Dev.	0.01003	0.02062	0.01519	0.01775	0.02363	0.01831	0.01609	0.01762	0.02046	0.02058	0.02280	0.02259	0.02004	0.01687	0.02002	0.01825
Skewness	-0.42093	-0.03391	0.09293	-2.87328	-0.04879	-1.56556	0.03495	-0.07874	0.03466	-0.04719	-1.03845	0.12073	-0.05265	0.02766	-0.75864	0.01622
Excess Kurtosis	4.73941	6.32342	3.70530	58.14641	3.82020	26.79062	1.61517	2.48255	1.91244	3.17316	17.72075	2.71957	1.88005	2.98447	15.50463	2.51612
Log Prices	**DJIA**	**IBM**	**MMM**	**PG**	**MSFT**	**UTX**	**JNJ**	**MRK**	**WMT**	**CAT**	**HD**	**C**	**GM**	**KO**	**MO**	**DD**
ADF test statistic level	-0.7894	0.0641	-0.7168	-1.1574	-1.6119	-0.3979	-0.5307	-1.1718	-0.7334	-0.8724	-1.6576	-0.4402	-1.3790	-1.9514	-1.2643	-1.3596
ADF test statistic 1st dif	-25.7802	-25.1024	-26.2018	-26.3229	-25.3279	-26.1123	-26.2340	-25.2362	-27.2319	-25.6927	-26.3996	-25.5707	-25.9069	-25.5096	-25.3160	-25.9636

Log Returns		**IP**	**GE**	**XOM**	**SBC**	**BA**	**JPM**	**AXP**	**AA**	**HON**	**INTC**	**EK**	**MCD**	**DIS**	**HWP**	**T**
Mean		0.00021	0.00066	0.00040	0.00043	0.00022	0.00057	0.00050	0.00052	0.00053	0.00110	0.00009	0.00037	0.00028	0.00040	-0.00008
Median		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00087	0.00000	0.00000	0.00000	0.00000	0.00000
Maximum		0.11233	0.11743	0.09243	0.10668	0.11000	0.14756	0.12018	0.13152	0.24860	0.18335	0.11212	0.10310	0.14203	0.15946	0.12386
Minimum		-0.11047	-0.11287	-0.07672	-0.13538	-0.19389	-0.12403	-0.14614	-0.11660	-0.19076	-0.24889	-0.28205	-0.10763	-0.20289	-0.20014	-0.29541
Std. Dev.		0.01926	0.01648	0.01398	0.01748	0.02012	0.02317	0.02206	0.02038	0.02139	0.02809	0.01920	0.01709	0.01998	0.02707	0.02077
Skewness		0.17372	-0.02005	0.15008	-0.07924	-0.64679	0.21881	-0.05145	0.30421	-0.04838	-0.33219	-1.45522	0.05251	-0.22271	-0.16494	-0.87169
Excess Kurtosis		2.82111	3.84247	1.80678	3.47744	8.78695	2.93187	2.43451	3.15468	13.31997	4.67986	21.23022	2.90969	8.22673	5.36547	18.53652
Log Prices		**IP**	**GE**	**XOM**	**SBC**	**BA**	**JPM**	**AXP**	**AA**	**HON**	**INTC**	**EK**	**MCD**	**DIS**	**HWP**	**T**
ADF test statistic level		-2.1910	-0.4111	-0.6201	-1.0114	-1.5366	-0.9265	-0.4873	-0.3749	-1.4228	-1.1055	-1.6145	-1.3118	-1.5149	-1.3511	-1.8608
ADF test statistic 1st dif		-25.6453	-26.5687	-28.8440	-27.2179	-25.7908	-23.8322	-26.6003	-25.2648	-25.0522	-25.7110	-24.2476	-25.8487	-25.8169	-26.2829	-25.1690

*MacKinnon critical values for rejection of hypothesis of a unit root.

1% Critical Value*	-3.4356
5% Critical Value	-2.8630
10% Critical Value	-2.5676

Appendix 3
Summary results for tracking portfolios

DJIA		
Average annual return		14.66%
Annual volatility		17.54%
Skewness		-0.53
Excess kurtosis		4.51
95% one sided confidence interval	0.00%	37.13%

p.a.

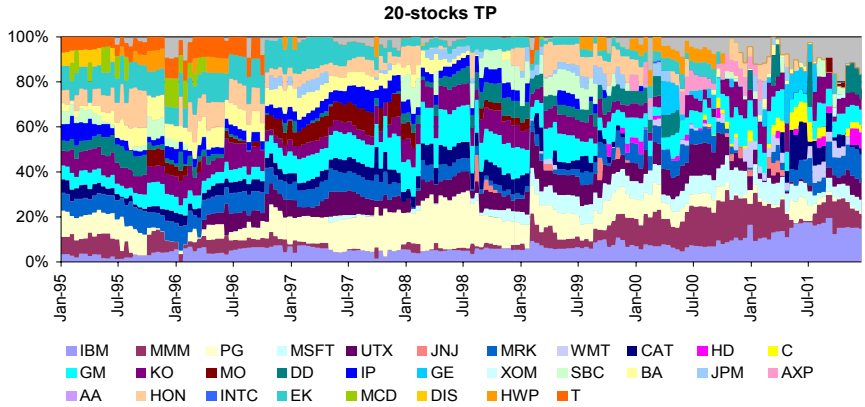
	Tracking portfolios - stock selection method RD						Tracking portfolios - stock selection method F1					
	No transaction costs			Transaction costs included at 0.2%			No transaction costs			Transaction costs included at 0.2%		
3-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-2.25%	0.31%	1.13%	-3.40%	-0.42%	0.81%	-1.29%	0.03%	1.13%	-1.76%	-0.39%	0.81%
Annual volatility of excess returns	4.31%	3.09%	2.12%	4.31%	3.09%	2.12%	4.11%	2.83%	2.12%	4.11%	2.83%	2.12%
Correlation with DJIA returns	0.97	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99
Skewness	0.29	0.29	0.54	0.27	0.27	0.54	-0.04	0.24	0.54	-0.04	0.24	0.54
Excess kurtosis	4.03	3.90	4.64	4.01	3.91	4.64	1.77	2.96	4.64	1.76	2.95	4.64
ADF	-6.60	-7.08	-7.47	-6.60	-7.08	-7.47	-6.59	-6.85	-7.47	-6.59	-6.85	-7.47
4-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-2.15%	0.25%	1.13%	-3.31%	-0.48%	0.84%	-1.46%	-0.09%	1.13%	-1.93%	-0.50%	0.84%
Annual volatility of excess returns	4.60%	3.39%	2.30%	4.60%	3.39%	2.30%	4.30%	3.18%	2.30%	4.30%	3.18%	2.30%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99
Skewness	0.09	0.20	0.30	0.07	0.19	0.29	0.01	0.12	0.30	0.00	0.12	0.29
Excess kurtosis	3.04	3.70	3.40	3.03	3.71	3.40	1.43	3.26	3.40	1.43	3.25	3.40
ADF	-6.79	-7.20	-7.61	-6.79	-7.20	-7.61	-6.85	-7.09	-7.61	-6.85	-7.09	-7.61
5-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-2.04%	-0.01%	0.98%	-3.21%	-0.74%	0.69%	-1.55%	0.06%	0.98%	-2.02%	-0.36%	0.69%
Annual volatility of excess returns	4.60%	3.60%	2.52%	4.60%	3.60%	2.52%	4.51%	3.50%	2.52%	4.51%	3.50%	2.52%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99
Skewness	0.06	0.16	0.29	0.04	0.14	0.29	0.10	0.05	0.29	0.09	0.04	0.28
Excess kurtosis	3.29	5.65	4.78	3.30	5.66	4.78	1.44	4.89	4.78	1.44	4.87	4.77
ADF	-6.75	-7.34	-7.87	-6.75	-7.34	-7.87	-6.90	-7.29	-7.87	-6.90	-7.29	-7.87

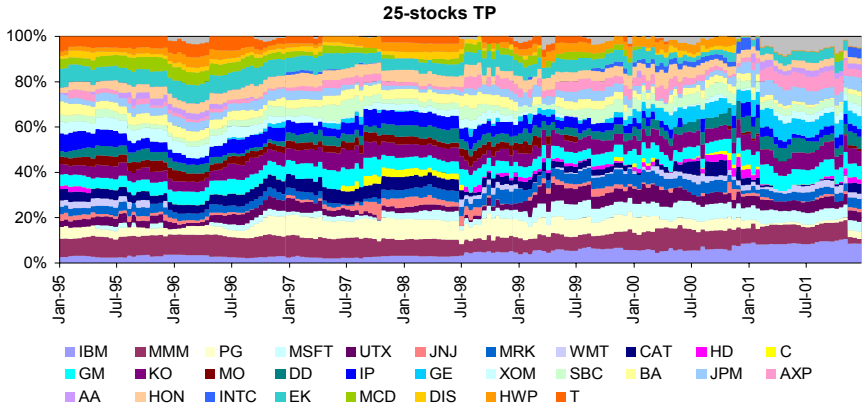
	Tracking portfolios - stock selection method RSA						Tracking portfolios - stock selection method F3					
	No transaction costs			Transaction costs included at 0.2%			No transaction costs			Transaction costs included at 0.2%		
3-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-1.00%	-0.55%	1.13%	-1.51%	-0.96%	0.81%	-1.58%	-0.17%	1.13%	-2.02%	-0.52%	0.81%
Annual volatility of excess returns	4.48%	3.05%	2.12%	4.48%	3.05%	2.12%	4.12%	2.82%	2.12%	4.12%	2.82%	2.12%
Correlation with DJIA returns	0.97	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99
Skewness	0.18	0.12	0.54	0.17	0.11	0.54	0.02	0.50	0.54	0.01	0.49	0.54
Excess kurtosis	3.30	2.75	4.64	3.27	2.75	4.64	3.07	3.92	4.64	3.07	3.92	4.64
ADF	-6.70	-6.99	-7.47	-6.70	-6.99	-7.47	-6.76	-6.80	-7.47	-6.76	-6.80	-7.47
4-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-0.97%	-0.92%	1.13%	-1.45%	-1.32%	0.84%	-2.48%	-0.41%	1.13%	-2.90%	-0.73%	0.84%
Annual volatility of excess returns	4.62%	3.35%	2.30%	4.62%	3.35%	2.30%	4.39%	3.08%	2.30%	4.39%	3.08%	2.30%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99
Skewness	0.09	0.04	0.30	0.08	0.03	0.29	-0.01	0.28	0.30	-0.01	0.28	0.29
Excess kurtosis	2.50	3.09	3.40	2.48	3.08	3.40	3.08	2.94	3.40	3.08	2.94	3.40
ADF	-6.74	-7.17	-7.61	-6.74	-7.17	-7.61	-6.81	-7.09	-7.61	-6.81	-7.09	-7.61
5-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-0.95%	-0.74%	0.98%	-1.42%	-1.13%	0.69%	-2.39%	-0.64%	0.98%	-2.80%	-0.96%	0.69%
Annual volatility of excess returns	4.71%	3.59%	2.52%	4.71%	3.59%	2.52%	4.66%	3.23%	2.52%	4.66%	3.23%	2.52%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99	0.97	0.98	0.99
Skewness	0.10	0.06	0.29	0.09	0.06	0.29	- 0.10	0.10	0.29	-0.10	0.09	0.29
Excess kurtosis	2.81	4.81	4.78	2.80	4.79	4.78	3.71	2.80	4.78	3.71	2.80	4.78
ADF	-6.75	-7.21	-7.87	-6.75	-7.21	-7.87	-6.93	-7.36	-7.87	-6.93	-7.36	-7.87

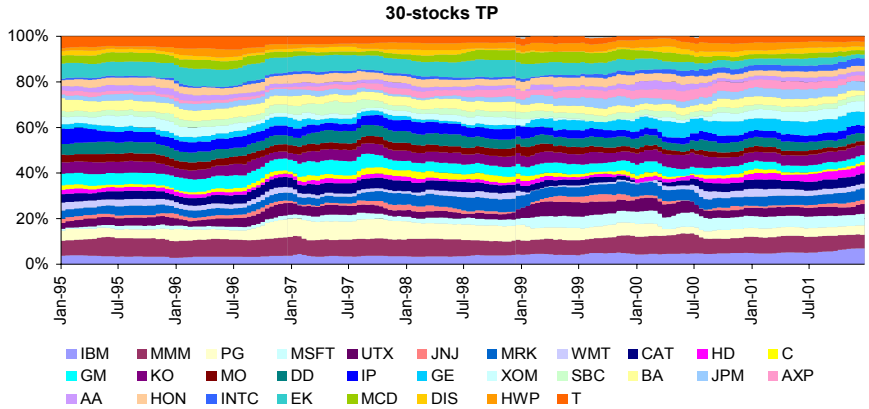
	Tracking portfolios - stock selection method RA						Tracking portfolios - stock selection method F5					
	No transaction costs			Transaction costs included at 0.2%			No transaction costs			Transaction costs included at 0.2%		
3-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-0.58%	-0.43%	1.13%	-1.00%	-0.82%	0.81%	-2.27%	0.00%	1.13%	-2.67%	-0.35%	0.81%
Annual volatility of excess returns	4.13%	3.04%	2.12%	4.13%	3.04%	2.12%	4.77%	2.99%	2.12%	4.77%	2.99%	2.12%
Correlation with DJIA returns	0.97	0.99	0.99	0.97	0.99	0.99	0.96	0.99	0.99	0.96	0.99	0.99
Skewness	0.14	0.25	0.54	0.13	0.24	0.54	0.01	0.33	0.54	0.01	0.32	0.54
Excess kurtosis	3.43	2.90	4.64	3.44	2.90	4.64	3.21	3.04	4.64	3.21	3.04	4.64
ADF	-6.60	-6.73	-7.47	-6.60	-6.73	-7.47	-6.61	-6.98	-7.47	-6.61	-6.98	-7.47
4-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-0.92%	-0.79%	1.13%	-1.31%	-1.16%	0.84%	-3.21%	-0.30%	1.13%	-3.57%	-0.62%	0.84%
Annual volatility of excess returns	4.10%	3.32%	2.30%	4.10%	3.32%	2.30%	4.91%	3.20%	2.30%	4.91%	3.20%	2.30%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.96	0.98	0.99	0.96	0.98	0.99
Skewness	0.02	0.18	0.30	0.02	0.17	0.29	0.06	0.19	0.30	0.06	0.19	0.29
Excess kurtosis	2.01	3.03	3.40	2.01	3.02	3.40	2.91	2.69	3.40	2.91	2.69	3.40
ADF	-6.59	-6.92	-7.61	-6.59	-6.92	-7.61	-6.95	-7.11	-7.61	-6.95	-7.11	-7.61
5-years	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**	**20 stocks**	**25 stocks**	**30 stocks**
Average annual excess return	-1.00%	-0.39%	0.98%	-1.39%	-0.75%	0.69%	-3.30%	-0.93%	0.98%	-3.64%	-1.25%	0.69%
Annual volatility of excess returns	4.42%	3.54%	2.52%	4.42%	3.54%	2.52%	5.07%	3.35%	2.52%	5.07%	3.35%	2.52%
Correlation with DJIA returns	0.97	0.98	0.99	0.97	0.98	0.99	0.96	0.98	0.99	0.96	0.98	0.99
Skewness	0.13	0.13	0.29	0.13	0.12	0.29	0.06	0.03	0.29	0.06	0.02	0.29
Excess kurtosis	3.07	4.71	4.78	3.08	4.71	4.78	2.92	2.64	4.78	2.92	2.64	4.78
ADF	-6.76	-7.17	-7.87	-6.76	-7.17	-7.87	-7.07	-7.30	-7.87	-7.07	-7.30	-7.87

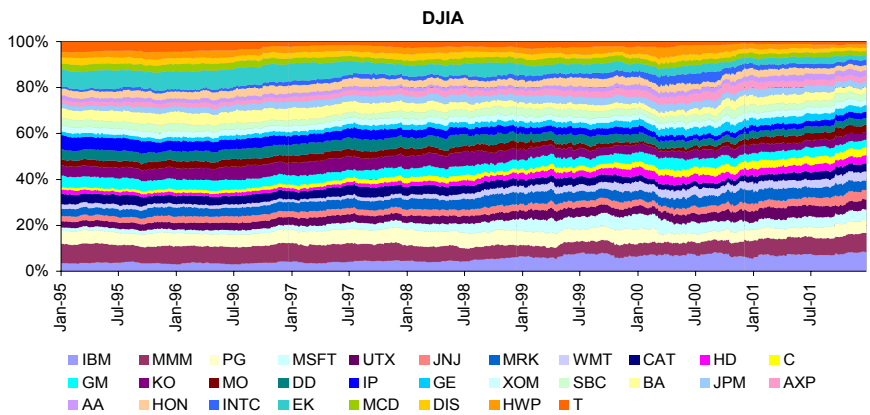
Period 1-Jan-95 to 31-Dec-01

Appendix 4
Stock weights in DJIA and some tracking portfolios









Appendix 5
Correlation of the tracking errors with the market returns

RD	20 stocks	25 stocks	30 stocks
3 years	0.01	0.07	0.01
4 years	0.02	0.07	-0.01
5 years	0.01	0.07	0.00

RSA	20 stocks	25 stocks	30 stocks
3 years	0.01	0.07	0.01
4 years	-0.01	0.06	-0.01
5 years	0.00	0.07	0.00

RA	20 stocks	25 stocks	30 stocks
3 years	0.08	0.06	0.01
4 years	0.05	0.04	-0.01
5 years	0.08	0.04	0.00

F1	20 stocks	25 stocks	30 stocks
3 years	-0.04	0.03	0.01
4 years	-0.06	0.03	-0.01
5 years	-0.04	0.03	0.00

F3	20 stocks	25 stocks	30 stocks
3 years	0.04	0.02	0.01
4 years	-0.01	-0.01	-0.01
5 years	0.01	-0.02	0.00

F5	20 stocks	25 stocks	30 stocks
3 years	-0.14	-0.03	0.01
4 years	-0.16	-0.05	-0.01
5 years	-0.17	-0.04	0.00

Appendix 6

Sharpe ratios for the tracking portfolios

avg interest rate	5.26%
DJIA	0.54

Transaction costs included at 0.2%

RD	20 stocks	25 stocks	30 stocks
3 years	0.33	0.49	0.57
4 years	0.33	0.49	0.57
5 years	0.34	0.47	0.56

RSA	20 stocks	25 stocks	30 stocks
3 years	0.43	0.46	0.57
4 years	0.43	0.44	0.57
5 years	0.43	0.45	0.56

RA	20 stocks	25 stocks	30 stocks
3 years	0.45	0.47	0.57
4 years	0.44	0.45	0.57
5 years	0.43	0.47	0.56

F1	20 stocks	25 stocks	30 stocks
3 years	0.42	0.50	0.57
4 years	0.41	0.49	0.57
5 years	0.41	0.50	0.56

F3	20 stocks	25 stocks	30 stocks
3 years	0.40	0.49	0.57
4 years	0.36	0.48	0.57
5 years	0.36	0.47	0.56

F5	20 stocks	25 stocks	30 stocks
3 years	0.38	0.51	0.57
4 years	0.33	0.49	0.57
5 years	0.33	0.45	0.56

Information ratios for the tracking portfolios

avg interest rate	5.26%
DJIA	0.84

Transaction costs included at 0.2%

RD	20 stocks	25 stocks	30 stocks
3 years	0.62	0.78	0.87
4 years	0.62	0.78	0.87
5 years	0.63	0.76	0.86

RSA	20 stocks	25 stocks	30 stocks
3 years	0.72	0.75	0.87
4 years	0.73	0.73	0.87
5 years	0.72	0.74	0.86

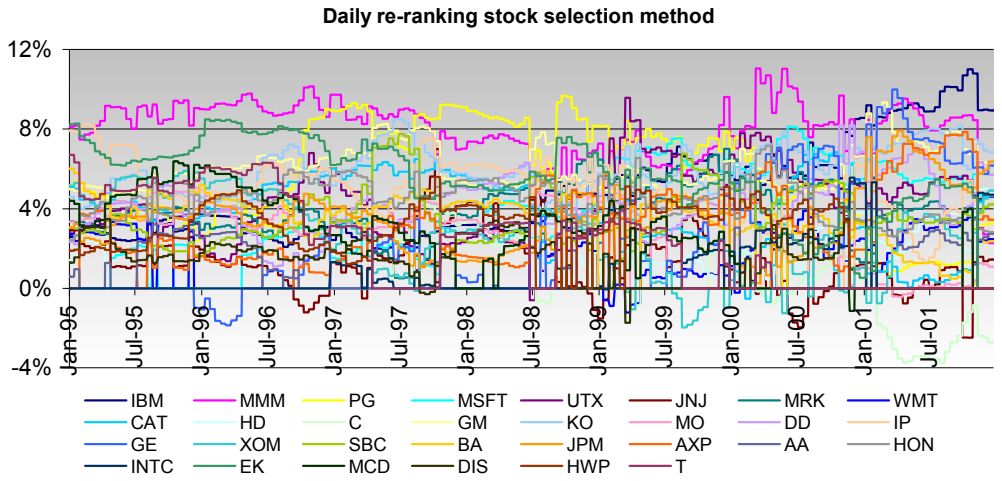
RA	20 stocks	25 stocks	30 stocks
3 years	0.74	0.77	0.87
4 years	0.73	0.75	0.87
5 years	0.72	0.77	0.86

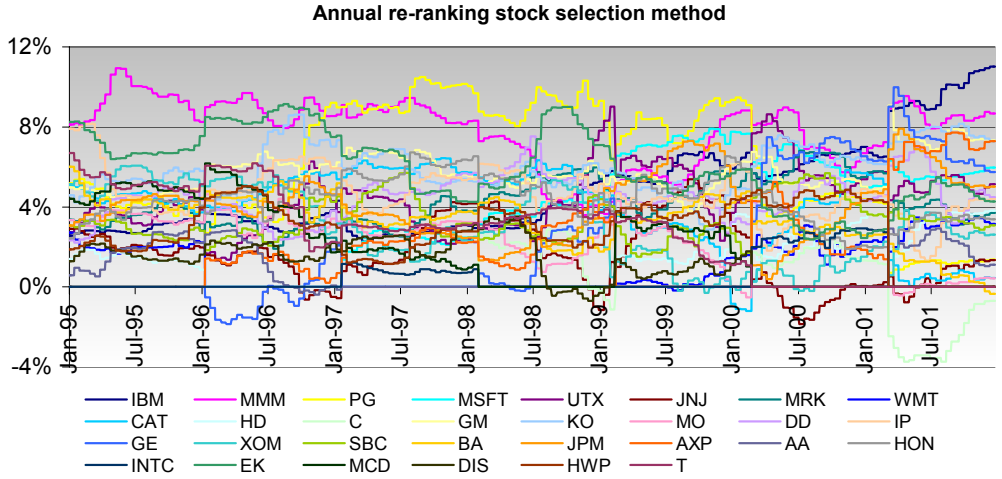
F1	20 stocks	25 stocks	30 stocks
3 years	0.72	0.79	0.87
4 years	0.71	0.79	0.87
5 years	0.70	0.79	0.86

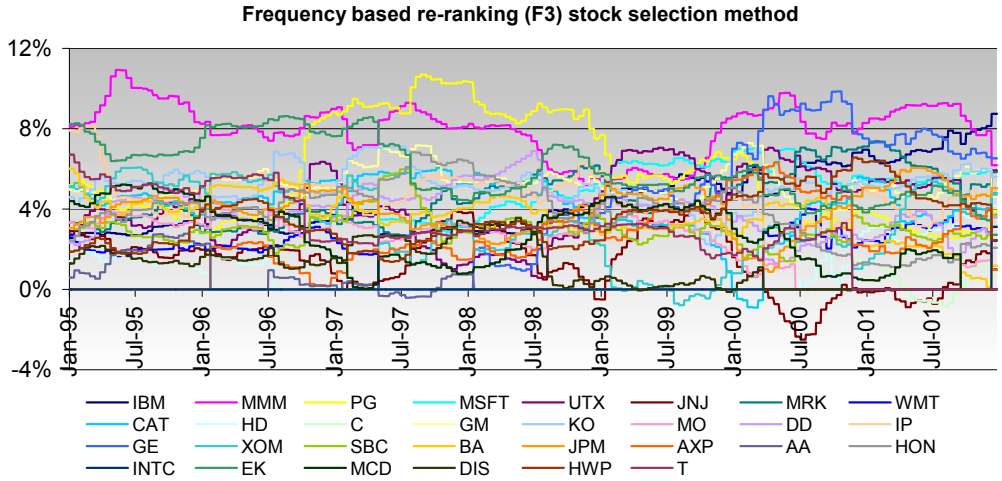
F3	20 stocks	25 stocks	30 stocks
3 years	0.69	0.79	0.87
4 years	0.65	0.78	0.87
5 years	0.65	0.77	0.86

F5	20 stocks	25 stocks	30 stocks
3 years	0.68	0.80	0.87
4 years	0.63	0.79	0.87
5 years	0.63	0.75	0.86

Appendix 7
Stock weights in the 25-stocks tracking portfolios constructed with different stock selection methods







Appendix 8
Stock selection strategies - closest to the benchmark, highest and lowest return

Transaction costs included at 0.2%

Closest tracking portfolio

3-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RSA	RA	F5
Average annual excess return	-0.16%	-1.00%	-0.35%
4-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RA	RA	RD
Average annual excess return	-2.20%	-1.31%	-0.48%
5-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RA	RA	F1
Average annual excess return	-2.09%	-1.39%	-0.36%

Highest return tracking portfolio

3-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RSA	RA	F3
Average annual excess return	-0.16%	-1.00%	-0.35%
4-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RA	RA	RD
Average annual excess return	-2.20%	-1.31%	-0.48%
5-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RA	RA	F1
Average annual excess return	-2.09%	-1.39%	-0.36%

Lowest return tracking portfolio

3-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RD	RD	RSA
Average annual excess return	-3.47%	-3.40%	-0.96%
4-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RD	F5	RSA
Average annual excess return	-4.62%	-3.57%	-1.32%
5-years	**15 stocks**	**20 stocks**	**25 stocks**
Strategy	RD	F5	F5
Average annual excess return	-4.96%	-3.64%	-1.25%

Appendix 9
Some stories about cointegration

a. UTX

United Technologies Corporation (UTX) is a well-diversified business with a global market presence. During the years 1995-1997 UTX experienced a steady increase in price while being with few exceptions over-weighted in the tracking portfolio as compared to the market index. Again the stock become strongly over-weighted in the tracking portfolio at the beginning of year 1999 and stayed there until mid-2000. In this latter case, the excess return came from the large increase in the cointegration coefficient of UTX, prior to the technology crash in 2000.

b. CAT

Caterpillar, a leading provider of construction and mining equipment, was significantly over-weighted in the tracking portfolio during the period 1996-1998. This period follows a series of tranquil bull years for both the market and this particular stock. However, the company's worldwide business was heavily affected by the global economic slowdown that followed the Asian and Russian crises. The end of 1998 was a volatile period for CAT, which led to a decrease in its cointegration coefficient. The stock became significantly under-weighted in the tracking portfolio further to a large price increase in January 1999 generated by improved profit expectations on highly volatile conditions. The stock continued to fall throughout the entire year 1999, its significant under-weighting generating excess returns for the tracking portfolio. Partly due to continuous price decrease, its relative underweight decreased, and when the stock started to rise again, it was already over-weighted in the tracking portfolio, generating excess returns throughout the second half of the year 2000 and the entire year 2001.

c. GM

General Motors, the world largest vehicle manufacturer, also has a special behaviour in the tracking portfolio weights. Until the end of 1997, it was significantly over-weighted in the tracking portfolio, due to its price pattern being very similar to the market index. During this time it experienced a steady increase, which generated excess returns for the tracking portfolio. From 1998, its cointegration weights remained quite steady, all significant changes in the relative over-weighting being the result of changes in the market price. For example, the sharp decrease of the stock price in May 2000, due to a combination of bad news regarding macro-economic and stock specific facts, has generated a significant decrease of its under-weighting in the tracking portfolio. Again, over the entire back-test period, GM weighting behaviour generated excess returns for the tracking portfolio.

d. GE

General Electric, a considerably more diversified corporation than GM, has had a different behaviour in the cointegration weights, and still generated excess returns for the tracking portfolio. As opposed to GM, GE stock prices were notably less volatile during the period under examination. The stock experienced a rather steady increase throughout the entire back-test period, and was continuously over-weighted in the tracking portfolio, with the singular exception of year 1998. Its overweight is due to a price behaviour very similar to the benchmark, which is the result of having a well-diversified business presence.

e. MCD

McDonalds is another successful story of the cointegration-based tracking. Its evolution during the years 1995-1998 was steady and highly similar to the market index. As a result, it was on average over-weighted, more significantly after 1998, which generated excess returns for the tracking portfolio, on an increasing trend of its price.

f. MMM

Minnesota Mining and Manufacturing is another fairly diversified business with a significant worldwide presence. It is considered to be one of the traditional stocks in the DJIA. Its weight in the

tracking portfolio increased relative to the market index weight only at the beginning of 1998, further to a consistent over-performance of the market during the year 1997. On the background of a rather volatile stock price evolution during the years 1998-2000, MMM remained significantly overweight in the tracking portfolio, all subsequent changes being induced mainly by the price evolution than by changes in the actual cointegration weights. During the bursting of the technology bubble in 2000, the price of MMM rose, relative to the index. Our portfolio's overweight in this stock therefore generated considerable excess return during this period.

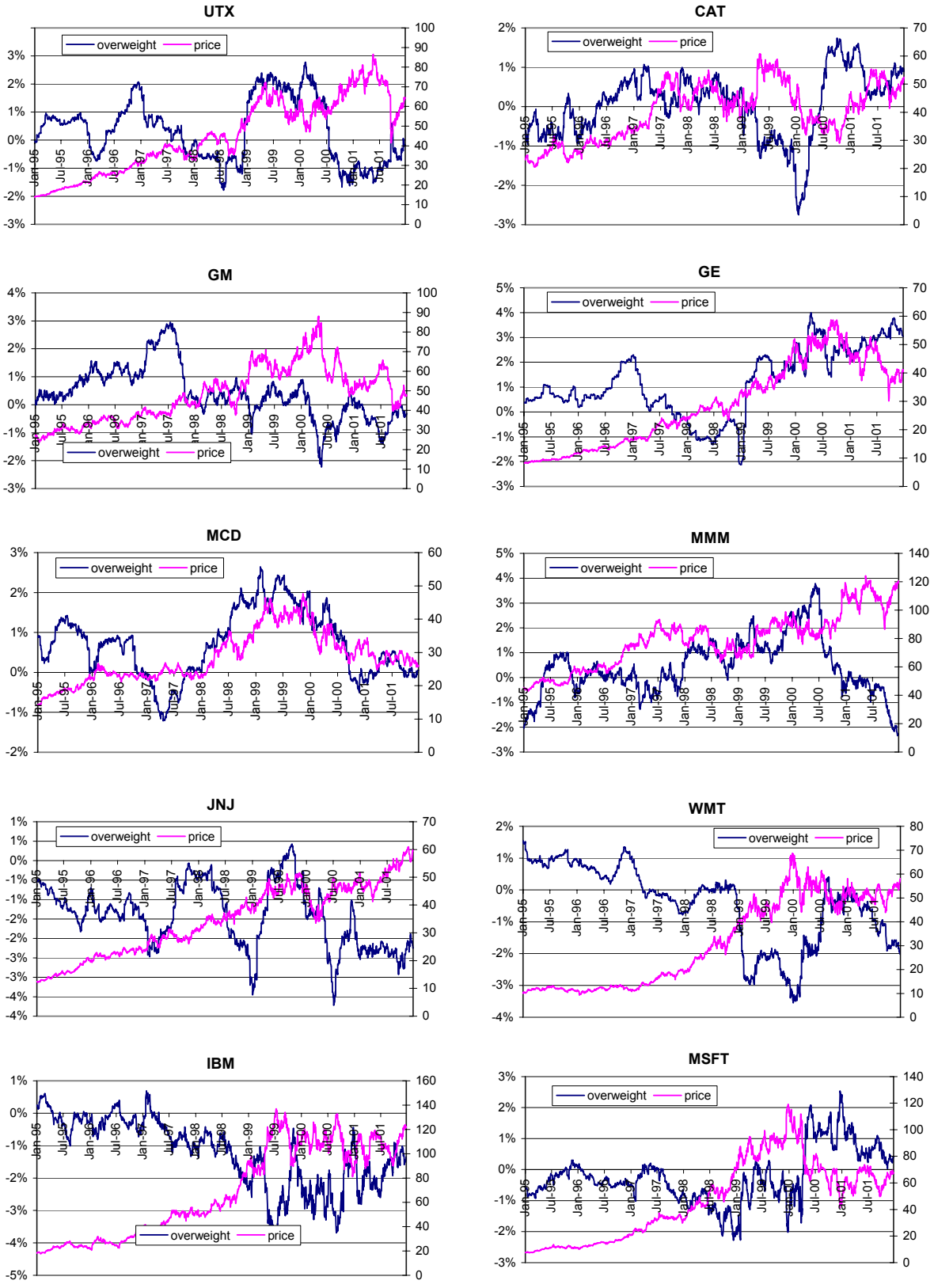
g. IBM and MSFT

Leading technology stocks, IBM and MSFT have both experienced strong upward trends at the beginning of 1998, which resulted in their relative under-weighting in the tracking portfolios as compared to the benchmark. The symmetric but opposite movements of stock prices/relative weights which can be easily detected on the graphs are the direct effect of price changes on the stock weights in the market index, while the tracking portfolio weights remain relatively constant. Therefore, while having the cointegration weight stable and the price increasing, these technology stocks were under-weighted in the tracking portfolios during the entire technology bubble, which generated relative losses. However, during the burst of the technology bubble, the under-weighting of these stocks in the tracking portfolio generated excess returns. Considering that the cointegration weights of the technology stocks remained stable over time, it can be assessed that the strategy treated the technology bubble as a temporary disequilibria, which had no long-term effects on the cointegration relationship.

However, the mean reversion does not hold for every stock and, if the tracking portfolio contained enough such stocks it could, in theory, under-perform the benchmark. Mean reversion does not occur when a particular stock is subject to a strong and persistent trend that affects its cointegration with the system and disables the mean reversion, that is, when the price disequilibria in not temporary, but determined by exogenous factors.

For example, Johnson & Johnson and Wal-Mart Stores are two stocks that generated relative losses for the tracking portfolio due to their under-weighting in the tracking portfolio during sustained up-trending price periods. Both stocks have experienced a steady increase in their prices even during bear markets, which has affected their cointegration with the benchmark. Therefore, their cointegration coefficients have decreased after including year 1998 in the calibration period. Moreover, the steady increase in price has affected their weights in the market index, and therefore the relative under-weighting in the tracking portfolios has increased.

Appendix 9 (cont'd)
Some stock weights in the tracking portfolios



Appendix 10
Summary results on long-short strategies – daily re-ranking stock selection method

calibration period: 3 years

20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-7.22%	-12.78%	-17.16%
annual volatility		0.00%	3.09%	5.68%	7.91%
correlation	0	NA	0.11	0.11	0.11
skewness		NA	-0.08	-0.07	-0.07
excess kurtosis		NA	1.19	1.28	1.39
total return		-9.66%	-16.86%	-22.41%	-26.75%
annual volatility		3.85%	6.93%	9.50%	11.71%
correlation	5%	0.11	0.11	0.11	0.11
skewness		-0.08	-0.08	-0.08	-0.07
excess kurtosis		1.13	1.11	1.15	1.20
total return		-23.41%	-30.56%	-36.05%	-40.34%
annual volatility		8.99%	12.04%	14.57%	16.75%
correlation	10%	0.10	0.10	0.11	0.11
skewness		-0.06	-0.07	-0.07	-0.07
excess kurtosis		1.36	1.19	1.12	1.10
total return		-46.16%	-53.21%	-58.61%	-62.80%
annual volatility		17.37%	20.27%	22.68%	24.74%
correlation	15%	0.10	0.10	0.10	0.10
skewness		0.01	-0.01	-0.02	-0.03
excess kurtosis		2.91	2.22	1.82	1.56

calibration period: 4 years

20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-5.92%	-10.61%	-14.50%
annual volatility		0.00%	3.27%	5.84%	7.94%
correlation	0	NA	0.00	0.00	0.00
skewness		NA	0.03	0.02	0.01
excess kurtosis		NA	1.57	1.66	1.73
total return		-8.14%	-14.05%	-18.71%	-22.57%
annual volatility		4.38%	7.63%	10.20%	12.28%
correlation	5%	0.00	0.00	0.00	0.00
skewness		0.04	0.03	0.03	0.03
excess kurtosis		1.39	1.43	1.49	1.55
total return		-21.30%	-27.15%	-31.76%	-35.56%
annual volatility		10.88%	14.09%	16.61%	18.65%
correlation	10%	0.00	0.00	0.00	0.00
skewness		0.04	0.04	0.04	0.04
excess kurtosis		1.54	1.42	1.38	1.37
total return		-52.29%	-58.02%	-62.50%	-66.18%
annual volatility		23.89%	26.88%	29.22%	31.11%
correlation	15%	0.00	0.00	0.00	0.00
skewness		0.10	0.09	0.08	0.08
excess kurtosis		4.31	3.42	2.88	2.53

calibration period: 5 years

20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-4.58%	-8.05%	-10.77%
annual volatility		0.00%	3.15%	5.57%	7.49%
correlation	0	NA	-0.01	-0.01	-0.01
skewness		NA	-0.02	-0.02	-0.01
excess kurtosis		NA	1.37	1.47	1.57
total return		-6.41%	-10.99%	-14.44%	-17.14%
annual volatility		4.38%	7.52%	9.93%	11.84%
correlation	5%	-0.01	-0.01	-0.01	-0.01
skewness		-0.01	-0.02	-0.02	-0.02
excess kurtosis		1.25	1.26	1.30	1.36
total return		-16.39%	-20.92%	-24.33%	-27.00%
annual volatility		11.15%	14.26%	16.63%	18.51%
correlation	10%	-0.01	-0.01	-0.01	-0.01
skewness		0.00	0.00	-0.01	-0.01
excess kurtosis		1.49	1.34	1.28	1.26
total return		-37.28%	-41.71%	-45.02%	-47.60%
annual volatility		25.10%	28.04%	30.27%	32.03%
correlation	15%	-0.01	-0.01	-0.01	-0.01
skewness		-0.01	-0.01	-0.01	-0.01
excess kurtosis		3.48	2.90	2.53	2.28

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-4.81%	-8.76%	-12.03%
annual volatility		0.00%	2.91%	5.38%	7.51%
correlation	0	NA	0.03	0.03	0.03
skewness		NA	-0.13	-0.14	-0.14
excess kurtosis		NA	0.93	0.91	0.91
total return		-5.92%	-10.73%	-14.67%	-17.92%
annual volatility		3.54%	6.45%	8.91%	11.02%
correlation	5%	0.03	0.03	0.03	0.03
skewness		-0.11	-0.12	-0.13	-0.14
excess kurtosis		1.02	0.96	0.93	0.91
total return		-13.36%	-18.16%	-22.08%	-25.31%
annual volatility		8.00%	10.89%	13.34%	15.44%
correlation	10%	0.03	0.03	0.03	0.03
skewness		-0.10	-0.11	-0.12	-0.12
excess kurtosis		1.14	1.05	0.99	0.94
total return		-23.17%	-27.92%	-31.81%	-35.00%
annual volatility		14.00%	16.86%	19.28%	21.35%
correlation	15%	0.03	0.03	0.03	0.03
skewness		-0.07	-0.08	-0.09	-0.10
excess kurtosis		1.47	1.28	1.16	1.08

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-6.04%	-11.05%	-15.31%
annual volatility		0.00%	3.23%	5.81%	7.91%
correlation	0	NA	-0.09	-0.09	-0.09
skewness		NA	0.00	-0.02	-0.03
excess kurtosis		NA	2.25	2.30	2.34
total return		-7.64%	-13.68%	-18.68%	-22.92%
annual volatility		4.22%	7.44%	10.01%	12.11%
correlation	5%	-0.08	-0.08	-0.09	-0.08
skewness		0.03	0.01	0.00	-0.01
excess kurtosis		2.11	2.16	2.20	2.24
total return		-18.20%	-24.21%	-29.19%	-33.42%
annual volatility		10.06%	13.27%	15.83%	17.91%
correlation	10%	-0.08	-0.08	-0.08	-0.08
skewness		0.03	0.03	0.02	0.01
excess kurtosis		2.04	2.05	2.07	2.10
total return		-36.14%	-42.11%	-47.03%	-51.20%
annual volatility		19.20%	22.35%	24.85%	26.89%
correlation	15%	-0.08	-0.08	-0.08	-0.08
skewness		0.01	0.02	0.02	0.02
excess kurtosis		2.20	2.09	2.03	2.00

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-4.92%	-9.22%	-12.95%
annual volatility		0.00%	3.36%	5.97%	8.05%
correlation	0	NA	-0.10	-0.10	-0.10
skewness		NA	0.12	0.12	0.11
excess kurtosis		NA	3.38	3.49	3.58
total return		-5.57%	-10.48%	-14.77%	-18.49%
annual volatility		4.55%	7.91%	10.50%	12.58%
correlation	5%	-0.09	-0.09	-0.09	-0.10
skewness		0.14	0.13	0.13	0.12
excess kurtosis		3.04	3.17	3.27	3.35
total return		-11.55%	-16.44%	-20.70%	-24.41%
annual volatility		11.25%	14.58%	17.15%	19.21%
correlation	10%	-0.09	-0.09	-0.09	-0.09
skewness		0.15	0.14	0.14	0.13
excess kurtosis		2.83	2.89	2.96	3.02
total return		-17.22%	-22.06%	-26.27%	-29.92%
annual volatility		23.27%	26.48%	28.96%	30.94%
correlation	15%	-0.08	-0.08	-0.08	-0.09
skewness		0.16	0.15	0.15	0.15
excess kurtosis		3.15	2.95	2.85	2.80

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.13%	-0.39%	-0.67%
annual volatility		0.00%	2.79%	5.22%	7.37%
correlation	0	NA	0.02	0.02	0.02
skewness		NA	-0.36	-0.35	-0.34
excess kurtosis		NA	1.98	2.00	2.03
total return		0.22%	0.10%	-0.16%	-0.43%
annual volatility		3.27%	6.06%	8.48%	10.62%
correlation	5%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.36	-0.35
excess kurtosis		1.95	1.95	1.96	1.97
total return		1.14%	1.04%	0.79%	0.53%
annual volatility		7.22%	10.00%	12.41%	14.54%
correlation	10%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.37	-0.36
excess kurtosis		1.94	1.93	1.92	1.92
total return		3.30%	3.22%	3.01%	2.78%
annual volatility		12.18%	14.94%	17.33%	19.43%
correlation	15%	0.02	0.02	0.02	0.02
skewness		-0.38	-0.37	-0.37	-0.37
excess kurtosis		1.95	1.91	1.88	1.87

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.21%	-0.51%	-0.89%
annual volatility		0.00%	3.07%	5.59%	7.71%
correlation	0	NA	-0.08	-0.08	-0.08
skewness		NA	-0.10	-0.11	-0.11
excess kurtosis		NA	1.22	1.32	1.41
total return		-0.15%	-0.36%	-0.65%	-1.02%
annual volatility		3.87%	6.94%	9.45%	11.56%
correlation	5%	-0.07	-0.08	-0.08	-0.08
skewness		-0.07	-0.08	-0.09	-0.10
excess kurtosis		0.98	1.07	1.16	1.24
total return		-0.52%	-0.71%	-1.00%	-1.36%
annual volatility		8.98%	12.03%	14.53%	16.62%
correlation	10%	-0.07	-0.07	-0.08	-0.08
skewness		-0.06	-0.07	-0.08	-0.08
excess kurtosis		0.89	0.93	0.99	1.04
total return		-2.41%	-2.58%	-2.84%	-3.17%
annual volatility		16.37%	19.38%	21.83%	23.89%
correlation	15%	-0.06	-0.07	-0.07	-0.07
skewness		-0.05	-0.06	-0.06	-0.06
excess kurtosis		1.01	0.93	0.91	0.90

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	0.65%	0.57%	0.11%
annual volatility		0.00%	3.28%	5.88%	8.01%
correlation	0	NA	-0.09	-0.09	-0.09
skewness		NA	-0.14	-0.15	-0.16
excess kurtosis		NA	1.80	1.95	2.09
total return		2.12%	2.77%	2.69%	2.24%
annual volatility		4.31%	7.58%	10.17%	12.29%
correlation	5%	-0.08	-0.09	-0.09	-0.09
skewness		-0.11	-0.12	-0.13	-0.14
excess kurtosis		1.47	1.58	1.70	1.81
total return		7.37%	8.04%	7.97%	7.54%
annual volatility		10.36%	13.61%	16.19%	18.29%
correlation	10%	-0.08	-0.08	-0.08	-0.08
skewness		-0.07	-0.09	-0.10	-0.11
excess kurtosis		1.38	1.41	1.47	1.53
total return		20.30%	21.01%	20.98%	20.57%
annual volatility		20.08%	23.27%	25.78%	27.83%
correlation	15%	-0.07	-0.07	-0.08	-0.08
skewness		0.00	-0.02	-0.04	-0.05
excess kurtosis		1.68	1.53	1.46	1.43

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		15.18%	17.03%	17.63%	17.94%
annual volatility		3.91%	4.55%	6.17%	7.95%
correlation	0	0.09	0.01	0.02	0.02
skewness		-0.06	-0.22	-0.25	-0.27
excess kurtosis		2.66	2.68	2.03	1.81
total return		7.51%	9.00%	9.54%	9.84%
annual volatility		6.08%	7.51%	9.26%	11.02%
correlation	5%	0.07	0.06	0.06	0.05
skewness		-0.28	-0.24	-0.24	-0.26
excess kurtosis		2.58	2.01	1.70	1.58
total return		-5.32%	-3.89%	-3.31%	-2.96%
annual volatility		10.57%	12.22%	13.95%	15.62%
correlation	10%	0.09	0.08	0.07	0.07
skewness		-0.18	-0.18	-0.19	-0.21
excess kurtosis		1.90	1.64	1.49	1.41
total return		-27.61%	-26.17%	-25.53%	-25.11%
annual volatility		18.56%	20.22%	21.83%	23.36%
correlation	15%	0.09	0.09	0.08	0.08
skewness		-0.06	-0.08	-0.11	-0.13
excess kurtosis		2.66	2.32	2.05	1.85

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		14.55%	16.06%	16.51%	16.62%
annual volatility		4.10%	5.02%	6.79%	8.57%
correlation	0	-0.01	-0.07	-0.08	-0.09
skewness		0.06	0.05	-0.03	-0.06
excess kurtosis		3.76	2.09	1.82	1.75
total return		8.39%	9.61%	10.02%	10.14%
annual volatility		6.83%	8.51%	10.38%	12.12%
correlation	5%	-0.02	-0.04	-0.05	-0.06
skewness		0.05	-0.02	-0.05	-0.07
excess kurtosis		2.36	1.84	1.63	1.57
total return		-3.87%	-2.67%	-2.22%	-2.05%
annual volatility		12.67%	14.55%	16.35%	17.98%
correlation	10%	-0.01	-0.03	-0.04	-0.04
skewness		0.01	-0.02	-0.03	-0.04
excess kurtosis		1.80	1.57	1.44	1.38
total return		-34.33%	-33.11%	-32.61%	-32.39%
annual volatility		25.24%	27.06%	28.68%	30.12%
correlation	15%	-0.01	-0.01	-0.02	-0.03
skewness		0.06	0.05	0.03	0.02
excess kurtosis		3.76	3.25	2.85	2.54

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		12.53%	14.77%	15.37%	15.35%
annual volatility		4.17%	5.11%	6.95%	8.74%
correlation	0	-0.01	-0.06	-0.08	-0.09
skewness		-0.04	0.19	0.15	0.10
excess kurtosis		3.06	2.84	2.91	2.87
total return		8.23%	10.18%	10.76%	10.74%
annual volatility		6.73%	8.54%	10.49%	12.25%
correlation	5%	-0.01	-0.04	-0.06	-0.07
skewness		-0.04	0.03	0.04	0.03
excess kurtosis		1.84	2.07	2.21	2.29
total return		-0.77%	1.14%	1.74%	1.75%
annual volatility		12.78%	14.82%	16.70%	18.35%
correlation	10%	-0.01	-0.03	-0.04	-0.05
skewness		-0.05	-0.02	-0.01	-0.01
excess kurtosis		1.49	1.61	1.69	1.75
total return		-21.04%	-19.09%	-18.44%	-18.37%
annual volatility		26.31%	28.29%	30.00%	31.46%
correlation	15%	-0.01	-0.02	-0.03	-0.03
skewness		-0.04	-0.03	-0.03	-0.03
excess kurtosis		3.06	2.83	2.62	2.46

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		1.13%	2.60%	2.93%	3.01%
annual volatility		2.44%	3.52%	5.54%	7.53%
correlation	0	0.02	-0.04	-0.02	-0.01
skewness		-0.09	-0.18	-0.22	-0.24
excess kurtosis		1.68	1.39	1.37	1.51
total return		-3.08%	-2.06%	-1.72%	-1.59%
annual volatility		4.63%	6.63%	8.72%	10.69%
correlation	5%	-0.02	0.00	0.00	0.01
skewness		-0.16	-0.17	-0.20	-0.22
excess kurtosis		2.07	1.20	1.08	1.11
total return		-10.09%	-9.10%	-8.70%	-8.50%
annual volatility		8.74%	10.91%	12.97%	14.88%
correlation	10%	0.01	0.01	0.01	0.01
skewness		-0.12	-0.15	-0.18	-0.19
excess kurtosis		1.64	1.18	1.03	0.99
total return		-19.72%	-18.68%	-18.22%	-17.95%
annual volatility		14.59%	16.78%	18.80%	20.64%
correlation	15%	0.02	0.02	0.02	0.02
skewness		-0.09	-0.12	-0.15	-0.16
excess kurtosis		1.68	1.34	1.16	1.05

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		1.62%	2.85%	3.10%	3.04%
annual volatility		2.69%	4.03%	6.14%	8.09%
correlation	0	-0.09	-0.12	-0.11	-0.11
skewness		0.04	-0.02	-0.04	-0.06
excess kurtosis		2.55	2.51	1.99	1.84
total return		-4.07%	-3.23%	-3.00%	-3.03%
annual volatility		5.57%	7.81%	9.98%	11.91%
correlation	5%	-0.11	-0.11	-0.11	-0.10
skewness		0.12	0.04	0.00	-0.02
excess kurtosis		3.52	2.57	2.14	1.97
total return		-14.10%	-13.30%	-13.01%	-12.95%
annual volatility		11.07%	13.47%	15.60%	17.47%
correlation	10%	-0.10	-0.10	-0.10	-0.10
skewness		0.09	0.05	0.02	0.01
excess kurtosis		2.81	2.35	2.11	1.98
total return		-31.81%	-30.98%	-30.64%	-30.53%
annual volatility		20.04%	22.45%	24.51%	26.30%
correlation	15%	-0.09	-0.09	-0.09	-0.09
skewness		0.04	0.02	0.02	0.01
excess kurtosis		2.55	2.27	2.09	1.97

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		2.29%	4.27%	4.72%	4.56%
annual volatility		2.91%	4.25%	6.40%	8.35%
correlation	0	-0.09	-0.12	-0.12	-0.12
skewness		0.15	-0.05	-0.01	-0.02
excess kurtosis		3.12	3.12	2.93	2.86
total return		-0.95%	0.62%	1.00%	0.84%
annual volatility		5.94%	8.30%	10.53%	12.45%
correlation	5%	-0.11	-0.12	-0.11	-0.11
skewness		0.03	0.08	0.07	0.05
excess kurtosis		3.67	3.37	3.15	3.05
total return		-6.18%	-4.65%	-4.24%	-4.36%
annual volatility		12.23%	14.76%	16.95%	18.81%
correlation	10%	-0.10	-0.10	-0.11	-0.11
skewness		0.12	0.12	0.11	0.09
excess kurtosis		3.13	3.02	2.94	2.90
total return		-11.54%	-10.00%	-9.54%	-9.60%
annual volatility		24.03%	26.53%	28.60%	30.34%
correlation	15%	-0.09	-0.09	-0.09	-0.09
skewness		0.15	0.14	0.14	0.13
excess kurtosis		3.12	2.98	2.86	2.79

Appendix 10 (cont'd)
Summary results on long-short strategies – annual re-ranking stock selection method

calibration period		3 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-2.65%	-4.64%	-6.04%
annual volatility		0.00%	3.01%	5.57%	7.81%
correlation	0	NA	0.13	0.13	0.13
skewness		NA	-0.05	-0.06	-0.06
excess kurtosis		NA	0.75	0.82	0.90
total return		-3.32%	-5.97%	-7.96%	-9.34%
annual volatility		3.67%	6.68%	9.22%	11.44%
correlation	5%	0.13	0.13	0.13	0.13
skewness		0.00	-0.02	-0.04	-0.05
excess kurtosis		0.71	0.69	0.71	0.76
total return		-7.26%	-9.88%	-11.85%	-13.22%
annual volatility		8.46%	11.43%	13.94%	16.13%
correlation	10%	0.12	0.12	0.13	0.13
skewness		0.06	0.03	0.01	-0.01
excess kurtosis		0.90	0.75	0.69	0.67
total return		-11.77%	-14.34%	-16.27%	-17.59%
annual volatility		15.91%	18.75%	21.15%	23.23%
correlation	15%	0.11	0.12	0.12	0.12
skewness		0.21	0.15	0.12	0.09
excess kurtosis		2.40	1.71	1.32	1.07

		4 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	1.12%	1.73%	1.94%
annual volatility		0.00%	3.17%	5.73%	7.88%
correlation	0	NA	0.06	0.06	0.06
skewness		NA	0.01	0.00	-0.01
excess kurtosis		NA	1.02	1.13	1.23
total return		1.70%	2.83%	3.44%	3.67%
annual volatility		4.10%	7.25%	9.81%	11.94%
correlation	5%	0.06	0.06	0.06	0.06
skewness		0.03	0.02	0.01	0.00
excess kurtosis		0.89	0.89	0.95	1.02
total return		3.59%	4.74%	5.37%	5.62%
annual volatility		9.99%	13.09%	15.59%	17.68%
correlation	10%	0.05	0.05	0.05	0.06
skewness		0.03	0.03	0.02	0.02
excess kurtosis		1.20	0.98	0.90	0.88
total return		1.09%	2.28%	2.97%	3.26%
annual volatility		21.37%	24.24%	26.55%	28.47%
correlation	15%	0.04	0.04	0.04	0.04
skewness		-0.08	-0.05	-0.04	-0.03
excess kurtosis		4.34	3.27	2.62	2.19

		5 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	2.03%	3.14%	3.71%
annual volatility		0.00%	3.15%	5.64%	7.66%
correlation	0	NA	0.03	0.03	0.03
skewness		NA	-0.09	-0.10	-0.12
excess kurtosis		NA	1.23	1.37	1.50
total return		3.71%	5.74%	6.86%	7.44%
annual volatility		4.22%	7.36%	9.83%	11.85%
correlation	5%	0.03	0.03	0.03	0.03
skewness		-0.04	-0.06	-0.08	-0.09
excess kurtosis		1.03	1.07	1.15	1.24
total return		11.03%	13.07%	14.20%	14.79%
annual volatility		10.47%	13.56%	16.00%	17.97%
correlation	10%	0.02	0.02	0.03	0.03
skewness		0.01	-0.01	-0.03	-0.04
excess kurtosis		1.26	1.10	1.05	1.05
total return		29.03%	31.11%	32.27%	32.89%
annual volatility		22.69%	25.60%	27.87%	29.72%
correlation	15%	0.02	0.02	0.02	0.02
skewness		0.11	0.08	0.07	0.05
excess kurtosis		3.55	2.81	2.35	2.05

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.28%	-0.61%	-0.94%
annual volatility		0.00%	2.91%	5.39%	7.54%
correlation	0	NA	0.02	0.02	0.02
skewness		NA	0.00	0.00	-0.01
excess kurtosis		NA	0.96	0.95	0.95
total return		-0.17%	-0.44%	-0.76%	-1.09%
annual volatility		3.52%	6.43%	8.90%	11.04%
correlation	5%	0.02	0.02	0.02	0.02
skewness		0.01	0.00	0.00	0.00
excess kurtosis		1.05	1.00	0.96	0.94
total return		-0.18%	-0.43%	-0.73%	-1.03%
annual volatility		7.93%	10.82%	13.28%	15.40%
correlation	10%	0.03	0.03	0.02	0.02
skewness		0.02	0.01	0.01	0.00
excess kurtosis		1.16	1.07	1.01	0.97
total return		-0.03%	-0.24%	-0.50%	-0.77%
annual volatility		13.86%	16.72%	19.14%	21.23%
correlation	15%	0.03	0.03	0.03	0.03
skewness		0.04	0.03	0.02	0.02
excess kurtosis		1.49	1.29	1.17	1.08

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	2.31%	3.96%	5.14%
annual volatility		0.00%	3.21%	5.80%	7.93%
correlation	0	NA	-0.06	-0.06	-0.06
skewness		NA	0.01	0.00	-0.01
excess kurtosis		NA	2.37	2.37	2.37
total return		3.25%	5.57%	7.22%	8.41%
annual volatility		4.13%	7.33%	9.91%	12.05%
correlation	5%	-0.07	-0.07	-0.06	-0.06
skewness		0.04	0.03	0.02	0.01
excess kurtosis		2.34	2.34	2.34	2.34
total return		7.77%	10.10%	11.76%	12.96%
annual volatility		9.76%	12.95%	15.51%	17.63%
correlation	10%	-0.07	-0.07	-0.07	-0.06
skewness		0.05	0.05	0.04	0.03
excess kurtosis		2.29	2.29	2.28	2.27
total return		13.39%	15.76%	17.46%	18.69%
annual volatility		18.39%	21.52%	24.02%	26.09%
correlation	15%	-0.07	-0.07	-0.07	-0.06
skewness		0.05	0.05	0.05	0.04
excess kurtosis		2.33	2.25	2.20	2.17

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	0.87%	0.97%	0.69%
annual volatility		0.00%	3.40%	6.06%	8.21%
correlation	0	NA	-0.08	-0.08	-0.08
skewness		NA	0.12	0.11	0.09
excess kurtosis		NA	3.11	3.19	3.25
total return		2.44%	3.31%	3.42%	3.14%
annual volatility		4.53%	7.92%	10.58%	12.72%
correlation	5%	-0.08	-0.08	-0.08	-0.08
skewness		0.16	0.14	0.13	0.12
excess kurtosis		2.86	2.95	3.02	3.08
total return		8.39%	9.27%	9.39%	9.13%
annual volatility		11.07%	14.44%	17.07%	19.19%
correlation	10%	-0.08	-0.08	-0.08	-0.08
skewness		0.20	0.18	0.17	0.15
excess kurtosis		2.74	2.76	2.80	2.84
total return		23.32%	24.24%	24.40%	24.16%
annual volatility		22.45%	25.70%	28.24%	30.28%
correlation	15%	-0.07	-0.08	-0.08	-0.08
skewness		0.26	0.24	0.23	0.21
excess kurtosis		3.26	2.99	2.85	2.76

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.13%	-0.39%	-0.67%
annual volatility		0.00%	2.79%	5.22%	7.37%
correlation	0	NA	0.02	0.02	0.02
skewness		NA	-0.36	-0.35	-0.34
excess kurtosis		NA	1.98	2.00	2.03
total return		0.22%	0.10%	-0.16%	-0.43%
annual volatility		3.27%	6.06%	8.48%	10.62%
correlation	5%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.36	-0.35
excess kurtosis		1.95	1.95	1.96	1.97
total return		1.14%	1.04%	0.79%	0.53%
annual volatility		7.22%	10.00%	12.41%	14.54%
correlation	10%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.37	-0.36
excess kurtosis		1.94	1.93	1.92	1.92
total return		3.30%	3.22%	3.01%	2.78%
annual volatility		12.18%	14.94%	17.33%	19.43%
correlation	15%	0.02	0.02	0.02	0.02
skewness		-0.38	-0.37	-0.37	-0.37
excess kurtosis		1.95	1.91	1.88	1.87

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.21%	-0.51%	-0.89%
annual volatility		0.00%	3.07%	5.59%	7.71%
correlation	0	NA	-0.08	-0.08	-0.08
skewness		NA	-0.10	-0.11	-0.11
excess kurtosis		NA	1.22	1.32	1.41
total return		-0.15%	-0.36%	-0.65%	-1.02%
annual volatility		3.87%	6.94%	9.45%	11.56%
correlation	5%	-0.07	-0.08	-0.08	-0.08
skewness		-0.07	-0.08	-0.09	-0.10
excess kurtosis		0.98	1.07	1.16	1.24
total return		-0.52%	-0.71%	-1.00%	-1.36%
annual volatility		8.98%	12.03%	14.53%	16.62%
correlation	10%	-0.07	-0.07	-0.08	-0.08
skewness		-0.06	-0.07	-0.08	-0.08
excess kurtosis		0.89	0.93	0.99	1.04
total return		-2.41%	-2.58%	-2.84%	-3.17%
annual volatility		16.37%	19.38%	21.83%	23.89%
correlation	15%	-0.06	-0.07	-0.07	-0.07
skewness		-0.05	-0.06	-0.06	-0.06
excess kurtosis		1.01	0.93	0.91	0.90

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	0.65%	0.57%	0.11%
annual volatility		0.00%	3.28%	5.88%	8.01%
correlation	0	NA	-0.09	-0.09	-0.09
skewness		NA	-0.14	-0.15	-0.16
excess kurtosis		NA	1.80	1.95	2.09
total return		2.12%	2.77%	2.69%	2.24%
annual volatility		4.31%	7.58%	10.17%	12.29%
correlation	5%	-0.08	-0.09	-0.09	-0.09
skewness		-0.11	-0.12	-0.13	-0.14
excess kurtosis		1.47	1.58	1.70	1.81
total return		7.37%	8.04%	7.97%	7.54%
annual volatility		10.36%	13.61%	16.19%	18.29%
correlation	10%	-0.08	-0.08	-0.08	-0.08
skewness		-0.07	-0.09	-0.10	-0.11
excess kurtosis		1.38	1.41	1.47	1.53
total return		20.30%	21.01%	20.98%	20.57%
annual volatility		20.08%	23.27%	25.78%	27.83%
correlation	15%	-0.07	-0.07	-0.08	-0.08
skewness		0.00	-0.02	-0.04	-0.05
excess kurtosis		1.68	1.53	1.46	1.43

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		9.10%	10.74%	10.92%	10.83%
annual volatility		3.53%	4.40%	6.17%	8.03%
correlation	0	0.09	-0.06	-0.03	-0.02
skewness		0.14	-0.17	-0.27	-0.30
excess kurtosis		2.07	2.29	1.91	1.83
total return		6.94%	8.04%	8.14%	8.07%
annual volatility		5.41%	7.07%	8.99%	10.85%
correlation	5%	0.03	0.03	0.03	0.03
skewness		-0.10	-0.18	-0.23	-0.26
excess kurtosis		1.30	1.35	1.38	1.42
total return		3.35%	4.35%	4.50%	4.48%
annual volatility		9.56%	11.40%	13.27%	15.04%
correlation	10%	0.07	0.07	0.06	0.06
skewness		-0.02	-0.08	-0.14	-0.18
excess kurtosis		1.01	1.05	1.09	1.12
total return		-1.02%	0.00%	0.22%	0.26%
annual volatility		16.68%	18.49%	20.23%	21.86%
correlation	15%	0.09	0.08	0.08	0.07
skewness		0.14	0.08	0.02	-0.03
excess kurtosis		2.07	1.75	1.53	1.37

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		11.76%	13.04%	13.11%	12.89%
annual volatility		3.43%	4.67%	6.65%	8.53%
correlation	0	0.02	-0.10	-0.10	-0.10
skewness		-0.05	0.01	-0.06	-0.10
excess kurtosis		4.16	2.19	2.17	2.10
total return		14.47%	15.28%	15.30%	15.09%
annual volatility		5.80%	7.77%	9.81%	11.67%
correlation	5%	0.00	-0.03	-0.05	-0.06
skewness		0.10	0.00	-0.06	-0.08
excess kurtosis		1.25	1.39	1.44	1.49
total return		16.63%	17.38%	17.43%	17.26%
annual volatility		11.13%	13.17%	15.10%	16.83%
correlation	10%	0.02	0.00	-0.01	-0.02
skewness		0.07	0.02	-0.02	-0.04
excess kurtosis		1.32	1.16	1.10	1.09
total return		14.26%	15.02%	15.10%	14.98%
annual volatility		22.22%	24.12%	25.81%	27.31%
correlation	15%	0.02	0.01	0.00	0.00
skewness		-0.05	-0.05	-0.05	-0.05
excess kurtosis		4.16	3.33	2.75	2.36

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		7.75%	9.47%	9.60%	9.23%
annual volatility		2.77%	4.23%	6.43%	8.41%
correlation	0	-0.08	-0.10	-0.10	-0.10
skewness		0.23	0.03	-0.01	-0.04
excess kurtosis		3.09	3.06	2.87	2.83
total return		11.13%	12.33%	12.44%	12.08%
annual volatility		5.76%	8.28%	10.58%	12.55%
correlation	5%	-0.09	-0.10	-0.10	-0.10
skewness		0.09	0.08	0.04	0.02
excess kurtosis		3.59	3.16	2.97	2.91
total return		17.28%	18.45%	18.60%	18.27%
annual volatility		11.90%	14.58%	16.86%	18.77%
correlation	10%	-0.09	-0.09	-0.09	-0.09
skewness		0.17	0.14	0.10	0.08
excess kurtosis		2.99	2.83	2.76	2.74
total return		32.28%	33.48%	33.67%	33.39%
annual volatility		23.07%	25.70%	27.87%	29.69%
correlation	15%	-0.08	-0.08	-0.08	-0.09
skewness		0.23	0.21	0.18	0.16
excess kurtosis		3.09	2.90	2.77	2.69

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		10.82%	13.27%	15.24%	16.94%
annual volatility		2.34%	3.51%	5.56%	7.56%
correlation	0	0.02	-0.02	-0.01	0.00
skewness		0.04	-0.20	-0.30	-0.32
excess kurtosis		1.67	1.45	1.58	1.73
total return		13.77%	16.21%	18.19%	19.89%
annual volatility		4.42%	6.52%	8.66%	10.65%
correlation	5%	-0.01	0.00	0.01	0.01
skewness		0.04	-0.13	-0.21	-0.24
excess kurtosis		2.17	1.26	1.25	1.33
total return		17.66%	20.11%	22.08%	23.78%
annual volatility		8.49%	10.72%	12.83%	14.77%
correlation	10%	0.01	0.01	0.01	0.02
skewness		0.03	-0.08	-0.14	-0.18
excess kurtosis		1.62	1.23	1.16	1.17
total return		23.10%	25.55%	27.52%	29.22%
annual volatility		14.27%	16.50%	18.56%	20.42%
correlation	15%	0.02	0.02	0.02	0.02
skewness		0.04	-0.03	-0.08	-0.12
excess kurtosis		1.67	1.39	1.26	1.20

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		11.48%	12.49%	12.44%	12.17%
annual volatility		2.57%	3.98%	6.13%	8.10%
correlation	0	-0.07	-0.10	-0.10	-0.10
skewness		0.06	0.00	-0.06	-0.09
excess kurtosis		2.90	3.02	2.32	2.09
total return		15.68%	16.17%	16.11%	15.86%
annual volatility		5.32%	7.66%	9.88%	11.83%
correlation	5%	-0.08	-0.09	-0.09	-0.09
skewness		0.11	0.02	-0.02	-0.05
excess kurtosis		4.37	3.05	2.50	2.27
total return		20.38%	20.83%	20.81%	20.60%
annual volatility		10.62%	13.10%	15.28%	17.19%
correlation	10%	-0.07	-0.08	-0.08	-0.08
skewness		0.08	0.03	0.01	-0.01
excess kurtosis		3.42	2.77	2.45	2.29
total return		26.07%	26.55%	26.58%	26.42%
annual volatility		19.10%	21.57%	23.69%	25.51%
correlation	15%	-0.07	-0.07	-0.08	-0.08
skewness		0.06	0.03	0.02	0.01
excess kurtosis		2.90	2.55	2.34	2.22

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		11.28%	13.25%	13.51%	13.20%
annual volatility		3.74%	5.04%	7.06%	8.93%
correlation	0	0.00	-0.12	-0.12	-0.12
skewness		0.12	-0.01	-0.02	-0.05
excess kurtosis		3.29	2.62	2.65	2.66
total return		16.02%	17.57%	17.78%	17.47%
annual volatility		6.06%	8.18%	10.30%	12.17%
correlation	5%	-0.04	-0.06	-0.07	-0.08
skewness		-0.05	-0.06	-0.08	-0.09
excess kurtosis		1.33	1.60	1.76	1.88
total return		23.64%	25.15%	25.39%	25.11%
annual volatility		11.65%	13.89%	15.92%	17.67%
correlation	10%	-0.01	-0.03	-0.04	-0.05
skewness		0.02	-0.01	-0.04	-0.06
excess kurtosis		1.23	1.29	1.34	1.39
total return		41.75%	43.29%	43.58%	43.36%
annual volatility		23.55%	25.66%	27.47%	29.02%
correlation	15%	0.00	-0.01	-0.02	-0.02
skewness		0.12	0.09	0.07	0.05
excess kurtosis		3.29	2.91	2.60	2.37

Appendix 10 (cont'd)

Summary results on long-short strategies – frequency based re-ranking stock selection method

calibration period		3 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-6.61%	-12.16%	-16.80%
annual volatility		0.00%	3.08%	5.77%	8.17%
correlation	0	NA	0.14	0.14	0.14
skewness		NA	-0.10	-0.10	-0.09
excess kurtosis		NA	1.29	1.52	1.79
total return		-7.83%	-14.44%	-19.99%	-24.62%
annual volatility		3.62%	6.69%	9.37%	11.76%
correlation	5%	0.14	0.14	0.14	0.14
skewness		-0.08	-0.09	-0.09	-0.09
excess kurtosis		0.95	1.07	1.23	1.42
total return		-17.12%	-23.71%	-29.24%	-33.87%
annual volatility		8.04%	11.09%	13.75%	16.11%
correlation	10%	0.14	0.14	0.14	0.14
skewness		-0.06	-0.07	-0.08	-0.08
excess kurtosis		0.88	0.91	1.00	1.11
total return		-28.14%	-34.70%	-40.21%	-44.81%
annual volatility		13.91%	16.90%	19.50%	21.82%
correlation	15%	0.14	0.14	0.14	0.14
skewness		-0.02	-0.03	-0.05	-0.06
excess kurtosis		1.19	1.01	0.94	0.94

		4 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-6.94%	-13.43%	-19.44%
annual volatility		0.00%	3.15%	5.74%	7.91%
correlation	0	NA	0.07	0.07	0.07
skewness		NA	-0.17	-0.20	-0.23
excess kurtosis		NA	1.36	1.52	1.67
total return		-7.22%	-14.15%	-20.64%	-26.65%
annual volatility		3.97%	7.12%	9.70%	11.87%
correlation	5%	0.07	0.07	0.07	0.07
skewness		-0.10	-0.13	-0.16	-0.18
excess kurtosis		1.01	1.14	1.27	1.40
total return		-14.28%	-21.19%	-27.66%	-33.66%
annual volatility		9.26%	12.39%	14.95%	17.09%
correlation	10%	0.07	0.07	0.07	0.07
skewness		-0.04	-0.07	-0.10	-0.12
excess kurtosis		0.87	0.94	1.02	1.11
total return		-20.19%	-27.07%	-33.51%	-39.47%
annual volatility		17.09%	20.16%	22.66%	24.76%
correlation	15%	0.07	0.07	0.07	0.07
skewness		0.04	0.01	-0.02	-0.04
excess kurtosis		0.90	0.85	0.84	0.86

		5 years			
20-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-1.22%	-3.01%	-4.99%
annual volatility		0.00%	3.32%	5.94%	8.08%
correlation	0	NA	0.00	-0.01	-0.01
skewness		NA	-0.12	-0.15	-0.16
excess kurtosis		NA	2.10	2.28	2.42
total return		0.35%	-0.86%	-2.65%	-4.62%
annual volatility		4.38%	7.69%	10.31%	12.44%
correlation	5%	0.00	0.00	0.00	0.00
skewness		-0.05	-0.08	-0.10	-0.12
excess kurtosis		1.67	1.84	1.98	2.11
total return		5.08%	3.89%	2.12%	0.17%
annual volatility		10.60%	13.88%	16.48%	18.58%
correlation	10%	0.00	0.00	0.00	0.00
skewness		0.03	-0.01	-0.04	-0.06
excess kurtosis		1.50	1.57	1.66	1.74
total return		25.35%	24.20%	22.47%	20.56%
annual volatility		21.02%	24.20%	26.72%	28.76%
correlation	15%	0.00	0.00	0.00	0.00
skewness		0.21	0.15	0.11	0.09
excess kurtosis		2.12	1.85	1.72	1.66

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-4.65%	-8.09%	-10.58%
annual volatility		0.00%	2.91%	5.43%	7.65%
correlation	0	NA	0.02	0.02	0.02
skewness		NA	-0.11	-0.11	-0.10
excess kurtosis		NA	1.15	1.16	1.18
total return		-6.22%	-10.86%	-14.30%	-16.78%
annual volatility		3.44%	6.34%	8.86%	11.07%
correlation	5%	0.02	0.02	0.02	0.02
skewness		-0.12	-0.12	-0.11	-0.11
excess kurtosis		1.15	1.14	1.13	1.14
total return		-14.64%	-19.26%	-22.68%	-25.14%
annual volatility		7.64%	10.53%	13.03%	15.23%
correlation	10%	0.03	0.03	0.02	0.02
skewness		-0.12	-0.12	-0.12	-0.12
excess kurtosis		1.18	1.14	1.11	1.10
total return		-26.40%	-30.99%	-34.37%	-36.80%
annual volatility		13.16%	16.00%	18.46%	20.62%
correlation	15%	0.03	0.03	0.03	0.03
skewness		-0.10	-0.11	-0.11	-0.11
excess kurtosis		1.46	1.28	1.18	1.12

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-3.52%	-6.47%	-9.01%
annual volatility		0.00%	3.19%	5.82%	8.04%
correlation	0	NA	-0.04	-0.04	-0.04
skewness		NA	-0.18	-0.20	-0.21
excess kurtosis		NA	1.31	1.40	1.47
total return		-4.45%	-7.97%	-10.90%	-13.44%
annual volatility		3.97%	7.15%	9.78%	12.00%
correlation	5%	-0.04	-0.04	-0.04	-0.04
skewness		-0.15	-0.16	-0.18	-0.19
excess kurtosis		1.07	1.16	1.24	1.32
total return		-10.66%	-14.17%	-17.09%	-19.61%
annual volatility		9.18%	12.34%	14.95%	17.15%
correlation	10%	-0.04	-0.04	-0.04	-0.04
skewness		-0.12	-0.14	-0.15	-0.16
excess kurtosis		0.95	1.00	1.06	1.12
total return		-21.21%	-24.67%	-27.56%	-30.05%
annual volatility		16.80%	19.90%	22.45%	24.60%
correlation	15%	-0.04	-0.04	-0.04	-0.04
skewness		-0.09	-0.10	-0.11	-0.12
excess kurtosis		1.10	0.99	0.95	0.95

25-stocks	-\+	0%	5%	10%	15%
total return		0.00%	2.16%	3.32%	3.91%
annual volatility		0.00%	3.39%	6.12%	8.37%
correlation	0	NA	-0.03	-0.03	-0.03
skewness		NA	-0.14	-0.15	-0.17
excess kurtosis		NA	1.55	1.60	1.66
total return		3.98%	6.14%	7.31%	7.90%
annual volatility		4.37%	7.75%	10.47%	12.72%
correlation	5%	-0.03	-0.03	-0.03	-0.03
skewness		-0.08	-0.11	-0.13	-0.14
excess kurtosis		1.50	1.49	1.52	1.55
total return		11.71%	13.87%	15.06%	15.66%
annual volatility		10.37%	13.73%	16.43%	18.66%
correlation	10%	-0.02	-0.03	-0.03	-0.03
skewness		-0.03	-0.06	-0.08	-0.10
excess kurtosis		1.68	1.56	1.51	1.49
total return		28.74%	30.95%	32.16%	32.80%
annual volatility		19.88%	23.15%	25.77%	27.93%
correlation	15%	-0.02	-0.02	-0.02	-0.02
skewness		0.05	0.02	0.00	-0.02
excess kurtosis		2.66	2.25	2.01	1.85

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.13%	-0.39%	-0.67%
annual volatility		0.00%	2.79%	5.22%	7.37%
correlation	0	NA	0.02	0.02	0.02
skewness		NA	-0.36	-0.35	-0.34
excess kurtosis		NA	1.98	2.00	2.03
total return		0.22%	0.10%	-0.16%	-0.43%
annual volatility		3.27%	6.06%	8.48%	10.62%
correlation	5%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.36	-0.35
excess kurtosis		1.95	1.95	1.96	1.97
total return		1.14%	1.04%	0.79%	0.53%
annual volatility		7.22%	10.00%	12.41%	14.54%
correlation	10%	0.02	0.02	0.02	0.02
skewness		-0.37	-0.37	-0.37	-0.36
excess kurtosis		1.94	1.93	1.92	1.92
total return		3.30%	3.22%	3.01%	2.78%
annual volatility		12.18%	14.94%	17.33%	19.43%
correlation	15%	0.02	0.02	0.02	0.02
skewness		-0.38	-0.37	-0.37	-0.37
excess kurtosis		1.95	1.91	1.88	1.87

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	-0.21%	-0.51%	-0.89%
annual volatility		0.00%	3.07%	5.59%	7.71%
correlation	0	NA	-0.08	-0.08	-0.08
skewness		NA	-0.10	-0.11	-0.11
excess kurtosis		NA	1.22	1.32	1.41
total return		-0.15%	-0.36%	-0.65%	-1.02%
annual volatility		3.87%	6.94%	9.45%	11.56%
correlation	5%	-0.07	-0.08	-0.08	-0.08
skewness		-0.07	-0.08	-0.09	-0.10
excess kurtosis		0.98	1.07	1.16	1.24
total return		-0.52%	-0.71%	-1.00%	-1.36%
annual volatility		8.98%	12.03%	14.53%	16.62%
correlation	10%	-0.07	-0.07	-0.08	-0.08
skewness		-0.06	-0.07	-0.08	-0.08
excess kurtosis		0.89	0.93	0.99	1.04
total return		-2.41%	-2.58%	-2.84%	-3.17%
annual volatility		16.37%	19.38%	21.83%	23.89%
correlation	15%	-0.06	-0.07	-0.07	-0.07
skewness		-0.05	-0.06	-0.06	-0.06
excess kurtosis		1.01	0.93	0.91	0.90

30-stocks	-\+	0%	5%	10%	15%
total return		0.00%	0.65%	0.57%	0.11%
annual volatility		0.00%	3.28%	5.88%	8.01%
correlation	0	NA	-0.09	-0.09	-0.09
skewness		NA	-0.14	-0.15	-0.16
excess kurtosis		NA	1.80	1.95	2.09
total return		2.12%	2.77%	2.69%	2.24%
annual volatility		4.31%	7.58%	10.17%	12.29%
correlation	5%	-0.08	-0.09	-0.09	-0.09
skewness		-0.11	-0.12	-0.13	-0.14
excess kurtosis		1.47	1.58	1.70	1.81
total return		7.37%	8.04%	7.97%	7.54%
annual volatility		10.36%	13.61%	16.19%	18.29%
correlation	10%	-0.08	-0.08	-0.08	-0.08
skewness		-0.07	-0.09	-0.10	-0.11
excess kurtosis		1.38	1.41	1.47	1.53
total return		20.30%	21.01%	20.98%	20.57%
annual volatility		20.08%	23.27%	25.78%	27.83%
correlation	15%	-0.07	-0.07	-0.08	-0.08
skewness		0.00	-0.02	-0.04	-0.05
excess kurtosis		1.68	1.53	1.46	1.43

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		15.81%	17.33%	17.49%	17.41%
annual volatility		3.76%	4.45%	6.11%	7.92%
correlation	0	0.13	-0.02	0.00	0.00
skewness		0.00	0.07	-0.12	-0.21
excess kurtosis		1.04	1.50	1.20	1.37
total return		9.53%	10.50%	10.58%	10.48%
annual volatility		4.81%	6.52%	8.48%	10.38%
correlation	5%	0.08	0.07	0.06	0.05
skewness		0.09	-0.08	-0.18	-0.23
excess kurtosis		1.22	1.04	1.25	1.43
total return		0.69%	1.62%	1.74%	1.70%
annual volatility		8.34%	10.29%	12.24%	14.08%
correlation	10%	0.12	0.10	0.09	0.08
skewness		-0.01	-0.10	-0.16	-0.20
excess kurtosis		0.77	0.94	1.12	1.27
total return		-10.16%	-9.19%	-9.01%	-8.99%
annual volatility		13.85%	15.80%	17.66%	19.40%
correlation	15%	0.13	0.12	0.11	0.10
skewness		0.00	-0.07	-0.11	-0.14
excess kurtosis		1.04	1.05	1.09	1.13

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		22.28%	23.56%	23.72%	23.56%
annual volatility		4.09%	4.86%	6.58%	8.35%
correlation	0	0.07	-0.04	-0.06	-0.07
skewness		0.09	0.28	0.13	0.04
excess kurtosis		0.87	3.10	2.22	1.94
total return		16.43%	17.32%	17.40%	17.22%
annual volatility		5.59%	7.36%	9.32%	11.13%
correlation	5%	0.06	0.01	-0.02	-0.03
skewness		0.24	0.06	-0.03	-0.07
excess kurtosis		1.92	1.21	1.09	1.13
total return		9.85%	10.69%	10.81%	10.67%
annual volatility		9.99%	12.00%	13.92%	15.66%
correlation	10%	0.07	0.04	0.02	0.00
skewness		0.10	0.00	-0.05	-0.08
excess kurtosis		0.84	0.78	0.80	0.83
total return		4.10%	4.95%	5.13%	5.07%
annual volatility		17.47%	19.49%	21.31%	22.93%
correlation	15%	0.07	0.05	0.04	0.03
skewness		0.09	0.04	0.00	-0.02
excess kurtosis		0.87	0.82	0.79	0.77

20-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		20.52%	22.57%	22.89%	22.62%
annual volatility		4.30%	5.20%	7.01%	8.80%
correlation	0	0.00	-0.07	-0.08	-0.09
skewness		0.30	0.28	0.25	0.19
excess kurtosis		2.09	3.73	3.48	3.27
total return		22.02%	23.64%	23.91%	23.63%
annual volatility		5.99%	8.00%	10.07%	11.92%
correlation	5%	-0.01	-0.05	-0.06	-0.07
skewness		0.30	0.26	0.18	0.12
excess kurtosis		2.58	2.39	2.30	2.31
total return		27.16%	28.74%	29.04%	28.81%
annual volatility		11.26%	13.51%	15.54%	17.32%
correlation	10%	-0.01	-0.03	-0.04	-0.05
skewness		0.22	0.17	0.12	0.09
excess kurtosis		1.54	1.60	1.65	1.70
total return		47.60%	49.22%	49.57%	49.39%
annual volatility		21.31%	23.54%	25.45%	27.08%
correlation	15%	0.00	-0.02	-0.02	-0.03
skewness		0.30	0.26	0.22	0.19
excess kurtosis		2.09	2.00	1.91	1.85

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		7.35%	8.47%	8.42%	8.22%
annual volatility		1.98%	3.35%	5.50%	7.55%
correlation	0	0.03	0.01	0.02	0.02
skewness		-0.10	-0.15	-0.22	-0.24
excess kurtosis		1.49	1.03	1.21	1.41
total return		2.29%	2.76%	2.67%	2.49%
annual volatility		3.88%	6.19%	8.43%	10.49%
correlation	5%	0.02	0.02	0.02	0.02
skewness		-0.07	-0.16	-0.20	-0.22
excess kurtosis		2.22	1.16	1.16	1.25
total return		-5.97%	-5.50%	-5.52%	-5.65%
annual volatility		7.79%	10.16%	12.36%	14.36%
correlation	10%	0.02	0.02	0.02	0.02
skewness		-0.10	-0.16	-0.19	-0.21
excess kurtosis		1.41	1.15	1.13	1.16
total return		-17.66%	-17.14%	-17.11%	-17.18%
annual volatility		13.18%	15.52%	17.64%	19.57%
correlation	15%	0.03	0.03	0.03	0.03
skewness		-0.10	-0.14	-0.17	-0.18
excess kurtosis		1.49	1.28	1.20	1.17

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		9.78%	11.36%	11.46%	11.08%
annual volatility		2.22%	4.00%	6.33%	8.35%
correlation	0	-0.02	-0.06	-0.07	-0.08
skewness		0.06	0.03	-0.03	-0.06
excess kurtosis		2.67	1.36	1.64	1.85
total return		14.70%	15.82%	15.88%	15.51%
annual volatility		4.90%	7.73%	10.16%	12.20%
correlation	5%	-0.01	-0.04	-0.06	-0.06
skewness		-0.03	-0.05	-0.07	-0.09
excess kurtosis		1.55	1.56	1.66	1.76
total return		22.56%	23.68%	23.79%	23.45%
annual volatility		10.60%	13.48%	15.87%	17.85%
correlation	10%	-0.02	-0.04	-0.05	-0.05
skewness		-0.02	-0.04	-0.06	-0.07
excess kurtosis		1.69	1.64	1.63	1.66
total return		39.64%	40.80%	40.97%	40.68%
annual volatility		20.00%	22.80%	25.09%	26.99%
correlation	15%	-0.02	-0.03	-0.04	-0.04
skewness		0.06	0.03	0.01	-0.01
excess kurtosis		2.67	2.35	2.15	2.02

25-stocks minus/ 30-stocks plus	-\+	0%	5%	10%	15%
total return		9.78%	11.36%	11.46%	11.08%
annual volatility		2.22%	4.00%	6.33%	8.35%
correlation	0	-0.02	-0.06	-0.07	-0.08
skewness		0.06	0.03	-0.03	-0.06
excess kurtosis		2.67	1.36	1.64	1.85
total return		14.70%	15.82%	15.88%	15.51%
annual volatility		4.90%	7.73%	10.16%	12.20%
correlation	5%	-0.01	-0.04	-0.06	-0.06
skewness		-0.03	-0.05	-0.07	-0.09
excess kurtosis		1.55	1.56	1.66	1.76
total return		22.56%	23.68%	23.79%	23.45%
annual volatility		10.60%	13.48%	15.87%	17.85%
correlation	10%	-0.02	-0.04	-0.05	-0.05
skewness		-0.02	-0.04	-0.06	-0.07
excess kurtosis		1.69	1.64	1.63	1.66
total return		39.64%	40.80%	40.97%	40.68%
annual volatility		20.00%	22.80%	25.09%	26.99%
correlation	15%	-0.02	-0.03	-0.04	-0.04
skewness		0.06	0.03	0.01	-0.01
excess kurtosis		2.67	2.35	2.15	2.02

Appendix 11
Sharpe ratios for the long-short strategies

calibration period: 3 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.33	-0.32	-0.31
5%	-0.36	-0.35	-0.34	-0.33
10%	-0.37	-0.36	-0.35	-0.34
15%	-0.38	-0.37	-0.37	-0.36

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.24	-0.23	-0.23
5%	-0.24	-0.24	-0.24	-0.23
10%	-0.24	-0.24	-0.24	-0.23
15%	-0.24	-0.24	-0.24	-0.23

+30/-30 stocks	0%	5%	10%	15%
0	NA	-0.01	-0.27	-0.28
5%	0.01	0.00	-0.00	-0.01
10%	0.02	0.01	0.01	0.01
15%	0.04	0.03	0.02	0.02

+30/-20 stocks	0%	5%	10%	15%
0	0.554195	0.53	0.41	0.32
5%	0.18	0.17	0.15	0.13
10%	-0.07	-0.05	-0.03	-0.03
15%	-0.21	-0.18	-0.17	-0.15

+30/-25 stocks	0%	5%	10%	15%
0	0.066446	0.11	0.08	0.06
5%	-0.10	-0.04	-0.03	-0.02
10%	-0.16	-0.12	-0.10	-0.08
15%	-0.19	-0.16	-0.14	-0.12

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.13	-0.12	-0.11
5%	-0.13	-0.13	-0.12	-0.12
10%	-0.12	-0.12	-0.12	-0.12
15%	-0.11	-0.11	-0.11	-0.11

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.01	-0.02	-0.02
5%	-0.01	-0.01	-0.01	-0.01
10%	-0.00	-0.01	-0.01	-0.01
15%	-0.00	-0.00	-0.00	-0.01

+30/-30 stocks	0%	5%	10%	15%
0	NA	-0.01	-0.01	-0.01
5%	0.01	0.00	-0.00	-0.01
10%	0.02	0.01	0.01	0.01
15%	0.04	0.03	0.02	0.02

+30/-20 stocks	0%	5%	10%	15%
0	0.37	0.35	0.25	0.19
5%	0.18	0.16	0.13	0.11
10%	0.05	0.05	0.05	0.04
15%	-0.01	-0.00	0.00	0.00

+30/-25 stocks	0%	5%	10%	15%
0	0.66	0.54	0.39	0.32
5%	0.44	0.36	0.30	0.27
10%	0.30	0.27	0.25	0.23
15%	0.23	0.22	0.21	0.20

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.31	-0.30	-0.29
5%	-0.31	-0.31	-0.30	-0.30
10%	-0.30	-0.31	-0.30	-0.30
15%	-0.29	-0.29	-0.29	-0.29

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.23	-0.21	-0.20
5%	-0.26	-0.24	-0.23	-0.22
10%	-0.27	-0.26	-0.25	-0.24
15%	-0.29	-0.28	-0.27	-0.25

calibration period: 4 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.26	-0.26	-0.26
5%	-0.27	-0.26	-0.26	-0.26
10%	-0.28	-0.28	-0.27	-0.27
15%	-0.31	-0.31	-0.31	-0.30

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.27	-0.27	-0.28
5%	-0.26	-0.26	-0.27	-0.27
10%	-0.26	-0.26	-0.26	-0.27
15%	-0.27	-0.27	-0.27	-0.27

+30/-30 stocks	0%	5%	10%	15%
0	NA	-0.01	-0.01	-0.02
5%	-0.01	-0.01	-0.01	-0.01
10%	-0.01	-0.01	-0.01	-0.01
15%	-0.02	-0.02	-0.02	-0.02

+30/-20 stocks	0%	5%	10%	15%
0	0.507325	0.46	0.35	0.28
5%	0.18	0.16	0.14	0.12
10%	-0.04	-0.03	-0.02	-0.02
15%	-0.19	-0.17	-0.16	-0.15

+30/-25 stocks	0%	5%	10%	15%
0	0.086091	0.10	0.07	0.05
5%	-0.10	-0.06	-0.04	-0.04
10%	-0.18	-0.14	-0.12	-0.11
15%	-0.23	-0.20	-0.18	-0.17

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	0.05	0.04	0.04
5%	0.06	0.06	0.05	0.04
10%	0.05	0.05	0.05	0.05
15%	0.01	0.01	0.02	0.02

+25/-25 stocks	0%	5%	10%	15%
0	NA	0.10	0.10	0.09
5%	0.11	0.11	0.10	0.10
10%	0.11	0.11	0.11	0.11
15%	0.10	0.10	0.10	0.10

+30/-30 stocks	0%	5%	10%	15%
0	NA	-0.01	-0.01	-0.02
5%	-0.01	-0.01	-0.01	-0.01
10%	-0.01	-0.01	-0.01	-0.01
15%	-0.02	-0.02	-0.02	-0.02

+30/-20 stocks	0%	5%	10%	15%
0	0.49	0.40	0.28	0.22
5%	0.36	0.28	0.22	0.18
10%	0.21	0.19	0.16	0.15
15%	0.09	0.09	0.08	0.08

+30/-25 stocks	0%	5%	10%	15%
0	0.64	0.45	0.29	0.21
5%	0.42	0.30	0.23	0.19
10%	0.27	0.23	0.19	0.17
15%	0.20	0.18	0.16	0.15

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.31	-0.33	-0.35
5%	-0.26	-0.28	-0.30	-0.32
10%	-0.22	-0.24	-0.26	-0.28
15%	-0.17	-0.19	-0.21	-0.23

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.16	-0.16	-0.16
5%	-0.16	-0.16	-0.16	-0.16
10%	-0.17	-0.16	-0.16	-0.16
15%	-0.18	-0.18	-0.18	-0.17

calibration period: 5 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.21	-0.21	-0.21
5%	-0.21	-0.21	-0.21	-0.21
10%	-0.21	-0.21	-0.21	-0.21
15%	-0.21	-0.21	-0.21	-0.21

+25/-25 stocks	0%	5%	10%	15%
0	NA	-0.21	-0.22	-0.23
5%	-0.17	-0.19	-0.20	-0.21
10%	-0.15	-0.16	-0.17	-0.18
15%	-0.11	-0.12	-0.13	-0.14

+30/-30 stocks	0%	5%	10%	15%
0	NA	0.03	0.01	0.00
5%	0.07	0.05	0.04	0.03
10%	0.10	0.08	0.07	0.06
15%	0.14	0.13	0.12	0.11

+30/-20 stocks	0%	5%	10%	15%
0	0.429216	0.41	0.32	0.25
5%	0.17	0.17	0.15	0.13
10%	-0.01	0.01	0.01	0.01
15%	-0.11	-0.10	-0.09	-0.08

+30/-25 stocks	0%	5%	10%	15%
0	0.112602	0.14	0.11	0.08
5%	-0.02	0.01	0.01	0.01
10%	-0.07	-0.04	-0.04	-0.03
15%	-0.07	-0.05	-0.05	-0.05

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	0.09	0.08	0.07
5%	0.13	0.11	0.10	0.09
10%	0.15	0.14	0.13	0.12
15%	0.18	0.17	0.17	0.16

+25/-25 stocks	0%	5%	10%	15%
0	NA	0.04	0.02	0.01
5%	0.08	0.06	0.05	0.04
10%	0.11	0.09	0.08	0.07
15%	0.15	0.13	0.12	0.11

+30/-30 stocks	0%	5%	10%	15%
0	NA	0.03	0.01	0.00
5%	0.07	0.05	0.04	0.03
10%	0.10	0.08	0.07	0.06
15%	0.14	0.13	0.12	0.11

+30/-20 stocks	0%	5%	10%	15%
0	0.43	0.38	0.27	0.21
5%	0.38	0.31	0.25	0.21
10%	0.29	0.26	0.23	0.20
15%	0.25	0.24	0.23	0.21

+30/-25 stocks	0%	5%	10%	15%
0	0.40	0.32	0.21	0.16
5%	0.28	0.21	0.17	0.14
10%	0.21	0.18	0.16	0.14
15%	0.20	0.19	0.17	0.16

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	-0.05	-0.07	-0.09
5%	0.01	-0.02	-0.04	-0.05
10%	0.07	0.04	0.02	0.00
15%	0.17	0.14	0.12	0.10

+25/-25 stocks	0%	5%	10%	15%
0	NA	0.09	0.08	0.07
5%	0.13	0.11	0.10	0.09
10%	0.16	0.14	0.13	0.12
15%	0.21	0.19	0.18	0.17

Appendix 12
Information ratios for the long-short strategies

calibration period: 3 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.37	0.60	0.36
5%	1.01	0.41	0.22	0.12
10%	0.21	0.07	0.01	-0.03
15%	-0.08	-0.12	-0.14	-0.15

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.57	0.75	0.47
5%	1.25	0.58	0.36	0.24
10%	0.42	0.24	0.16	0.11
15%	0.14	0.08	0.04	0.01

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.88	1.00	0.70
5%	1.62	0.87	0.62	0.49
10%	0.75	0.54	0.43	0.37
15%	0.47	0.38	0.33	0.29

+30/-20 stocks	0%	5%	10%	15%
0	1.90	1.69	1.26	0.98
5%	1.04	0.87	0.72	0.60
10%	0.43	0.38	0.34	0.31
15%	0.07	0.08	0.07	0.07

+30/-25 stocks	0%	5%	10%	15%
0	2.22	1.60	1.02	0.76
5%	1.04	0.75	0.57	0.47
10%	0.44	0.36	0.31	0.27
15%	0.17	0.15	0.14	0.13

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.62	0.82	0.56
5%	1.30	0.66	0.45	0.34
10%	0.50	0.34	0.26	0.21
15%	0.22	0.17	0.14	0.12

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.79	0.75	0.47
5%	1.49	0.81	0.58	0.46
10%	0.66	0.48	0.39	0.33
15%	0.38	0.31	0.27	0.24

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.88	1.00	0.70
5%	1.62	0.87	0.62	0.49
10%	0.75	0.54	0.43	0.37
15%	0.47	0.38	0.33	0.29

+30/-20 stocks	0%	5%	10%	15%
0	1.86	1.55	1.11	0.85
5%	1.16	0.91	0.71	0.59
10%	0.60	0.52	0.44	0.39
15%	0.31	0.28	0.26	0.24

+30/-25 stocks	0%	5%	10%	15%
0	2.90	2.04	1.34	1.02
5%	1.63	1.16	0.91	0.76
10%	0.92	0.76	0.66	0.59
15%	0.60	0.54	0.50	0.46

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.40	0.61	0.35
5%	1.15	0.48	0.26	0.15
10%	0.35	0.17	0.08	0.03
15%	0.09	0.02	-0.02	-0.05

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.58	0.76	0.49
5%	1.27	0.58	0.36	0.26
10%	0.41	0.24	0.15	0.11
15%	0.11	0.05	0.02	0.00

calibration period: 4 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.35	0.64	0.40
5%	0.94	0.43	0.25	0.17
10%	0.20	0.10	0.04	0.01
15%	-0.09	-0.11	-0.13	-0.13

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.36	0.63	0.39
5%	0.99	0.44	0.26	0.16
10%	0.26	0.14	0.07	0.03
15%	0.01	-0.03	-0.06	-0.08

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.70	0.93	0.67
5%	1.35	0.75	0.55	0.44
10%	0.58	0.43	0.35	0.30
15%	0.30	0.25	0.22	0.20

+30/-20 stocks	0%	5%	10%	15%
0	1.79	1.51	1.12	0.89
5%	0.95	0.78	0.64	0.55
10%	0.37	0.34	0.30	0.28
15%	0.01	0.02	0.02	0.02

+30/-25 stocks	0%	5%	10%	15%
0	2.04	1.41	0.93	0.70
5%	0.84	0.61	0.48	0.41
10%	0.29	0.25	0.22	0.20
15%	0.04	0.04	0.04	0.03

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.71	0.96	0.70
5%	1.34	0.78	0.59	0.48
10%	0.58	0.45	0.39	0.34
15%	0.25	0.23	0.21	0.20

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.74	0.63	0.39
5%	1.39	0.83	0.63	0.54
10%	0.65	0.52	0.45	0.40
15%	0.39	0.35	0.32	0.30

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.70	0.93	0.67
5%	1.35	0.75	0.55	0.44
10%	0.58	0.43	0.35	0.30
15%	0.30	0.25	0.22	0.20

+30/-20 stocks	0%	5%	10%	15%
0	2.02	1.52	1.07	0.83
5%	1.26	0.96	0.76	0.64
10%	0.69	0.59	0.51	0.46
15%	0.33	0.31	0.29	0.27

+30/-25 stocks	0%	5%	10%	15%
0	2.69	1.77	1.15	0.86
5%	1.41	0.99	0.77	0.64
10%	0.77	0.63	0.54	0.48
15%	0.47	0.42	0.38	0.35

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.35	0.58	0.31
5%	1.06	0.45	0.24	0.12
10%	0.35	0.18	0.09	0.03
15%	0.14	0.07	0.02	-0.02

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.49	0.75	0.49
5%	1.16	0.58	0.38	0.28
10%	0.41	0.26	0.19	0.14
15%	0.13	0.09	0.06	0.04

calibration period: 5 years

RD

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.46	0.74	0.50
5%	0.99	0.49	0.32	0.24
10%	0.26	0.16	0.11	0.08
15%	-0.00	-0.02	-0.04	-0.05

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.36	0.66	0.42
5%	0.98	0.48	0.30	0.21
10%	0.32	0.20	0.13	0.09
15%	0.12	0.08	0.05	0.03

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.63	0.91	0.66
5%	1.29	0.75	0.55	0.45
10%	0.61	0.47	0.40	0.35
15%	0.41	0.36	0.32	0.29

+30/-20 stocks	0%	5%	10%	15%
0	1.69	1.44	1.07	0.85
5%	0.96	0.79	0.65	0.55
10%	0.40	0.37	0.33	0.30
15%	0.09	0.09	0.09	0.08

+30/-25 stocks	0%	5%	10%	15%
0	1.92	1.38	0.93	0.71
5%	0.86	0.64	0.51	0.43
10%	0.36	0.31	0.27	0.25
15%	0.15	0.14	0.14	0.13

RA

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.76	1.01	0.76
5%	1.37	0.83	0.63	0.53
10%	0.65	0.53	0.46	0.41
15%	0.41	0.38	0.35	0.34

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.59	0.66	0.42
5%	1.24	0.72	0.54	0.45
10%	0.58	0.46	0.39	0.34
15%	0.38	0.34	0.31	0.29

+30/-30 stocks	0%	5%	10%	15%
0	NA	1.63	0.91	0.66
5%	1.29	0.75	0.55	0.45
10%	0.61	0.47	0.40	0.35
15%	0.41	0.36	0.32	0.29

+30/-20 stocks	0%	5%	10%	15%
0	2.30	1.56	1.03	0.78
5%	1.19	0.85	0.67	0.56
10%	0.65	0.54	0.47	0.42
15%	0.43	0.39	0.36	0.34

+30/-25 stocks	0%	5%	10%	15%
0	1.84	1.42	1.02	0.80
5%	1.25	0.95	0.76	0.64
10%	0.74	0.64	0.56	0.50
15%	0.48	0.45	0.42	0.39

F3

+20/-20 stocks	0%	5%	10%	15%
0	NA	1.53	0.81	0.56
5%	1.21	0.67	0.47	0.37
10%	0.56	0.42	0.34	0.28
15%	0.42	0.36	0.32	0.29

+25/-25 stocks	0%	5%	10%	15%
0	NA	1.64	0.94	0.69
5%	1.33	0.79	0.60	0.50
10%	0.67	0.53	0.45	0.40
15%	0.47	0.42	0.38	0.36

Appendix 13
The impact of repo costs

To account for impact on the performance of our long-short strategies of repo costs on the equity short positions, we have estimated them for a number of selected strategies, i.e. annual re-ranking stock selection method, 3 years of calibration period, benchmarks up to 15% away from the market index.

The repo costs were computed at 0.25% of the increase in the short position in case of a bull market for a particular stock (defined as an increase in price over the last 10 trading days) and at 0.35% on the same amount in case of a bear market for that particular stock (defined as a decrease in price over the last 10 trading days).

The repo costs for the selected strategies are reported below. They range from 1% for the +/-0% strategy to 8% for +/-15%, over the entire back-test period. As shown previously in respect of the transaction costs, the strategies tracking wide spreads attract high repo costs, which significantly erodes their profitability.

As expected, the impact of the repo costs turned out to be relevant only in case of the strategies' returns and did not affect their volatility, correlation with the market index, skewness or kurtosis. The new Sharpe ratios (average over the period Jan 95 to Dec 01), after including the repo costs are reported below. We have also reported the average annual Sharpe ratios (Jan-95 to Dec-01) for the strategies combining index tracking with long-short strategies presented in Section 5.

Repo costs on selected long-short strategies

+30/-25 stocks	0%	5%	10%	15%
0	1.06%	2.05%	3.66%	5.87%
5%	1.60%	2.76%	4.36%	6.52%
10%	2.43%	3.59%	5.17%	7.34%
15%	3.24%	4.38%	5.95%	8.06%

Average annual Sharpe ratios on selected long-short strategies

+30/-25 stocks	0%	5%	10%	15%
0	0.52	0.48	0.33	0.26
5%	0.31	0.29	0.24	0.21
10%	0.24	0.21	0.19	0.17
15%	0.17	0.16	0.16	0.15

Average annual Sharpe ratios on strategies combining index tracking excess return with long-short market neutral

+30/-25 stocks	0%	5%	10%
0	0.62	0.53	0.37
5%	0.76	0.48	0.32
10%	0.55	0.38	0.27

Average annual Sharpe ratios on alpha transport strategies

+30/-25 stocks	0%	5%	10%
0	0.64	0.64	0.63
5%	0.66	0.65	0.61
10%	0.66	0.64	0.59

Appendix 14
Some stock weights in the long-short strategies