Hedging Effectiveness of VIX Futures

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Abstract

The introduction of VIX futures and options has been a major financial innovation that will facilitate to a great extent the hedging of volatility risk. Using VIX futures, S&P 500 futures, S&P 500 options and S&P 500 futures options, this study examines alternative models within a *delta-vega* neutral strategy. VIX futures are found to outperform vanilla options in hedging a short position in S&P 500 *futures call options*. In particular, incorporating stochastic volatility and price jumps enhances hedging performance.

Keywords: VIX futures, S&P 500 futures, S&P 500 options, S&P 500 futures options, delta-vega neutral strategy, stochastic volatility, price jumps

Classification code: G12, G13

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1. Introduction

The 1987 crash brought volatility products to the attention of academics and practitioners and the Chicago Board Options Exchange (CBOE) successively launched Volatility Index (VIX) futures on March 26, 2004 and VIX options on February 24, 2006. These were the first of an entire family of volatility products to be traded on exchanges. As shown in Figure 1, their trading volume and open interest grew significantly over the period of March 2004 to February 2009 for VIX futures, and February 2006 to September 2008 for VIX options, reflecting their economic importance.

[Figure 1 about here]

The VIX calculation isolates expected volatility from other factors that could affect option prices such as dividends, interest rates, changes in the underlying price and time to expiration. VIX options and VIX futures consequently offer a way for investors to buy and sell option volatility without having to deal with factors that have an impact on the value of an S&P 500 index (SPX) option position.

The introduction of VIX futures and options has been a major financial innovation that will facilitate to a great extent the hedging of volatility risk. Traditionally, volatility hedging has been executed by market makers and other market participants using vanilla options. For example, Carr and Madan (1998) suggest options on a straddle and Brenner, Ou and Zhang (2006) construct a straddle from vanilla call and put options to hedge volatility risk. However, under this approach delta and volatility must be hedged simultaneously. A slightly dissenting focus is Rebonato (1999), who constructs two wide strangles with different maturities so that the changes of underlying stock price will not affect the payoff of the portfolio. The *forward-start* strangle hedges *forward* volatility risk without exposure to delta and gamma risk.[1] Other than using vanilla options, Neuberger (1994) adopts the log contract to hedge volatility. Finally, Psychoyios and Skiadopoulos (2006) in their simulation hedge instantaneous volatility using a volatility call option. They conclude that a vanilla option is a more efficient instrument than a volatility option to hedge volatility risk.

VIX futures and options offer pure exposure to volatility dynamics and, at least in theory, should provide a more effective hedge. The hedging effectiveness of the new VIX derivatives is an important question that has not been concluded in the literature. This paper addresses this question by using VIX futures, SPX futures, SPX options and SPX futures options to examine alternative models within a *delta-vega* neutral strategy.

The risks of an option writer can be partitioned into price risk and volatility risk.

[1] Since the present value profile of a straddle as a function of spot around the at-the-money level is less flat than a strangle, the delta and gamma of a straddle are less close to zero than a strangle. This study thus uses a *forward-start* strangle, instead of a *forward-start* straddle, to hedge the *forward* volatility risk.

For the futures option writer, the volatility risk he faces includes *spot* and *forward* volatility risk. The *forward* volatility risk refers to the risk exposure induced by volatility randomness between the futures option's expiry and the futures' expiry. Since VIX futures settle to the 30-day *forward* volatility of the S&P 500, they are natural to hedge the *forward vega* risk of SPX futures options.[2] A short position on SPX futures call options is chosen as our target instrument, since it consists of the volatility randomness between option's expiry and its underlying futures' expiry. In contrast, the traditional straddle or strangle strategy mainly hedges the volatility risk between current day and option's expiry, denoted *spot* volatility risk.

In the present literature, there are at least four sources of stochastic variations for SPX options: diffusive price risk, price-jump risk, volatility risk and interest rate risk. Bakshi, Cao and Chen (1997) find that once the stochastic volatility is modeled, the hedging performance may be improved by incorporating neither price jumps, nor stochastic interest rates into the SPX option pricing framework. Bakshi and Kapadia (2003) use Heston's (1993) stochastic-volatility option pricing model to construct a delta-hedged strategy for a long position on SPX call options. They find that the volatility risk is priced and the price jump affects the hedging efficiency. Vishnevskaya

[2] Lin (2007) demonstrates the fair value to VIX futures is the *forward* VIX. Hence, the current price of VIX futures reflects the market's expectation of the VIX level at expiration, that is, *forward* VIX.

(2004) follows the structure of Bakshi and Kapadia (2003) and constructs a *delta-vega*-hedged portfolio for a long position on the SPX call option, consisting of the underlying stock, another option and the money-market fund. His result suggests the existence of some other sources of risk.

Guided by previous studies, the price risk, stochastic-volatility risk and price-jump risk apparently become the key factors when constructing a hedging strategy for SPX option writers. Hence, this study examines the SPX futures option model that allows volatility and price jumps to be stochastic, abbreviated as the SVJ model. The setup contains competing futures option formulas as special cases, including the constant-volatility (CONST) model and the stochastic-volatility (SV) model. In reality most futures option contracts are American-style. It is important, in principle, to take into account the extra value accruing from the ability to exercise the options prior to maturity. One can follow such a nonparametric approach as in Aït-Sahalia and Lo (1998) and Broadie, Detemple, Ghysels and Torrés (2000) to price American options. Closed-form option pricing formulas, however, make it possible to derive hedge ratios analytically. Therefore, for options with early exercise potential this paper computes a quadratic approximation for evaluating American futures options. The approximation is based on the one developed by MacMillan (1987), examined by Barone-Adesi and

Whaley (1987) for the CONST process, extended by Bates (1991) for the jump-diffusion process, and modified by Bates (1996) for the SV and SVJ processes. For the CONST process this approximation for the early exercise premium reconciles Whaley's (1986) American index futures option pricing formula. For the SV and SVJ processes this approximation is consistent with Bates (1996) for evaluating American currency futures options.

For the purpose of comparison, a *forward-start* strangle portfolio is proposed to manage *forward* volatility risk. This study then constructs the hedged portfolio by coupling these models with two hedging schemes that use either VIX futures or the *forward-start* strangle portfolio as the instruments to manage *forward* volatility risk. Our finding reveals that the VIX futures generally outperform the *forward-start* strangle portfolio over the hedging period October 20, 2004–June 30, 2005. Based on our results, this paper concludes that the VIX futures contract is a better hedging instrument than vanilla options if the target asset is a SPX *futures call option*. Hedging performance can be also improved further by incorporating price jumps into the American futures option pricing framework.

The rest of this paper proceeds as follows. Next section illustrates hedging strategies. Pricing models for calculating delta and vega hedge ratios are presented in

Section 3. Section 4 summarizes data and model parameter estimation. Section 5 analyzes empirical results. Section 6 finally concludes.

2. Hedging Strategies

A time-*t* short position on the T_1-matured call option written on T_2-matured SPX futures is used as the target portfolio, i.e. $TAR_t = -C_t^A(F)$ for $t < T_1 < T_2$. This study then constructs two hedging schemes to hedge the target portfolio.

Hedging Scheme 1 (HS1): *The instrument portfolio consists of* $N_{1,t}$ *shares of underlying SPX futures, and* $N_{2,t}$ *shares of forward-start strangle portfolios. The forward-start strangle portfolio consists of a short position on a* T_1*-matured strangle and a long position on a* T_2*-matured strangle, denoted as*

$$INST_t = -c_t^E(S,T_1,K_2) - p_t^E(S,T_1,K_1) + c_t^E(S,T_2,K_2) + p_t^E(S,T_2,K_1) \tag{1}$$

where $c_t^E(S,T_1,K_2)$ *and* $c_t^E(S,T_2,K_2)$ *are* K_2*-strike SPX call options with maturities* T_1 *and* T_2*, respectively.* $p_t^E(S,T_1,K_1)$ *and* $p_t^E(S,T_2,K_1)$ *are* K_1*-strike SPX put options with maturities* T_1 *and* T_2*, respectively.*

Hedging Scheme 2 (HS2): *The instrument portfolio consists of* $N_{1,t}$ *shares of* *underlying SPX futures, and* $N_{2,t}$ *shares of the VIX futures, i.e.,*

$$INST_t = \mathrm{F}_t^{\mathrm{VIX}}(T_1) \tag{2}$$

where $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ *is the time-t price of the VIX futures with expiry* T_1.

The gain or loss of this hedged portfolio is expressed by

$$\pi_t = N_{1,t} F_t(T_2) + N_{2,t} INST_t - C_t^A(F) \tag{3}$$

Further add the constraints of delta-neutral and vega-neutral by

$$\frac{\partial \pi_t}{\partial F_t} = N_{1,t} + N_{2,t} \frac{\partial INST_t}{\partial F_t} - \frac{\partial C_t^A(F)}{\partial F_t} = 0 \tag{4}$$

$$\frac{\partial \pi_t}{\partial \nu_t} = N_{2,t} \frac{\partial INST_t}{\partial \nu_t} - \frac{\partial C_t^A(F)}{\partial \nu_t} = 0 \tag{5}$$

The shares of instrument assets are computed as

$$N_{1,t} = \frac{\partial C_t^A(F)}{\partial F_t} - N_{2,t} \frac{\partial INST_t}{\partial F_t} \tag{6}$$

$$N_{2,t} = \left(\frac{\partial C_t^A(F)}{\partial \nu_t} \right) / \left(\frac{\partial INST_t}{\partial \nu_t} \right) \tag{7}$$

The formulas of $\partial C_t^A(F)/\partial F_t$, $\partial INST_t/\partial F_t$, $\partial C_t^A(F)/\partial \nu_t$ and $\partial INST_t/\partial \nu_t$ for alternate models are provided in the following section.

Next, this study couples these two hedging schemes with the CONST, the SV and the SVJ option models to construct six hedging strategies: HS1-CONST, HS1-SV, HS1-SVJ, HS2-CONST, HS2-SV, and HS2-SVJ. Assuming that there are no arbitrage opportunities, the hedged portfolio π_t should earn the risk-free interest rate *r*. In other words, the change in the value of this hedged portfolio over Δt is expressed as

$$\Delta\pi_{t+\Delta t} = \pi_{t+\Delta t} - \pi_t = \pi_t(e^{r\Delta t} - 1) \tag{8}$$

where $\Delta\pi_{t+\Delta t} = N_{1,t}[F_{t+\Delta t}(T_2) - F_t(T_2)] + N_{2,t}[INST_{t+\Delta t} - INST_t] - [C^A_{t+\Delta t}(F) - C^A_t(F)]$.

Hence, the hedging error is defined as the additional profit (loss) over the risk-free return and it can be written as

$$\begin{aligned} HE_t(t+\Delta t) &= \Delta\pi_{t+\Delta t} - \pi_t(e^{r\Delta t} - 1) \\ &= N_{1,t}[F_{t+\Delta t}(T_2) - F_t(T_2)] + N_{2,t}[INST_{t+\Delta t} - INST_t] - [C^A_{t+\Delta t}(F) - C^A_t(F)] \\ &\quad - (e^{r\Delta t} - 1)[N_{1,t}F_t(T_2) + N_{2,t}INST_t - C^A_t(F)] \end{aligned} \tag{9}$$

And the absolute hedging error through a hedging period ($T_1 - t$) is calculated as

$$TDHE(t,T_1) = \sum_{l=1}^{M} | HE_{t+(l-1)\Delta t}(t + l\Delta t)\, e^{r\Delta t(M-l)} | \tag{10}$$

where $M = (T_1 - t)/\Delta t$ and T_1 is the expiry of the target SPX futures call option.

3. Empirical Pricing Models

Hedging strategies are constructed using SPX futures, SPX options, SPX futures options and VIX futures. Therefore, their fair value and related Greeks are required for further empirical analyses. The most general process considered in this paper is the jump-diffusion and stochastic volatility (SVJ) process of Bates (1996) and Bakshi et al. (1997). This general process contains stochastic volatility (SV) of Heston (1993) and constant volatility (CONST) of Black and Scholes (1973) and Merton (1973) as special cases. Consequently, pricing formulas and related Greeks for the SV model obtain as a special case of the general model with price jumps restricted to zero, i.e., $J_t dN_t = 0$ and thus $\lambda_J = \kappa^* = \sigma_J$ =0. Further setting stochastic volatility to constant volatility, pricing formulas and related hedge ratios for the CONST model are obtained.

3.1 SVJ Process for the SPX Price

Contingent claims are priced as if investors were risk-neutral and under the SVJ model the SPX price follows the jump-diffusion with stochastic volatility

$$dS_t = (b - \lambda_J \mu_J) S_t + \sqrt{v_t} S_t \, d\omega_{S,t} + J_t S_t dN_t \qquad (11)$$

where b is the cost of carry coefficient (0 for futures options and $r - \delta$ for stock options with a cash dividend yield δ). J_t is the percentage jump size with mean κ^*. The jumps in the asset log-price are assumed to be normally distributed, i.e., $\ln(1 + J_t) \sim N(\mu_J, \sigma_J^2)$. Satisfying the no-arbitrage condition, $\kappa^* = \exp(\mu_J + \sigma_J^2 / 2) - 1$. N_t is the jump frequency following a Poisson process with mean λ_J. The instantaneous variance v_t of the index follows a mean-reverting square root process

$$dv_t = (\theta_v - \kappa_v v_t) dt + \sigma_v \sqrt{v_t} \, d\omega_{v,t} \qquad (12)$$

where κ_v is the speed of mean-reverting adjustment of v_t; θ_v / κ_v is the long-run mean of v_t; σ_v is the variation coefficient of v_t; and $\omega_{S,t}$ and $\omega_{v,t}$ are two correlated Brownian motions with the correlation coefficient $\rho\, dt = corr(d\omega_{S,t}, d\omega_{v,t})$.

3.2 Fair Value to SPX Options

SPX options are European-style. Bakshi et al. (1997) provide the time-*t* value of SPX call and put options with strike *K* and maturity T for the SVJ model:

$$c_t^E(S,K,T) = S_t e^{-\delta(T-t)}\Pi_1 - Ke^{-r(T-t)}\Pi_2 \tag{13}$$

$$p_t^E(S,K,T) = S_t e^{-\delta(T-t)}(\Pi_1 - 1) - Ke^{-r(T-t)}(\Pi_2 - 1) \tag{14}$$

where Π_1 and Π_2 are risk-neutral probabilities that are recovered from inverting the characteristic functions f_1 and f_2, respectively,

$$\Pi_j(T-t, K; S_t, r, v_t) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\varphi \ln K} f_j(t, T-t; S_t, r, v_t; \varphi)}{i\varphi}\right] d\varphi \tag{15}$$

for *j* = 1, 2. The characteristic functions f_1 and f_2 for the SVJ model are given in equations (A12) and (A13) of Bakshi et al. (1997). Delta and vega of the European SPX options are given in equation (13) of Bakshi et al. (1997). Finally, delta and vega of the *forward-start* strangle portfolio can be calculated straightforward.

3.3 Fair Value to SPX Futures Options

Since SPX futures options are American-style, it is important, in principle, to take into account the extra value accruing from the ability to exercise the options prior to maturity. Referred to Bates (1996), the futures call option is

$$C_t^A(F,K,T_1) \equiv \begin{cases} C_t^E(F,K,T_1) + KA_2\left(\dfrac{F_t(T_2)/K}{y_c^*}\right)^{q_2} & \text{if} \quad F_t(T_2)/K < y_c^* \\ F_t(T_2) - K & \text{if} \quad F_t(T_2)/K \geq y_c^* \end{cases} \tag{16}$$

where $C_t^E(F,K,T_1) = e^{-r(T_1-t)}[F_t(T_2)\Pi_1' - K\Pi_2']$ is the time-*t* price of a European-style futures call option with strike K and expiry T_1. $F_t(T_2)$ is the time-*t* futures price with maturity T_2. For *j*=1, 2, Π_j' are the relevant tail probabilities for evaluating futures call options, which are the same to Π_j in equation (15) except replacing *K* with $Ke^{-(r-\delta)(T_2-T_1)}$. $A_2 = (y_c^*/q_2)[1 - \partial C_t^E(y_c^*,1,T_1)/\partial F_t]$. For the SVJ process, q_2 is the positive root to the equation of *q* given as follows

$$\frac{1}{2}\bar{V}q^2 + \left(-\lambda_J\kappa^* - \frac{1}{2}\bar{V}\right)q - \frac{r}{1-e^{-r(T_1-t)}} + \lambda_J[(1+\kappa^*)^q e^{q(q-1)\sigma_J^2/2} - 1] = 0 \tag{17}$$

$\bar{V}$ is the expected average variance over the lifetime of the option conditional on no jumps, i.e., $\bar{V} = \dfrac{\theta_V}{\kappa_V} + \left(V_t - \dfrac{\theta_V}{\kappa_V}\right)\dfrac{[1-e^{-\kappa_V(T_1-t)}]}{\kappa_V(T_1-t)}$. The critical futures price/strike price ratio $y_c^* \geq 1$ above which the futures call is exercised immediately is given implicitly by

$$y_c^* - 1 = C_t^E(y_c^*,1,T_1) + \left(\frac{y_c^*}{q_2}\right)\left[1 - \frac{\partial C_t^E(y_c^*,1,T_1)}{\partial F_t}\right] \tag{18}$$

The closed form solutions to the parameters q_2 and y_c^* are provided for given model parameters and for given maturity T_1. Since linear homogeneity in underlying asset and strike holds for European options, by Euler theorem the following equations sustain:

$$C_t^E(y_c,1,T_1)=\frac{1}{K}C_t^E(F,K,T_1)=e^{-r(T_1-t)}(y_c\Pi_1'-\Pi_2') \tag{19}$$

$$\frac{\partial C_t^E(y_c,1,T_1)}{\partial F_t}=\frac{1}{K}\frac{\partial C_t^E(F,K,T_1)}{\partial F_t}=\frac{1}{K}e^{-r(T_1-t)}\Pi_1' \tag{20}$$

where $y_c=F_t(T_2)/K$. Replacing $C_t^E(y_c,1,T_1)$ and $\frac{\partial C_t^E(y_c,1,T_1)}{\partial F_t}$ of y_c^* in equation (18) with equations (19) and (20), the solution to y_c^* is given by

$$y_c^*=\frac{1-e^{-r(T_1-t)}\Pi_2'}{1-\frac{1}{q_2}+\left(\frac{1}{q_2K}-1\right)e^{-r(T_1-t)}\Pi_1'} \tag{21}$$

Thus, the solution to A_2 is computed as

$$A_2=\frac{[1-e^{-r(T_1-t)}\Pi_2']}{\left[q_2-1+\left(\frac{1}{K}-q_2\right)e^{-r(T_1-t)}\Pi_1'\right]}\left[1-\frac{1}{K}e^{-r(T_1-t)}\Pi_1'\right] \tag{22}$$

Finally, the calculation for delta and vega of the futures call option is straightforward.

3.4 Fair Value to VIX Futures

From Lin (2007), the time-*t* fair price of the VIX futures expiring at *T* under the SVJ model is given by

$$\mathrm{F}_t^{\mathrm{VIX}}(T)\equiv\sqrt{\zeta_2+\frac{1}{\tau}(a_\tau\alpha_{T-t}v_t+a_\tau\beta_{T-t}+b_\tau)}-\frac{\left(\frac{a_\tau}{\tau}\right)^2(C_{T-t}v_t+D_{T-t})}{8\left[\zeta_2+\frac{1}{\tau}(a_\tau\alpha_{T-t}v_t+a_\tau\beta_{T-t}+b_\tau)\right]^{3/2}} \tag{23}$$

where $\tau=\frac{30}{365}$, $\zeta_2=2\lambda_J(\kappa^*-\mu_J)$, $\alpha_{T-t}=e^{-\kappa_v(T-t)}$, $\beta_{T-t}=\frac{\theta_v}{\kappa_v}(1-\alpha_{T-t})$, $a_\tau=\frac{(1-e^{-\kappa_v\tau})}{\kappa_v}$,

$$b_\tau = \frac{\theta_v}{\kappa_v}(\tau - a_\tau)\,, C_{T-t} = \frac{\sigma_v^2}{\kappa_v}(\alpha_{T-t} - \alpha_{T-t}^2)\,, D_{T-t} = \frac{\sigma_v^2 \theta_v}{2\kappa_v^2}(1-\alpha_{T-t})^2\,,\;\; v_t \equiv \frac{\tau(\mathrm{VIX}_t^2 - \zeta_2) - b_\tau}{a_\tau}\,.$$

4. Data and Parameter Estimation

The hedging period of this study is from October 20, 2004 to June 30, 2005. Intraday prices for SPX futures and SPX futures options are obtained from CME. Daily prices for SPX options and VIX futures are retrieved from CBOE. Further, the contracts that are selected for empirical analyses are described as follows: First, the selected SPX futures contracts expire in March, June, September and December. Second, the SPX futures call options that expire in February, May, August and November are selected as the target portfolio. Third, the *forward-start* strangle portfolio involves in two strangles. This study uses the SPX options contracts that expire in February, May, August and November to construct a short-term strangle, and that expire in March, June, September, and December for another long-term strangle. Finally, the VIX futures that expire in February, May, August and November are selected as the hedging instrument. The interest rate data are daily annualized Treasury-bill rates obtained from Datastream database. The daily dividend-yield ratio data are obtained from the S&P Corporation. The data of SPX options, SPX futures options and SPX futures that violate the upper and lower boundaries are not included in the sample.

The SPX futures options that expire in the February quarterly cycle consist of 29,804 intraday observations. This study employs the last reported quote of each contract for each day. Hence, there are in total 7,231 observations remained. After coupling with VIX futures that expire in the February quarterly cycle, there are 7,003 observations in the sample. Since the SPX options available for constructing the *forward-start* strangle only cover the period from October 20, 2004 to June 30, 2005, this study further filters out the 4,521 SPX futures options and 2,482 SPX futures options observations remain. Table 1 reports descriptive properties of the SPX futures call options for each moneyness-maturity category where moneyness is defined as $F_t(T)/K$. Out of 2,482 SPX futures call option observations, about 56% is out-of-the-money (OTM) and 40% is at-the-money (ATM). The average futures call price ranges from 0.1827 points for short-term (<30 days) deep out-of-the-money (DOTM) call options to 117.7 points for medium-term (30–60 days) deep in-the-money (DITM) call options.

[Table 1 added here]

For the *forward-start* strangle strategy, this study uses SPX options that expire in the February quarterly cycle as T_1-strangle and that expire in the March quarterly cycle as T_2-strangle. Therefore, the pair of (T_1, T_2) data must be February–March, May–June, August–September and November–December. The maximum and the minimum strikes of SPX options available for each pair of (T_1, T_2) data on each day are selected as the two strike prices K_1 and K_2. Hence, there are four pairs of SPX options with strikes (K_1, K_2) corresponding to SPX options that expire on the four pairs of (T_1, T_2). There are 692 SPX option observations selected. Hence, the strike K_1 is the minimum strike that is available in the options which expire in the February and March quarterly cycles simultaneously, and are traded on each trading date. The result shows that the selected strike K_1 of the SPX options is 700 index points. The strike K_2 of the SPX options that expire in November 2004 and February, May, and August 2005 are 1,250, 1,250, 1,300 and 1,350, respectively. Table 2 reports sample properties of those SPX options that expire on the pairs of (T_1, T_2) from October 20, 2004 to June 30, 2005. It reports the average point of the SPX option and the observations for each moneyness-maturity category where moneyness is defined as *S/K*. There are in total 30,166 option observations, consisting of 15,083 SPX calls and 15,083 SPX puts. The average call prices range from 0.1628 points for short-term DOTM call options to 301.1628 points for long-term DITM call options. The average put prices range from 0.3092 points for short-term DOTM put options to 176.2546 points for long-term DITM put options.

[Table 2 added here]

The vector of structural parameters Φ for alternate processes is backed out by minimizing the sum of the squared pricing errors between option model and market prices over the period, April 21, 2004 to October 19, 2004. The minimization is given by

$$\min_{\Phi} \sum_{t=1}^{N_T} \sum_{n=1}^{N_t} [C_n - C_n^*(\Phi)]^2 \tag{24}$$

where N_T is the number of trading days in the estimation sample, N_t is the number of VIX futures, SPX options and SPX futures call options on day *t*, and C_n and C_n^* are the observed and model futures or option prices, respectively. The parameters of the CONST, SV and the SVJ models are estimated separately each month and thus Φ are assumed to be constant over a month. The assumption that the structural parameters are constant over a month is justified by an appeal to parameter stability (Bates, 1996; Eraker, 2004; Zhang and Zhu, 2006). The estimation period is chosen because the settlement day of the VIX futures is the third Wednesday, and the last trading day is Tuesday. Hence, the month is defined as the period from the third Wednesday of prior calendar month to the third Tuesday of this calendar month. The risk-neutral parameters κ_V, θ_V, σ_V and ρ of the SV model are on average 5.63, 0.69, 0.53 and –0.50, respectively. The risk-neutral parameters κ_V, θ_V, σ_V, ρ, λ_J, μ_J and σ_J of the SVJ model are on average 8.77, 0.65, 0.44, –0.42, 2.17, –0.35, and 0.31.

5. Empirical Results

This study follows two steps to assess the hedging performance of writing a SPX futures call option using two hedging schemes under three SPX price processes. First, this study uses the *previous month's* structural parameters and the *current day's* (t) SPX, SPX futures, SPX futures options, SPX options, VIX futures and U.S. Treasury-bill rates to construct the hedged portfolio. Second, this study calculates the hedging error of day $t + n$, where n is the available trading dates till SPX futures call option's expiry, and also rebalance the hedging portfolio. Since the quotes of each futures option are not all available for each day until its expiry, this study only takes rebalance on the day with available quote data after day t. These steps are repeated for each futures option contract that expires in February quarterly cycle on every trading date with quote data available in the sample. The hedging performance is reported in Tables 3 and 4. The average points of absolute hedging errors, defined as $\sum_{l=1}^{M} | HE_{t+(l-1)\Delta t}(t+l\Delta t)e^{r\Delta t(M-l)} | / M$, are presented in Table 3, where $M = (T_1 - t)/\Delta t$ and T_1 is the maturity date of SPX futures call options. Table 4 reports the average points of hedging errors defined as $\sum_{l=1}^{M} HE_{t+(l-1)\Delta t}(t+l\Delta t)e^{r\Delta t(M-l)} / M$. This study illustrates the hedging errors in points and each point represents $250.

In Table 3, under the CONST model, the hedging errors of HS2 range from 0.16 points (DOTM short-term) to 4.64 points (DITM long-term), whereas HS1 has hedging errors from 0.83 points (OTM short-term) to 11.57 points (DITM long-term). For the SV model, the hedging errors of HS2 are from 0.01 points (DOTM short-term) to 4.56 points (DITM long-term), whereas the hedging errors of HS1 range from 0.71 points (OTM short-term) to 7.79 points (ATM1 medium-term). For the SVJ model, HS2 has hedging errors from 0.01 points (DOTM short-term) to 4.53 points (DITM long-term), whereas HS1 has hedging errors from 0.61 points (OTM short-term) to 7.17 points (ATM1 medium-term). For all moneyness-maturity categories, HS2 performs better than HS1 and short-term SPX futures calls have smaller errors. The results are robust across models. The results indicate that the *forward-start* strangle portfolio is a less efficient instrument to hedge *forward* volatility risk than VIX futures.

The results also show that the absolute hedging errors of the SVJ model are less than that of the SV model. It represents the random price jump feature commonly exists in the SPX price process. However, this result seems not to be consistent with those of Bakshi et al. (1997) and Bakshi and Kapadia (2003), which show the hedging superiority of the SV model relative to the SVJ model. Note that the parameter of jump-frequency intensity λ_J in Bakshi et al. (1997) is 0.59, i.e. one year and half for a price jump to occur. Their hedging portfolio is rebalanced daily or every five days. They conclude that the reason for the SV model dominates the SVJ model in terms of hedging performance is the chance for a price jump to occur is small in the daily or five-day rebalancing period. Other than the uncertain rebalance frequency in our empirical work,[3] the estimated parameter λ_J in our empirical work is 2.17 larger than that of Bakshi et al. (1997). Therefore, their reason does not hold for our empirical result. One possible reason is that the SPX futures options used in this study are American-style, while SPX options are European-style for prior research. Since the traders with American-style options positions have early-exercise choice and thus can take caution to prevent any loss from the potential jump events than the ones with European-style options. Thus, American-style option buyers (sellers) may even favor (hate) volatility risk than the ones with European-style options. In addition, given the possibility of price jumps, the specification of SVJ can provide more accurate parameter estimates than SV (Bates, 1996). Thus, the *delta-vega*-neutral strategy could be constructed in a more effective way under SVJ than SV. Thus, it is not surprising for our results showing that SVJ outperforms SV in terms of hedging efficiency.

[3] There are in total 169 unique SPX futures call options contracts over our hedging period, 20 October 2004–30 June 2005. Among these data, there are 18 unique contracts can be daily rebalanced. The maximum rebalancing period is 26 days for only one unique contract (with May-2005 maturity on its first trading date, 8 March 2005). On average, the rebalancing period is 4.44 days.

About the maturity impact on hedging performance, the absolute hedging errors in general increase with maturity and the difference between HS1 and HS2 increases with maturity. Most of short-term options have smaller absolute hedging errors than medium- and long-term options. Except for the HS1 strategy under ATM1 and ATM2, medium-term options have smaller absolute hedging errors than long-term options. This result consists with Psychoyios and Skiadopoulos (2006) for ITM and OTM target options. They also show that the difference between hedging schemes decreases with maturity in case of ITM and OTM, and increases with maturity in case of ATM.

In terms of the moneyness effect, Psychoyios and Skiadopoulos (2006) find that the options perform best for ATM and worse for ITM, and the difference between hedging schemes is minimized for ATM and maximized for ITM. Our results show that the absolute hedging errors of HS2 increase with moneyness except for ATM2 long-term options, and the relationship between the absolute hedging errors of HS1 and moneyness is uncertain. Therefore, the difference between HS1 and HS2 across moneyness remains uncertain in this study.

[Table 3 added here]

Theoretically, if a portfolio is perfectly hedged, it should earn the risk-free rate of interest, and the average hedging errors should be close to zero. In this study, the hedging error is defined as the changes in the value of the hedged portfolio minus the risk-free return. Table 4 reports the average hedging errors. If the figure is greater (less) than zero, it means that the strategy gets more (less) profits than risk-free return. For most moneyness-maturity categories, the hedging performance through all hedging periods is less than risk-free rate. The average hedging error of HS2−SVJ strategy is the smallest in most moneyness-maturity categories.

Noticeably, HS2 scheme is superior to HS1 scheme for the CONST model. In the equity market the volatility is non-constant and stock-volatility correlation is markedly negative. Hence, the position hedged with CONST Greeks still has unhedged exposures. That will let this strategy incurs additional risk exposure and incurs losses. The losses are most apparent for the cases when using the HS1-CONST strategy and for the options across medium and long maturities when using the HS2-CONST strategy. The findings for the CONST model come into the following conclusions. First, the VIX futures that are volatility sensitive can help reduce model misspecification to hedge the volatility risk. Second, the magnitude of error reduction works best for short-dated options. This is consistent with Psychoyios and Skiadopoulos' (2006) result that the volatility is more stable in long-term than short-term. Compared with the SV model, most hedging performance of the SVJ model is smaller or comparable. It is consistent with the results in Table 3 and represents the existence of the random price jump feature for SPX futures options. Still, the hedging performance of HS2 is better than HS1.

[Table 4 added here]

6. Conclusion

This study examines the hedging performance of the VIX futures against the forward volatility risk. For the purpose of comparison, a *forward-start* strangle portfolio is also constructed for managing forward volatility. A short position on the SPX futures option is chosen as our target asset because its vega risk is related to forward volatility between the option's expiry and the underlying futures' expiry. This study then couples two hedging schemes (HS1 and HS2) with three SPX price processes (CONST, SV and SVJ) to hedge the target asset. On the one hand, SPX futures and the *forward-start* strangle portfolio are used to construct three hedging strategies (HS1−CONST, HS1−SV and HS1−SVJ). On the other hand, SPX futures and VIX futures are used to construct the other three hedging strategies (HS2−CONST, HS2−SV and HS2−SVJ).

There are some interesting empirical findings. First, HS2 dominates HS1 in most moneyness-maturity categories. That is, the VIX futures contract is a more efficient instrument to hedge forward volatility risk than a *forward-start* strangle portfolio. Second, gauged by the absolute hedging errors, the SVJ model is the best overall performer, followed by the SV model, and then by the CONST model. Third, VIX futures can help reduce constant-volatility model misspecification to manage the forward volatility risk. Our findings are in sharp contrast with that obtained by Psychoyios and Skiadopoulos (2006). They find that when applied to hedging a short position on a call option, volatility options are not better hedging instruments than plain-vanilla options, and that the most naïve volatility option-pricing model can be reliably used for pricing and hedging purposes. Further, the hedging-based ranking of the models is in contrast with that obtained in Bakshi et al. (1997). Bakshi et al. (1997) find that the SVJ does not improve over the SV's hedging performance for a short position on a SPX call option. Combined with prior studies and based on our results, this paper concludes that the VIX futures is a better hedging instrument than standard options if the target option is a traditional *futures option*, or equivalently if the risk exposure is the *forward* volatility risk. Hedging performance can also be improved further by incorporating price jumps into the American-style futures option pricing framework.

The contributions of this paper are threefold. First, closed-form solutions to the target American-style futures options under alternate SPX price processes are examined. Second, the concept of *forward* volatility risk applied to VIX futures and a *forward-start* strangle portfolio is introduced. Third, this study derives the hedging weights of VIX futures and the *forward-start* strangle portfolio that will be convenient to practical participants for risk management purposes.

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Table 1 Sample Properties of SPX Futures Call Options

The average points of futures options ($250 per point), the average points of underlying futures ($250 per point) and the total number of futures options are presented in each moneyness-maturity category. The futures option contracts listed at CME are four quarterly and two serial, and the longest time to maturity for the chosen futures options is 120 days. Therefore, this study classifies those observations into short-term (<30 days), medium-term (30–60 days), and long-term (≧60 days). Moneyness is defined as $F_t(T)/K$, where $F_t(T)$ is the price of the SPX futures and K is the strike price of SPX futures options. This study classifies those futures options into deep out-of-the-money (DOTM) if $F_t(T)/K$ ≦0.94; out-of-the-money (OTM) if $F_t(T)/K \in [0.94,0.97)$; at-the-money 1 (ATM1) if $F_t(T)/K \in [0.97,1)$; at-the-money 2 (ATM2) if $F_t(T)/K \in [1,1.03)$; in-the-money (ITM) if $F_t(T)/K \in [1.03,1.06)$; and deep in-the-money (DITM) if $F_t(T)/K$ >1.06. The data period is from 20 October 2004 to 30 June 2005.

	Moneyness $F_t(T)/K$	Maturity <30	Maturity 30–60	Maturity ≧60	Subtotal
		0.1827	0.5007	1.3969	
DOTM	≦0.94	1156.245	1175.539	1200.946	
		231	238	113	582
		0.7433	2.5717	4.7168	
OTM	0.94–0.97	1162.678	1184.388	1203.839	
		345	289	184	818
		4.9073	10.5415	12.7566	
ATM1	0.97–1	1168.642	1181.149	1205.591	
		424	196	61	681
		19.4346	24.2600	29.6692	
ATM2	1–1.03	1171.62	1176.276	1211.154	
		257	50	13	320
		48.7425	52.0571	NA	
ITM	1.03–1.06	1171.179	1166.5	NA	
		60	7	NA	67
		81.6333	117.7000	74.2000	
DITM	>1.06	1171.55	1216	1198.8	
		12	1	1	14
Subtotal		1329	781	372	2482

Table 2 Sample Properties of SPX Options

The average points of the SPX options ($100 per point) and the total number of the SPX options are presented in each moneyness-maturity category. The SPX options are traded in CBOE. Its expiration months are three near-term months followed by three additional months from the March quarterly cycle. The longest time to maturity for our observations is 723 days. Therefore, this study classifies those observations into short-term (<30 days), medium-term (30–60 days), and long-term (≧60 days). Moneyness is defined as S/K where S is the price of the SPX and K is the strike price of SPX options. This study classifies those observations into deep out-of-the-money (DOTM) if $S/K \leqq 0.94$; out-of-the-money (OTM) if $S/K \in [0.94, 0.97)$; at-the-money 1 (ATM1) if $S/K \in [0.97, 1)$; at-the-money 2 (ATM2) if $S/K \in [1, 1.03)$; in-the-money (ITM) if $S/K \in [1.03, 1.06)$; and deep in-the-money (DITM) if $S/K > 1.06$. The data period is from 20 October 2004 to 30 June 2005.

		All				Call				Put			
	Moneyness	Maturity				Maturity				Maturity			
	S/K	<30	30–60	≧60	Subtotal	<30	30–60	≧60	Subtotal	<30	30–60	≧60	Subtotal
DOTM	≦0.94	65.1301	83.5214	88.5615		0.1628	0.4002	0.8683		130.0973	166.6426	176.2546	
		668	1852	1992	4512	334	926	996	2256	334	926	996	2256
OTM	0.94-0.97	27.4918	29.8519	33.4532		0.6334	2.9607	7.2748		54.3503	56.7430	59.6316	
		782	1290	842	2914	391	645	421	1457	391	645	421	1457
ATM1	0.97-1	13.6321	20.3251	27.0631		4.5939	11.5286	19.8819		22.6703	29.1216	34.2442	
		940	1294	1110	3344	470	647	555	1672	470	647	555	1672
ATM2	1-1.03	14.2759	21.7983	29.6033		22.8446	30.6625	39.0762		5.7072	12.9341	20.1303	
		876	1086	842	2804	438	543	421	1402	438	543	421	1402
ITM	1.03-1.06	26.9440	31.2917	38.2495		52.1228	56.4900	65.0060		1.7651	6.0934	11.4929	
		806	888	552	2246	403	444	276	1123	403	444	276	1123
DITM	>1.06	109.4576	131.7505	151.3518		218.6059	262.5436	301.1628		0.3092	0.9574	1.5408	
		3346	4992	6008	14346	1673	2496	3004	7173	1673	2496	3004	7173
Subtotal		7418	11402	11346	30166	3709	5701	5673	15083	3709	5701	5673	15083

Table 3 Absolute Hedging Errors

The figures in this table denote the average points of absolute hedging errors ($250 per point): $\sum_{l=1}^{M} | HE_{t+(l-1)\Delta t}(t+l\Delta t)e^{r\Delta t(M-l)} | / M$ where $M=(T-t)/\Delta t$ and T is the maturity date of SPX futures options. The hedging error between time t and time $t+\Delta t$ is defined as $HE_t(t+\Delta t)$. The instrument portfolio of hedging scheme 1 (HS1) consists of $N_{1,t}$ shares of underlying SPX futures, and $N_{2,t}$ shares of *forward-start* strangle portfolios. The *forward-start* strangle portfolio consists of a short position on a T_1-matured strangle and a long position on a T_2-matured strangle. The instrument portfolio of hedging scheme 2 (HS2) consists of $N_{1,t}$ shares of underlying SPX futures, and $N_{2,t}$ shares of the VIX futures $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ with expiry T_1. The hedging period is from 20 October 2004 to 30 June 2005. The SPX futures option contracts listed at CME are four quarterly and two serial, and the longest time to maturity for the empirical observations is 120 days. Therefore, this study classifies those observations into short-term (<30 days), medium-term (30–60 days), and long-term (≧60 days). Moneyness is defined as $F_t(T)/K$ where $F_t(T)$ is the price of the SPX futures and K is the strike price of SPX futures options. This study classifies those observations into deep out-of-the-money (DOTM) if $F_t(T)/K$ ≦0.94; out-of-the-money (OTM) if $F_t(T)/K \in [0.94, 0.97)$; at-the-money 1 (ATM1) if $F_t(T)/K \in [0.97, 1)$; at-the-money 2 (ATM2) if $F_t(T)/K \in [1, 1.03)$; in-the-money (ITM) if $F_t(T)/K \in [1.03, 1.06)$; and deep in-the-money (DITM) if $F_t(T)/K$ >1.06.

Moneyness			Maturity		
$F_t(T)/K$			<30	30–60	≧60
DOTM	HS1	CONST	2.40	5.43	5.85
		SV	1.85	2.40	2.90
		SVJ	1.12	1.91	2.23
	HS2	CONST	0.16	0.19	0.28
		SV	0.01	0.08	0.35
		SVJ	0.01	0.08	0.35
OTM	HS1	CONST	0.83	6.43	6.68
		SV	0.71	2.39	2.63
		SVJ	0.61	2.19	2.47
	HS2	CONST	0.23	0.35	0.32
		SV	0.07	0.72	0.79
		SVJ	0.07	0.70	0.76
ATM1	HS1	CONST	4.10	9.28	10.46
		SV	1.97	7.79	5.85
		SVJ	1.56	7.17	2.97
	HS2	CONST	0.44	1.59	1.72
		SV	0.34	0.72	0.82
		SVJ	0.34	0.62	0.78
ATM2	HS1	CONST	1.49	3.24	10.85
		SV	1.33	2.52	1.89
		SVJ	1.20	2.24	1.66

	HS2	CONST	1.05	1.04	1.07
		SV	0.94	1.03	1.02
		SVJ	0.50	0.70	0.86
ITM	HS1	CONST	2.95	6.35	NA
		SV	1.81	4.45	NA
		SVJ	1.65	2.89	NA
	HS2	CONST	1.24	2.49	NA
		SV	1.40	2.73	NA
		SVJ	1.36	2.62	NA
DITM	HS1	CONST	3.85	NA	11.57
		SV	2.44	NA	7.11
		SVJ	2.38	NA	5.06
	HS2	CONST	2.64	NA	4.64
		SV	2.20	NA	4.56
		SVJ	2.09	NA	4.53

Table 4 Average Hedging Errors

The figures in this table denote the average points of hedging errors ($250 per point): $\sum_{l=1}^{M} HE_{t+(l-1)\Delta t}(t+l\Delta t)e^{r\Delta t(M-l)}/M$ where $M=(T-t)/\Delta t$ and T is the maturity date of SPX futures options. The hedging error between time t and time $t+\Delta t$ is defined as $HE_t(t+\Delta t)$. The instrument portfolio of hedging scheme 1 (HS1) consists of $N_{1,t}$ shares of underlying SPX futures, and $N_{2,t}$ shares of *forward-start* strangle portfolios. The *forward-start* strangle portfolio consists of a short position on a T_1-matured strangle and a long position on a T_2-matured strangle. The instrument portfolio of hedging scheme 2 (HS2) consists of $N_{1,t}$ shares of underlying SPX futures, and $N_{2,t}$ shares of the VIX futures $\mathrm{F}_t^{\mathrm{VIX}}(T_1)$ with expiry T_1. The hedging period is from 20 October 2004 to 30 June 2005. The SPX futures option contracts listed at CME are four quarterly and two serial, and the longest time to maturity for our observations is 120 days. Therefore, this study classifies those observations into short-term (<30 days), medium-term (30–60 days), and long-term (≧60 days). Moneyness is defined as $F_t(T)/K$ where $F_t(T)$ is the price of the SPX futures and K is the strike price of SPX futures options. This study classifies those observations into deep out-of-the-money (DOTM) if $F_t(T)/K$ ≦0.94; out-of-the-money (OTM) if $F_t(T)/K \in [0.94,0.97)$; at-the-money 1 (ATM1) if $F_t(T)/K \in [0.97,1)$; at-the-money 2 (ATM2) if $F_t(T)/K \in [1,1.03)$; in-the-money (ITM) if $F_t(T)/K \in [1.03,1.06)$; and deep in-the-money (DITM) if $F_t(T)/K >1.06$.

Moneyness $F_t(T)/K$			Maturity <30	30–60	≧60
DOTM	HS1	CONST	−2.32	−3.25	−5.79
		SV	0.27	−0.24	0.53
		SVJ	−0.15	−0.19	0.29
	HS2	CONST	0.09	−0.04	−0.19
		SV	0.00	0.00	0.17
		SVJ	0.00	0.00	0.17
OTM	HS1	CONST	−7.42	−1.37	−4.32
		SV	−0.16	0.34	−0.53
		SVJ	−0.11	0.40	−0.26
	HS2	CONST	0.11	−0.32	−0.30
		SV	0.02	−0.30	0.28
		SVJ	0.02	−0.30	0.28
ATM1	HS1	CONST	−3.36	−1.97	−6.84
		SV	−0.45	−1.48	1.13
		SVJ	−0.20	−1.40	0.94
	HS2	CONST	0.15	−0.54	−0.56
		SV	0.10	−0.69	−0.65
		SVJ	0.10	−0.60	−0.52
ATM2	HS1	CONST	−1.43	−3.00	−10.84
		SV	−0.88	−1.65	1.79
		SVJ	−0.67	−0.38	1.27
	HS2	CONST	0.42	−0.93	−0.75
		SV	0.44	−0.86	−0.85
		SVJ	0.43	−0.25	−0.65
ITM	HS1	CONST	−2.94	−6.28	NA
		SV	−1.75	2.80	NA
		SVJ	−1.44	1.99	NA
	HS2	CONST	0.98	−1.78	NA

		SV	1.11	−2.04	NA
		SVJ	1.09	−1.94	NA
DITM	HS1	CONST	−3.82	NA	−5.72
		SV	2.10	NA	−5.13
		SVJ	1.99	NA	−4.51
	HS2	CONST	2.32	NA	−4.07
		SV	1.91	NA	−3.96
		SVJ	1.83	NA	−3.53

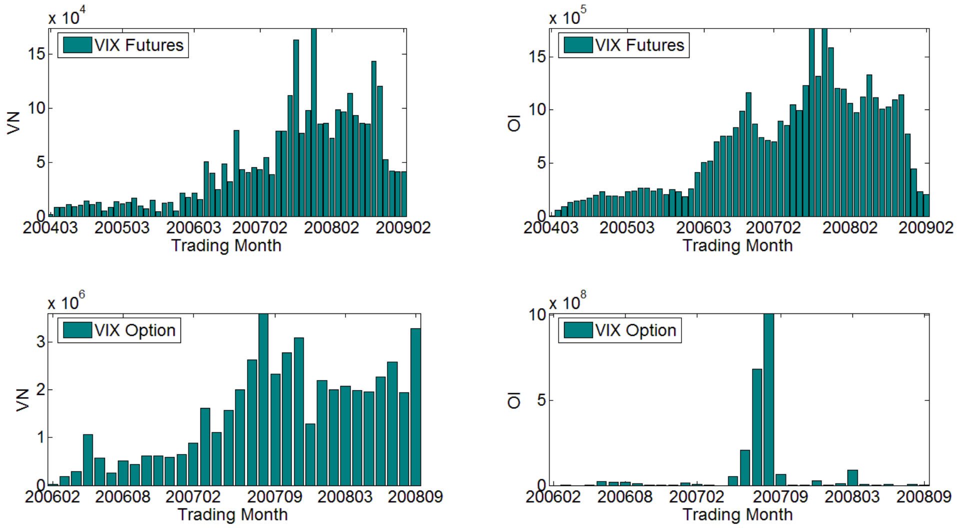

Figure 1 Trading volume (VN) and open interest (OI) of VIX futures and VIX options across trading months, March 2004 – February 2009 and February 2006 – September 2008, respectively.