FX Options and Structured Products

Second Edition

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9 June 2017

To Ansua

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0 Preface

0.1 Scope of this Book

Treasury management of international corporates involves dealing with cash flows in different currencies. Therefore the natural service of an investment bank consists of a variety of money market and foreign exchange products. This book explains the most popular products and strategies with a focus on everything beyond vanilla options.

It explains all the FX derivatives including options, common structures and tailor-made solutions in examples with a special focus on the application including views from traders and sales as well as from a corporate treasurer's perspective.

It contains actually traded deals with corresponding motivations explaining why the structures have been traded. This way the reader gets a feeling how to build new structures to suit clients' needs. We will also cover some examples of 'bad deals', deals that traded and led to dramatic losses.

Several sections deal with some basic quantitative aspect of FX options, such as quanto adjustment, deferred delivery, vanna-volga pricing, settlement issues.

One entire chapter is devoted to hedge accounting, where after the foundations a typical structured FX forward is examined in a case study.

0.1.1 Why I Decided to Write a Second Edition

There are numerous books on quantitative finance, and I am myself originally a quant. However, very few of these illustrate why certain products trade. There are also many books on options or derivatives in general. However, most of the options books are written in an equity options context. In my opinion, the key to really understand options is the Foreign Exchange market. No other asset class makes the symmetries so obvious, and no other asset class has underlyings as liquid as the major currency pairs. With this book I am taking the effort to go beyond common literature on options, and also pure textbook material on options. Anybody can write a book on options after spending a few days on an internet search engine. Any student can learn about options doing the same thing (and save a lot of tuition going to business schools). This book on FX options enables experts in the field to become more credible. My motivation to write this book is to share what I have learned in the many decades of dealing with FX derivatives in my various roles of a quant coding pricing libraries and handling market data, a structurer who deals with products from the trading and sales perspective, a risk manager who runs an options position, a consultant dealing with special topics in FX markets, an expert who resolves legal conflicts in the area of derivatives, an adviser to the public sector and politicians how to deal with currency risk, and last but not least as a trainer, who has been teaching FX options now for the second generation, a job, during which I have received so much of valuable feedback, that many sections of the First Edition need to be updated or extended. And obviously, ten years have passed since the First Edition had appeared, new products have been trading and new standards have been set. So it is about time. I really couldn't leave the first edition as it is. Moreover, many have asked me over the years to make solutions to the exercises available. This book now contains about 75 exercises, which I believe are very good practice material and support further learning and reflection, and all of the exercises have come with solutions in a separate book. It is now possible for trainers to use this book for teaching and exam preparation. Supplementary material will be published on the web page of the book, fxoptions.mathfinance.com.

0.1.2 What is not Contained in this Book

This book is not on valuation of financial engineering from a programmer's or quant's point of view. I will explain the relevance and cover some basics on vanilla options. For the quantitative matters I refer to my book on Modeling Foreign Exchange Options [141], which you may consider a second volume to this book. This does not mean that this book is not suitable for quants. On the contrary for a quant (front-office or market risk) it may help to learn the trader's view, the buy-side view and get an overview where all the programming may lead to.

0.2 The Readership

Prerequisite is some basic knowledge of FX markets as for example taken from the Book *Foreign Exchange Primer* by Shami Shamah, Wiley 2003, see [118]. For quantitative sections some knowledge of Stochastic Calculus as in Steven E. Shreve's volumes on Stochastic Calculus for Finance [120] are useful, but not essential for most of this book. The target readers are

- Graduate students and Faculty of Financial Engineering Programs, who can use this book as a textbook for a course named *structured products* or *exotic currency options*.
- Traders, Trainee Structurers, Product Developers, Sales and Quants with interest in the FX product line. For them it can serve as a source of ideas and as well as a reference

guide.

- Treasurers of corporates interested in managing their books. With this book at hand they can structure their solutions themselves.

The readers more interested in the quantitative and modeling aspects are recommended to read *Foreign Exchange Risk* by J. Hakala and U. Wystup, Risk Publications, London, 2002, see [65]. This book explains several exotic FX options with a special focus on the underlying models and mathematics, but does not contain any structures or corporate clients' or investors' view.

0.3 About the Author

Uwe Wystup is the founder and managing director of MathFinance AG, a consulting and software company specializing in quantitative finance, implementation of derivatives models, valuation and validation services. Previously he was a financial engineer and structurer in the FX Options trading desk at Commerzbank. Before that he worked for Deutsche Bank, Citibank, UBS and Sal. Oppenheim jr. & Cie. Uwe holds a PhD in mathematical finance from Carnegie Mellon University and is professor of financial option price modeling and foreign exchange derivatives at University of Antwerp and honorary professor of quantitative finance at Frankfurt School of Finance & Management, and lecturer at National University of Singapore. He has given several seminars on exotic options, numerical methods in finance and volatility modeling. His area of specialization are the quantitative aspects and the design of structured products of foreign exchange markets. He published a book on *Foreign Exchange Risk* and articles in many journals including *Finance and Stochastics*, *Review of Derivatives Research*, *European Actuarial Journal*, *Journal of Risk*, *Quantitative Finance*, *Applied Mathematical Finance*, *Wilmott*, *Annals of Finance*, and the *Journal of Derivatives*. He also edited the section on Foreign Exchange Derivatives in Wiley's *Encyclopedia of Quantitative Finance*. Uwe has given many presentations at both universities and banks around the world. Further information and a detailed publication list is available at www.mathfinance.com.

0.4 Acknowledgments

I would like to thank *Frankfurt School of Finance & Management* to support the first edition of this book project by allocating the necessary time.

I would like to thank my former colleagues on the trading floor, most of all Michael Braun, Jürgen Hakala, Tamás Korchmáros, Behnouch Mostachfi, Bereshad Nonas, Gustave Rieunier, Ingo Schneider, Jan Schrader, Noel Speake, Roman Stauss, Andreas Weber, and all my colleagues and co-authors, specially Christoph Becker, Susanne Griebsch, Christoph Kühn, Sebastian Krug, Marion Linck, Wolfgang Schmidt and Robert Tompkins.

Special thanks to Tino Senge and his many talks on long dated FX, parts of which have become part of this book.

I would like to thank Steve Shreve for his training in stochastic calculus and continuously supporting my academic activities.

Chris Swain, Rachael Wilkie and many others of Wiley publications deserve respect as they were dealing with my rather slow speed in completing the first edition of this book. Similar respect applies to Werner Coetzee, Jennie Kitchin, Lori Laker, Thomas Hyrkiel, Jeremy Chia, Viv Church (the copy editor), Abirami Srikandan (the production editor), and their colleagues for the second edition.

Many readers sent me valuable feedback, suggestions for improvement, error reports and questions; they include but are not limited to Anupam Banerji, David Bannister, Lluis Blanc, Charles Brown, Harold Cataquet, Sven Foulon, Steffen Gregersen, Federico Han, Rupesh Mishra, Daniele Moroni, Allan Mortensen, Josua Müller, Alexander Stromilo, Yanhong Zhao. Thank you all.

Nicole van de Locht and Choon Peng Toh deserve a medal for serious detailed proof reading of the first edition, and similarly Lars Helfenstein, Archita Mishra, Armin Wendel, Miroslav Svoboda, and again Choon Peng Toh for the second edition.

1 Foreign Exchange Derivatives

The FX derivatives market consists of FX swaps, FX forwards, FX or currency options, and other more general derivatives. FX structured products are either standardized or tailor-made linear combinations of simple FX derivatives including both vanilla and exotic options, or more general structured derivatives that can't be decomposed into simple building blocks. The market for structured products is restricted to the market of the necessary ingredients. Hence, typically there are mostly structured products traded in the currency pairs that can be formed between USD, JPY, EUR, CHF, GBP, CAD and AUD. In this chapter we start with a brief history of options, followed by a technical section on vanilla options and volatility, and deal with commonly used linear combinations of vanilla options. Then we will illustrate the most important ingredients for FX structured products: the first and second generation exotics.

1.1 Literature Review

While there are tons of books on options and derivatives in general, there are only very few dedicated specifically to FX options. After the 2008 financial crisis, more such books appeared. The book by Shamah [118] as a source to learn about FX Markets with a focus on market conventions, spot, forward and swap contracts, and vanilla options. For pricing and modeling of exotic FX options I (obviously) suggest Hakala and Wystup's *Foreign Exchange Risk* [65] or its translation into Mandarin [68] as useful companions to this book. One of the first books dedicated to *Mathematical Models of Foreign Exchange* is by Lipton [92]. In 2010, Iain Clark published a book on Foreign Exchange Option Pricing [28], and Antonio Castagna one on FX Options and Smile Risk [25], which both make a valuable contribution to the FX derivatives literature. A classic book on Managing Currency Risk Using Foreign Exchange Options is by Alan Hicks [76]. It provides a good overview of FX options mainly from the corporates point of view. An introductory book on *Options on Foreign Exchange* is by de Rosa [38]. The *Handbook of Exchange Rates* [82] provides a comprehensive compilation of articles on the FX market structure, products, policies and economic models.

1.2 A Journey through the History of Options

The very first options and futures were traded in ancient Greece, when olives were sold before they had reached ripeness. Thereafter the market evolved in the following way.

16th century Ever since the 15th century tulips, which were liked for their exotic appearance, were grown in Turkey. The head of the royal medical gardens in Vienna, Austria, was the first to cultivate those Turkish tulips successfully in Europe. When he fled to Holland because of religious persecution, he took the bulbs along. As the new head of the botanical gardens of Leiden, Netherlands, he cultivated several new strains. It was from these gardens that avaricious traders stole the bulbs to commercialize them, because tulips were a great status symbol.

17th century The first futures on tulips were traded in 1630. As of 1634, people could buy special tulip strains by the weight of their bulbs – the bulbs had the same value as gold. Along with the regular trading, speculators entered the market and the prices skyrocketed. A bulb of the strain "Semper Octavian" was worth two wagonloads of wheat, four loads of rye, four fat oxen, eight fat swine, twelve fat sheep, two hogsheads of wine, four barrels of beer, two barrels of butter, 1,000 pounds of cheese, one marriage bed with linen and one sizable wagon. People left their families, sold all their belongings, and even borrowed money to become tulip traders. When in 1637, this supposedly risk-free market crashed, traders as well as private individuals went bankrupt. The Dutch government prohibited speculative trading; the period became famous as Tulipmania.

18th century In 1728, the Royal West-Indian and Guinea Company, the monopolist in trading with the Caribbean Islands and the African coast issued the first stock options. Those were options on the purchase of the French Island of Ste. Croix, on which sugar plantings were planned. The project was realized in 1733 and paper stocks were issued in 1734. Along with the stock, people purchased a relative share of the island and the valuables, as well as the privileges and the rights of the company.

19th century In 1848, 82 businessmen founded the Chicago Board of Trade (CBOT). Today it is the biggest and oldest futures market in the entire world. Most written documents were lost in the great fire of 1871, however, it is commonly believed that the first standardized futures were traded as of 1860. CBOT now trades several futures and forwards, not treasury bonds, but also options and gold.

In 1870, the New York Cotton Exchange was founded. In 1880, the gold standard was introduced.

20th century

- In 1914, the gold standard was abandoned because of the war.
- In 1919, the Chicago Produce Exchange, in charge of trading agricultural products was renamed to Chicago Mercantile Exchange. Today it is the most important futures market for Eurodollar, foreign exchange, and livestock.
- In 1944, the Bretton Woods System was implemented in an attempt to stabilize the currency system.
- In 1970, the Bretton Woods System was abandoned for several reasons.
- In 1971, the Smithsonian Agreement on fixed exchange rates was introduced.

- In 1972, the International Monetary Market (IMM) traded futures on coins, currencies and precious metal.
- In 1973, the CBOE (Chicago Board of Exchange) firstly traded call options; four years later also put options. The Smithsonian Agreement was abandoned; the currencies followed managed floating.
- In 1975, the CBOT sold the first interest rate future, the first future with no "real" underlying asset.
- In 1978, the Dutch stock market traded the first standardized financial derivatives.
- In 1979, the European Currency System was implemented, and the European Currency Unit (ECU) was introduced.
- In 1991, the Maastricht Treaty on a common currency and economic policy in Europe was signed.
- In 1999, the Euro was introduced, but the countries still used cash of their old currencies, while the exchange rates were kept fixed.

21st century
- In 2002, the Euro was introduced as new money in the form of cash.

FX forwards and options originate from the need of corporate treasury to hedge currency risk. This is the key to understand FX options. Originally, FX options didn't start off as speculative products, but as hedging products. This is why they trade over the counter (OTC). They are tailored, i.e. cash-flow matching currency risk hedging instruments for corporates. The way to think about an option is that a corporate treasurer in the EUR zone has income in USD and needs a hedge to sell the USD and to buy EUR for these USD. He would go long a forward or a EUR call option. At maturity he would exercise the option if it is in the money and receive EUR and pay USD. FX options are by default delivery settled. Whilst FX derivatives have been used later also as investment products or speculative instruments, the key to understand FX options is corporate treasury.

1.3 Currency Options

Let us start with a definition of a currency option:

Definition 1.3.1 *A* Currency Option Transaction *means a transaction entitling the Buyer, upon Exercise, to purchase from the Seller at the Strike Price a specified quantity of Call Currency and to sell to the Seller at the Strike Price a specified quantity of Put Currency.*

This is the definition taken from the *1998 FX and Currency Option Definitions* published by the International Swaps and Derivatives Association (ISDA) in 1998 [77]. This definition was the result of a process of standardization of currency options in the industry and is now widely accepted. Note that the key feature of an option is that the holder has a right to exercise. The definition also demonstrates clearly that calls and puts are equivalent, i.e. a call on one currency is always a put on the other currency. The definition is designed for a treasurer, where an actual cashflow of two currency is triggered upon exercise. The definition also shows that the terms *derivative* and *option* are *not* synonyms. Derivative is a much wider term for financial transactions that depend on an underlying traded instrument. Derivatives include forwards, swaps, options, and exotic options. But not any derivative is also an option. For a currency option there is always a holder, the Buyer after buying the option, equipped with the right to exercise, and upon exercise a cash flow of pre-specified two currencies is triggered. Anything outside this definition does not constitute a currency option. I highly recommend reading the 1998 ISDA definitions. It is a text using legal language, but it does make all the terms around FX and currency options very clear and is the benchmark in the industry. It covers only put and call options, options that are typically referred to as *vanilla* options, because they are the most common and simple products. The definition allows for different exercise styles: European for exercise permitted only at maturity, American for exercise permitted at any time between inception and maturity, as well as Bermudan for exercise permitted as finitely many pre-specified points in time. Usually, FX options are European. If you don't mention anything, they are understood to be of European exercise style. Features like cash settlement are possible: in this case one would have to make the Call Currency Amount the net payoff and the Put Currency Amount equal to zero. There are a number of exotic options, which we will cover later in this book, that still fit into this framework: in particular, barrier options. Whilst they have special features not covered by the 1998 ISDA Definitions, they still can be considered Currency Option Transactions. However, Variance Swaps, Volatility Swaps, Correlation Swaps, Combination of Options, Structured Products, Target Forwards, just to mention a few obvious transactions do not constitute Currency Option Transactions.

1.4 Technical Issues for Vanilla Options

It is a standard in the FX options market to quote prices for FX options in terms of their implied volatility. The one-to-one correspondence between volatilities and options values rests on the convex payoff function of both call and put options. The conversion firmly rests on the Black-Scholes model. It is well-known in the financial industry and academia that the Black-Scholes model has many weaknesses in modeling the underlying market properly. Strictly speaking, it is inappropriate. And there are in fact many other models like local volatility or stochastic volatility models or their hybrids, which reflect the dynamics much better than the Black-Scholes model. Nevertheless, as a basic tool to convert volatilities into values and values into volatilities, it is the market standard for dealers, brokers and basically all risk management systems. This means: good news for those who have already learned it - it was not a waste of time and effort. And also bad news for the quant-averse: you need to deal with it to a certain extend, as otherwise the FX volatility surface and the FX smile construction will not be accessible to you. Therefore, I do want to get the basic math done, even in this book, which I don't intend to be a quant book. However, I don't want to scare away much of my potential readership. If you don't like the math, you can still read most of this book.

We consider the model *geometric Brownian motion*

$$dS_t = (r_d - r_f)S_t\,dt + \sigma S_t\,dW_t \tag{1}$$

for the underlying exchange rate quoted in FOR-DOM (foreign-domestic), which means that one unit of the foreign currency costs FOR-DOM units of the domestic currency. In case of EUR-USD with a spot of 1.2000, this means that the price of one EUR is 1.2000 USD. The notion of *foreign* and *domestic* do not refer to the location of the trading entity, but only to this quotation convention. There are other terms used for FOR, which are *underlying*, *CCY1*, *base*; there are also other terms for DOM, which are *base*, *CCY2*, *counter* or *term*, respectively. For the quants, DOM is also considered the *numeraire* currency. I leave it to you, which one you wish to use. I find 'base' a bit confusing, because it refers sometimes to FOR and sometimes to DOM. I also find 'CCY1' and 'CCY2' not very conclusive. The term 'numeraire' doesn't have an established counterpart for FOR. So I prefer FOR and DOM. You may also stick to the most liquid currency pair EUR/USD, and think of FOR as EUR and DOM as USD.

We denote the (continuous) foreign interest rate by r_f and the (continuous) domestic interest rate by r_d. In an equity scenario, r_f would represent a continuous dividend rate. Note that r_f is *not* the interest rate that is typically used to discount cash flows in foreign currency, but is the (artificial) foreign interest rate that ensures that the forward price calculated in Equation (9) matches the market forward price. The volatility is denoted by σ, and W_t is a standard Brownian motion. The sample paths are displayed in Figure 1.1. We consider this standard model, not because it reflects the statistical properties of the exchange rate (in fact, it doesn't), but because it is widely used in practice and front office systems and mainly serves as a tool to communicate prices of vanilla call and put options and switch between quotations in price and in terms of implied volatility. Currency option prices are commonly quoted in terms of volatility in the sense of this model. Model (1) is sometimes referred to as the Garman-Kohlhagen model [54]. However, all that happened there was adding the foreign interest rate r_f to the Black-Scholes model [15]. For this reason Model (1) is generally and in this book referred to as the Black-Scholes model.

Applying Itô's rule to $\ln S_t$ yields the following solution for the process S_t

$$S_t = S_0 \exp\left\{(r_d - r_f - \frac{1}{2}\sigma^2)t + \sigma W_t\right\}, \tag{2}$$

which shows that S_t is log-normally distributed, more precisely, $\ln S_t$ is normal with mean $\ln S_0 + (r_d - r_f - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$. Further model assumptions are:

1. There is no arbitrage.
2. Trading is frictionless, no transaction costs.
3. Any position can be taken at any time, short, long, arbitrary fraction, no liquidity constraints.

The payoff for a vanilla option (European put or call) is given by

$$F = [\phi(S_T - K)]^+, \tag{3}$$

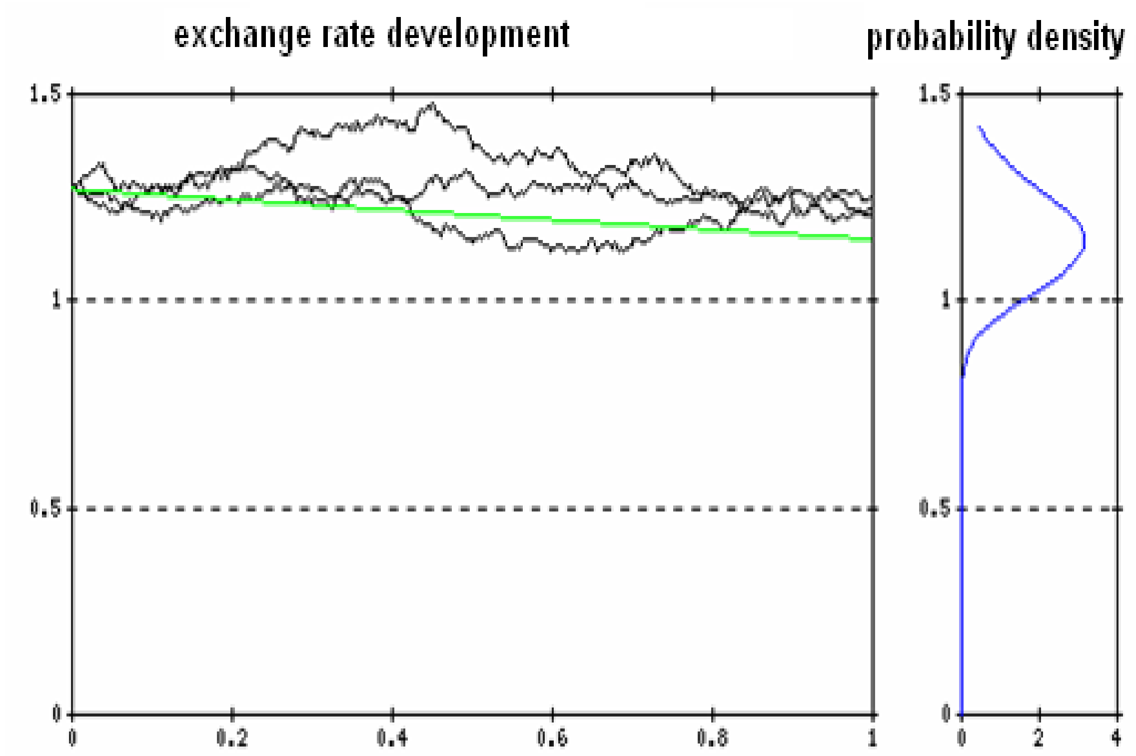

Figure 1.1: *Simulated paths of a geometric Brownian motion. The distribution of the spot S_T at time T is log-normal. The light green line reflects the average spot movement.*

where the contractual parameters are the strike K, the expiration time T and the type ϕ, a binary variable which takes the value $+1$ in the case of a call and -1 in the case of a put. The symbol x^+ denotes the positive part of x, i.e., $x^+ \stackrel{\Delta}{=} \max(0,x) \stackrel{\Delta}{=} 0 \vee x$. We generally use the symbol $\stackrel{\Delta}{=}$ to *define* a quantity. Most commonly, vanilla options on foreign exchange are of *European style*, i.e. the holder can only exercise the option at time T. *American style options*, where the holder can exercise any time or *Bermudan style options*, where the holder can exercise at selected times, are not used very often except for *time options*, see Section 2.1.19.

1.4.1 Valuation in the Black-Scholes Model

In the Black-Scholes model the value of the payoff F at time t if the spot is at x is denoted by $v(t,x)$ and can be computed either as the solution of the *Black-Scholes partial differential equation* (see [15])

$$v_t - r_d v + (r_d - r_f) x v_x + \frac{1}{2}\sigma^2 x^2 v_{xx} = 0, \tag{4}$$

$$v(T,x) = F. \tag{5}$$

or equivalently (*Feynman-Kac Theorem*) as the discounted expected value of the payoff-function

$$v(x,K,T,t,\sigma,r_d,r_f,\phi) = e^{-r_d\tau}\mathbb{E}[F]. \tag{6}$$

This is the reason why basic financial engineering is mostly concerned with solving partial differential equations or computing expectations (numerical integration). The result is the *Black-Scholes formula*

$$v(x,K,T,t,\sigma,r_d,r_f,\phi) = \phi e^{-r_d\tau}[f\mathcal{N}(\phi d_+) - K\mathcal{N}(\phi d_-)]. \tag{7}$$

The result of this formula is the value of a vanilla option in USD for one unit of EUR nominal. We abbreviate

$$
\begin{aligned}
x &: \text{ current price of the underlying,} \\
\tau &\stackrel{\Delta}{=} T-t: \text{ time to maturity,} && (8)\\
f &\stackrel{\Delta}{=} \mathbb{E}[S_T|S_t = x] = xe^{(r_d-r_f)\tau}: \text{ forward price of the underlying,} && (9)\\
\theta_\pm &\stackrel{\Delta}{=} \frac{r_d-r_f}{\sigma} \pm \frac{\sigma}{2}, && (10)\\
d_\pm &\stackrel{\Delta}{=} \frac{\ln\frac{x}{K} + \sigma\theta_\pm\tau}{\sigma\sqrt{\tau}} = \frac{\ln\frac{f}{K} \pm \frac{\sigma^2}{2}\tau}{\sigma\sqrt{\tau}}, && (11)\\
n(t) &\stackrel{\Delta}{=} \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}t^2} = n(-t) \text{ normal density,} && (12)\\
\mathcal{N}(x) &\stackrel{\Delta}{=} \int_{-\infty}^{x} n(t)\,dt = 1-\mathcal{N}(-x) \text{ normal distribution function.} && (13)
\end{aligned}
$$

We observe that some authors use d_1 for d_+ and d_2 for d_-, which requires extra memory and completely ruins the beautiful symmetry of the formula.

The Black-Scholes formula can be derived using the integral representation of Equation (6)

$$
\begin{aligned}
v &= e^{-r_d\tau}\mathbb{E}[F] \\
&= e^{-r_d\tau}\mathbb{E}[[\phi(S_T-K)]^+] \\
&= e^{-r_d\tau}\int_{-\infty}^{+\infty}\left[\phi\left(xe^{(r_d-r_f-\frac{1}{2}\sigma^2)\tau+\sigma\sqrt{\tau}y} - K\right)\right]^+ n(y)\,dy. && (14)
\end{aligned}
$$

Next one has to deal with the positive part and then complete the square to get the Black-Scholes formula. A derivation based on the partial differential equation can be done using results about the well-studied *heat-equation*. For valuation of options it is very important to ensure that the interest rates are chosen such that the forward price (9) matches the market, as otherwise the options may not satisfy the put-call-parity (41).

1.4.2 A Note on the Forward

The *forward price* f is the pre-agreed exchange rate which makes the time zero value of the *forward contract* with payoff

$$F = S_T - f \tag{15}$$

equal to zero. It follows that $f = I\!E[S_T] = xe^{(r_d - r_f)T}$, i.e. the forward price is the expected price of the underlying at time T in a risk-neutral measure (drift of the geometric Brownian motion is equal to cost of carry $r_d - r_f$). The situation $r_d > r_f$ is called *contango*, and the situation $r_d < r_f$ is called *backwardation*. Note that in the Black-Scholes model the class of forward price curves is quite restricted. For example, no seasonal effects can be included. Note that the post-trade value of the forward contract after time zero is usually different from zero, and since one of the counterparties is always short, there may be settlement risk of the short party. A *futures contract* prevents this dangerous affair: it is basically a forward contract, but the counterparties have to maintain a *margin account* to ensure the amount of cash or commodity owed does not exceed a specified limit.

1.4.3 Vanilla Greeks in the Black-Scholes Model

Greeks are derivatives of the value function with respect to model and contract parameters. They are an important information for traders and have become standard information provided by front-office systems. More details on Greeks and the relations among Greeks are presented in Hakala and Wystup [65] or Reiss and Wystup [107]. Initially there was a desire to use Greek letters for all these mathematical derivatives. It turned out that since the early days of risk management many higher order Greeks have been added whose terms no longer reflect Greek letters. Even vega is not a Greek letter, but we needed a Greek sounding term that starts with a 'v' to reflect volatility; but Greek doesn't have such a letter. For vanilla options we list some of them now.

(Spot) Delta.

$$\frac{\partial v}{\partial x} = \phi e^{-r_f \tau} \mathcal{N}(\phi d_+) \tag{16}$$

This spot delta ranges between 0% and a *discounted* $\pm 100\%$. The interpretation of this quantity is the amount of FOR the trader needs to buy to delta hedge a short option. So for instance, if you sell a call on 1 M EUR, that has a 25% delta, you need to buy 250,000 EUR to delta hedge the option. The corresponding forward delta ranges between 0% and $\pm 100\%$ and is symmetric in the sense that a 60-delta call is a 40-delta put, a 75-delta put is a 25-delta call etc. I had wrongly called it "driftless delta" in the first edition of this book.

Forward Delta.

$$\phi \mathcal{N}(\phi d_+) \tag{17}$$

The interpretation of forward delta is the number of units of FOR of forward contracts a trader needs to buy to delta hedge a short option. See Section 1.4.7 for a justification.

Future Delta.

$$\phi e^{-r_d \tau} \mathcal{N}(\phi d_+) \tag{18}$$

Gamma.

$$\frac{\partial^2 v}{\partial x^2} = e^{-r_f \tau} \frac{n(d_+)}{x\sigma\sqrt{\tau}} \tag{19}$$

The interpretation of gamma is the change of delta as spot changes. A high gamma means that the delta hedge must be adapted very frequently and will hence cause transaction costs. Gamma is typically high when the spot is near a strike of a barrier, generally whenever the payoff has a kink or more dramatically a jump. Trading systems usually quote gamma as a *traders' gamma*, using a 1% *relative* change in the spot price. For example, if gamma is quoted as 10,000 EUR, then delta will increase by that amount if the spot rises from 1.3000 to $1.3130 = 1.3000 \cdot (1 + 1\%)$. This can be approximated by $\frac{\partial^2 v}{\partial x^2} \cdot \frac{x}{100}$.

Speed.

$$\frac{\partial^3 v}{\partial x^3} = -e^{-r_f \tau} \frac{n(d_+)}{x^2\sigma\sqrt{\tau}} \left(\frac{d_+}{\sigma\sqrt{\tau}} + 1 \right) \tag{20}$$

The interpretation of speed is the change of gamma as spot changes.

Theta.

$$\begin{aligned} \frac{\partial v}{\partial t} &= -e^{-r_f \tau} \frac{n(d_+) x \sigma}{2\sqrt{\tau}} \\ &+ \phi[r_f x e^{-r_f \tau} \mathcal{N}(\phi d_+) - r_d K e^{-r_d \tau} \mathcal{N}(\phi d_-)] \end{aligned} \tag{21}$$

Theta reflects the change of the option value as the clock ticks. The *traders' theta* that you spot in a risk management system usually refers to a change of the option value in one day, i.e. the traders' theta can be approximated by $365 \frac{\partial v}{\partial t}$.

Charm.

$$\frac{\partial^2 v}{\partial x \partial \tau} = -\phi r_f e^{-r_f \tau} \mathcal{N}(\phi d_+) + \phi e^{-r_f \tau} n(d_+) \frac{2(r_d - r_f)\tau - d_- \sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}} \tag{22}$$

Color.

$$\frac{\partial^3 v}{\partial x^2 \partial \tau} = -e^{-r_f \tau} \frac{n(d_+)}{2x\tau\sigma\sqrt{\tau}} \left[2 r_f \tau + 1 + \frac{2(r_d - r_f)\tau - d_- \sigma\sqrt{\tau}}{2\tau\sigma\sqrt{\tau}} d_+ \right] \tag{23}$$

Vega.

$$\frac{\partial v}{\partial \sigma} = x e^{-r_f \tau} \sqrt{\tau} n(d_+) \tag{24}$$

Trading and risk management systems usually quote vega as a *traders' vega*, using a 1% *absolute* change in the volatility. For example, if vega is quoted as 4,000 EUR, then the option value will increase by that amount if the volatility rises from 10% to $11\% = 10\% + 1\%$. This can be approximated by $\frac{\partial v}{\partial \sigma} \cdot 100$.

Volga.

$$\frac{\partial^2 v}{\partial \sigma^2} = x e^{-r_f \tau} \sqrt{\tau} n(d_+) \frac{d_+ d_-}{\sigma} \tag{25}$$

Volga is also sometimes called *vomma* or *volgamma* or *dvega/dvol*. Volga reflects the change of vega as volatility changes. Traders' volga assumes again a 1% absolute change in volatility.

Vanna.

$$\frac{\partial^2 v}{\partial \sigma \partial x} = -e^{-r_f \tau} n(d_+) \frac{d_-}{\sigma} \tag{26}$$

Vanna is also sometimes called *dvega/dspot*. It reflects the change of vega as spot changes. Traders' vanna assumes again a 1% relative change in spot. The origin of the term vanna is not clear. I suspect it goes back to an article in Risk Magazine by Tim Owens in the 1990s, where he asked "wanna loose a lot of money?" and then explained how a loss may occur if second order Greeks such as vanna and volga are not hedged.

Volunga.

$$\frac{\partial^3 v}{\partial \sigma^3} = \frac{\text{vega}}{\sigma^2} \left((d_+ d_-)^2 - d_+^2 - d_+ d_- - d_-^2 \right) \tag{27}$$

This is actually not a joke.

Vanunga.

$$\frac{\partial^3 v}{\partial x \partial \sigma^2} = \frac{\text{vega}}{\sigma^2 x \sqrt{\tau}} \left(d_+ + d_+ d_- - d_+ d_-^2 \right) \tag{28}$$

This one isn't a joke either.

Rho.

$$\frac{\partial v}{\partial r_d} = \phi K \tau e^{-r_d \tau} \mathcal{N}(\phi d_-) \tag{29}$$

$$\frac{\partial v}{\partial r_f} = -\phi x \tau e^{-r_f \tau} \mathcal{N}(\phi d_+) \tag{30}$$

Trading and risk management systems usually quote rho as a *traders' rho*, using a 1% *absolute* change in the interest rate. For example, if rho is quoted as 4,000 EUR, then the option value will increase by that amount if the interest rate rises from 2% to $3\% = 2\% + 1\%$. This can be approximated by $\frac{\partial v}{\partial \rho} \cdot 100$. **Warning**: FX options always involve two currencies. Therefore, there will be two interest rates, a domestic interest rate r_d, and a foreign interest rate r_f. The value of the option can be represented in both DOM and FOR units. This means that you can have a change of the option value in FOR as the FOR rate changes, a change of the value of the option in FOR as the DOM rate changes, a change of the value of the option in DOM as the FOR rate changes, and a change of the value of the option in DOM as the DOM rate changes. Some systems add to the confusion as they list one rho, which refers to the change of the option value as the *difference of the interest rates* changes, and again possibly in both DOM and FOR terms.

Dual Delta.

$$\frac{\partial v}{\partial K} = -\phi e^{-r_d \tau} \mathcal{N}(\phi d_-) \tag{31}$$

The non-discounted version of the dual delta, also referred to as the forward dual delta also represents the risk-neutral exercise probability of the option.

Dual Gamma.

$$\frac{\partial^2 v}{\partial K^2} = e^{-r_d \tau} \frac{n(d_-)}{K\sigma\sqrt{\tau}} \tag{32}$$

Dual Theta.

$$\frac{\partial v}{\partial T} = -v_t \tag{33}$$

Dual Greeks refer to changes of the option value as contractual parameters change. This has no application in market risk management, because the contractual parameters are fixed between counterparts and cannot be changed on the way. However, the dual Greeks contribute a lot to understanding of derivatives. The dual gamma (on the strike space) for example - up to a discount factor - is identical to the probability density of the underlying exchange rate.

1.4.4 Reoccurring Identities

$$\frac{\partial d_\pm}{\partial \sigma} = -\frac{d_\mp}{\sigma} \tag{34}$$

$$\frac{\partial d_\pm}{\partial r_d} = \frac{\sqrt{\tau}}{\sigma} \tag{35}$$

$$\frac{\partial d_\pm}{\partial r_f} = -\frac{\sqrt{\tau}}{\sigma} \tag{36}$$

$$xe^{-r_f\tau}n(d_+) = Ke^{-r_d\tau}n(d_-) \tag{37}$$

$$\mathcal{N}(\phi d_-) = I\!P[\phi S_T \geq \phi K] \tag{38}$$

$$\mathcal{N}(\phi d_+) = I\!P\left[\phi S_T \leq \phi \frac{f^2}{K}\right] \tag{39}$$

The *put-call-parity* is a way to express the trivial equation $x = x^+ - x^-$ in financial terms and is the relationship on the payoff level

$$\begin{aligned} \text{call} - \text{put} &= \text{forward} \\ (S_T - K)^+ - (K - S_T)^+ &= S_T - K, \end{aligned} \tag{40}$$

which translates to the value functions of these products via

$$v(x, K, T, t, \sigma, r_d, r_f, +1) - v(x, K, T, t, \sigma, r_d, r_f, -1) = xe^{-r_f\tau} - Ke^{-r_d\tau}. \tag{41}$$

A forward contract that is constructed using a long call and a short put option is called a *synthetic forward*.

The *put-call delta parity* is

$$\frac{\partial v(x, K, T, t, \sigma, r_d, r_f, +1)}{\partial x} - \frac{\partial v(x, K, T, t, \sigma, r_d, r_f, -1)}{\partial x} = e^{-r_f\tau}. \tag{42}$$

In particular, we learn that the absolute value of a spot put delta and a spot call delta are not exactly adding up to 100%, but only to a positive number $e^{-r_f\tau}$. They add up to one approximately if either the time to expiration τ is short or if the foreign interest rate r_f is close to zero. The corresponding forward deltas do add up to 100%.

Whereas the choice $K = f$ produces identical values for call and put, we seek the *delta-symmetric strike* or *delta-neutral strike* K_+ which produces absolutely identical deltas (spot, forward or future). This condition implies $d_+ = 0$ and thus

$$K_+ = fe^{+\frac{\sigma^2}{2}\tau}, \tag{43}$$

in which case the absolute spot delta is $e^{-r_f\tau}/2$. In particular, we learn, that always $K_+ > f$, i.e., there can't be a put and a call with identical values *and* deltas. Note that the strike K_+

is usually chosen as the middle strike when trading a straddle or a butterfly. Similarly the dual-delta-symmetric strike $K_- = fe^{-\frac{\sigma^2}{2}T}$ can be derived from the condition $d_- = 0$. Note that the delta-symmetric strike K_+ also maximizes gamma and vega of a vanilla option and is thus often considered a center of symmetry.

1.4.5 Homogeneity based Relationships

We may wish to measure the value of the underlying in a different unit. This will obviously effect the option pricing formula as follows.

$$av(x,K,T,t,\sigma,r_d,r_f,\phi) = v(ax,aK,T,t,\sigma,r_d,r_f,\phi) \text{ for all } a > 0. \tag{44}$$

Differentiating both sides with respect to a and then setting $a = 1$ yields

$$v = xv_x + Kv_K. \tag{45}$$

Comparing the coefficients of x and K in Equations (7) and (45) leads to suggestive results for the delta v_x and dual delta v_K. This *space-homogeneity* is the reason behind the simplicity of the delta formulas, whose tedious computation can be saved this way.

Time Homogeneity

We can perform a similar computation for the time-affected parameters and obtain the obvious equation

$$v(x,K,T,t,\sigma,r_d,r_f,\phi) = v\left(x,K,\frac{T}{a},\frac{t}{a},\sqrt{a}\sigma,ar_d,ar_f,\phi\right) \text{ for all } a > 0. \tag{46}$$

Differentiating both sides with respect to a and then setting $a = 1$ yields

$$0 = \tau v_t + \frac{1}{2}\sigma v_\sigma + r_d v_{r_d} + r_f v_{r_f}. \tag{47}$$

Of course, this can also be verified by direct computation. The overall use of such equations is to generate double checking benchmarks when computing Greeks. These homogeneity methods can easily be extended to other more complex options.

Put-Call Symmetry

By *put-call symmetry* we understand the relationship (see [9], [10],[19] and [22])

$$v(x,K,T,t,\sigma,r_d,r_f,+1) = \frac{K}{f}v\left(x,\frac{f^2}{K},T,t,\sigma,r_d,r_f,-1\right). \tag{48}$$

The strike of the put and the strike of the call result in a geometric mean equal to the forward f. The forward can be interpreted as a *geometric mirror* reflecting a call into a certain number of puts. Note that for at-the-money options ($K = f$) the put-call symmetry coincides with the special case of the put-call parity where the call and the put have the same value.

Rates Symmetry

Direct computation shows that the *rates symmetry*

$$\frac{\partial v}{\partial r_d} + \frac{\partial v}{\partial r_f} = -\tau v \tag{49}$$

holds for vanilla options. This relationship, in fact, holds for all European options and a wide class of path-dependent options as shown in [107].

Foreign-Domestic Symmetry

One can directly verify the *foreign-domestic symmetry* as relationship

$$\frac{1}{x}v(x, K, T, t, \sigma, r_d, r_f, \phi) = Kv\left(\frac{1}{x}, \frac{1}{K}, T, t, \sigma, r_f, r_d, -\phi\right). \tag{50}$$

This equality can be viewed as one of the faces of put-call symmetry. The reason is that the value of an option can be computed both in units of domestic currency as well as in units of foreign currency. We consider the example of S_t modeling the exchange rate of EUR/USD. In New York, the call option $(S_T - K)^+$ costs $v(x, K, T, t, \sigma, r_{usd}, r_{eur}, 1)$ USD and hence $v(x, K, T, t, \sigma, r_{usd}, r_{eur}, 1)/x$ EUR. This EUR-call option can also be viewed as a USD-put option with payoff $K\left(\frac{1}{K} - \frac{1}{S_T}\right)^+$. This option costs $Kv(\frac{1}{x}, \frac{1}{K}, T, t, \sigma, r_{eur}, r_{usd}, -1)$ EUR in Frankfurt, because S_t and $\frac{1}{S_t}$ have the same volatility. Of course, the New York value and the Frankfurt value must agree, which leads to (50). We will also learn later, that this symmetry is just one possible result based on *change of numeraire*.

1.4.6 Quotation Conventions

Quotation of the Underlying Exchange Rate

Equation (1) is a model for the exchange rate. The quotation is a permanently confusing issue, so let us clarify this here. The exchange rate means how many units of the *domestic* currency are needed to buy one unit of *foreign* currency. For example, if we take EUR/USD as an exchange rate, then the default quotation is EUR-USD, where USD is the domestic currency and EUR is the foreign currency. The term *domestic* is in no way related to the location of the trader or any country. It merely means the *numeraire* currency. The terms *domestic*, *numeraire*, *currency two* or *base currency* are synonyms as are *foreign*, *currency one* and *underlying* . Some market participants even refer to the foreign currency as the base currency, one of the reasons why I prefer to avoid the term base currency altogether. Throughout this book we denote with the slash (/) the currency pair and with a dash (-) the quotation. The slash (/) does *not* mean a division. For instance, EUR/USD can also be quoted in either EUR-USD, which then means how many USD are needed to buy one EUR, or in USD-EUR, which then means how many EUR are needed to buy one USD. There are certain market standard quotations listed in Table 1.1.

currency pair	default quotation	sample quote
GBP/USD	GPB-USD	1.6000
GBP/CHF	GBP-CHF	2.2500
EUR/USD	EUR-USD	1.3000
EUR/GBP	EUR-GBP	0.8000
EUR/JPY	EUR-JPY	135.00
EUR/CHF	EUR-CHF	1.2000
USD/JPY	USD-JPY	108.00
USD/CHF	USD-CHF	1.0100

Table 1.1: *Standard market quotation of major currency pairs with sample spot prices*

Trading Floor Language

We call one million a *buck*, one billion a *yard*. This is because a billion is called 'milliarde' in French, German and other languages. For the British Pound one million is also often called a *quid*.

Certain currencies also have names, e.g. the New Zealand Dollar NZD is called a *Kiwi* , the Australian Dollar AUD is called *Aussie*, the Canadian Dollar CAD is called *Loonie*, the Scandinavian currencies DKK, NOK (*Nokkies*) and SEK (*Stockies*) are collectively called *Scandies*.

Exchange rates are generally quoted up to five relevant figures, e.g. in EUR-USD we could observe a quote of 1.2375. The last digit '5' is called the *pip*, the middle digit '3' is called the *big figure*, as exchange rates are often displayed in trading floors and the big figure, which is displayed in bigger size, is the most relevant information. The digits left to the big figure are known anyway. If a trader doesn't know these when getting to the office in the morning, he may most likely not have the right job. The pips right of the big figure are often negligible for general market participants of other asset classes and are only highly relevant for currency spot traders. To make it clear, a rise of USD-JPY 108.25 by 20 pips will be 108.45 and a rise by 2 big figures will be 110.25.

Cable Currency pairs are often referred to by nicknames. The price of one pound sterling in US dollars, denoted by GBP/USD is known by traders as the *cable*, which has its origins from the time when a communications cable under the Atlantic Ocean synchronized the GBP/USD quote between the London and New York markets. So where is the cable?

I tumbled upon a small town called Porthcurno near Land's End on the south-western Cornish coast and by mere incident spotted a small hut called the 'cable house', admittedly a strange object to find on a beautiful sandy beach. Trying to find Cornish cream tea I ended up at a telegraphic museum, which had all I ever wanted to know about the cable, see the photographs in Figure 1.2. Indeed, telegraphic news transmission was introduced in 1837, typically along the railway lines. Iron was rapidly replaced by copper. A new insulating material, gutta percha, which is similar to rubber, allowed cables to function under the sea, and as Britain neared the height of its international power, submarine cables started to be laid, gradually creating a global network of cables, which included the first long-term successful trans-Atlantic cable of 1865 laid by the Great Eastern ship.

The entrepreneur of that age was John Pender, founder of the Eastern Telegraph Company. He had started as a cotton trader and needed to communicate quickly with various ends of the world. In the 1860s telegraphic messaging was the new and only way to do this. He quickly discovered the value of fast communication. In the 1870s, an annual traffic of around 200,000 words went through Porthcurno. By 1900, cables connected Porthcurno with India (via Gibraltar and Malta), Australia and New Zealand. Looking at the cable network charts of the late 1800s reflects the financial trading centers of today very closely: Tokyo, Sydney, Singapore, Mumbai, London, New York.

Fast communication is ever so important for the financial industry. You can still go to Purthcurno and touch the cables. They have been in the sea for more than a 100 years, but they still work. However, they have been replaced by fiber glass cables, and communications has been extended by radio and satellites. Algorithmic trading relies on getting all the market information within milliseconds.

The word 'cable' itself is still used as GBP/USD rate, reflecting the importance of fast information.

Crosses Currency pairs not involving the USD such as EUR/JPY are called a *cross* , because it is the cross rate of the more liquidly traded USD/JPY and EUR/USD. If the cross is illiquid such as ILS/MYR, it is called an illiquid cross. Spot transactions would then happen in two steps via USD, and options on an illiquid cross are rare or traded at very high bid-offer spreads.

Quotation of Option Prices

Values and prices of vanilla options may be quoted in the six ways explained in Table 1.2.

Figure 1.2: *The Cable at Porthcurno, in the telegraphic museum and on the beach near the cable house.*

The Black-Scholes formula quotes **d pips**. The others can be computed using the following instruction.

$$\textbf{d pips} \xrightarrow{\times \frac{1}{S_0}} \%f \xrightarrow{\times \frac{S_0}{K}} \%d \xrightarrow{\times \frac{1}{S_0}} \textbf{f pips} \xrightarrow{\times S_0 K} \textbf{d pips} \quad (51)$$

Delta and Premium Convention

The spot delta of a European option assuming premium is paid in DOM is well known. It will be called *raw spot delta* δ_{raw} now. It can be quoted in either of the two currencies involved. The relationship is

$$\delta_{raw}^{reverse} = -\delta_{raw}\frac{S}{K}. \quad (52)$$

The delta is used to buy or sell spot in the corresponding amount in order to hedge the option up to first order. The raw spot delta, multiplied by the FOR nominal amount, represents the amount of FOR currency the traders needs to buy in order to delta hedge a short option. How do we get to the reverse delta: it rests firmly on the symmetry of currency options. A FOR call is a DOM put. Hence, buying FOR amount in the delta hedge is equivalent to selling DOM amount multiplied by the spot S. The negative sign reflects the change from buying to selling. This explains the negative sign and the spot factor. A right to buy 1 FOR (and pay for this K DOM) is equivalent to the right to sell K DOM and receive for that 1 DOM. Therefore, viewing the FOR call as a DOM put and applying the delta hedge to one unit of DOM (instead of K units of DOM), requires a division by K. Now read this paragraph again and again and again, until it clicks. Sorry.

For consistency the premium needs to be incorporated into the delta hedge, since a premium in foreign currency will already hedge part of the option's delta risk. In a stock options context

name	symbol	value in units of	example
domestic cash	**d**	DOM	29,148 USD
foreign cash	**f**	FOR	24,290 EUR
% domestic	**% d**	DOM per unit of DOM	2.3318% USD
% foreign	**% f**	FOR per unit of FOR	2.4290% EUR
domestic pips	**d pips**	DOM per unit of FOR	291.48 USD pips per EUR
foreign pips	**f pips**	FOR per unit of DOM	194.32 EUR pips per USD

Table 1.2: *Standard market quotation types for option values. In the example we take FOR=EUR, DOM=USD,* $S_0 = 1.2000$, $r_d = 3.0\%$, $r_f = 2.5\%$, $\sigma = 10\%$, $K = 1.2500$, $T = 1$ *year,* $\phi = +1$ *(call), notional* $= 1,000,000$ *EUR* $= 1,250,000$ *USD. For the pips, the quotation 291.48 USD pips per EUR is also sometimes stated as 2.9148% USD per 1 EUR. Similarly, the 194.32 EUR pips per USD can also be quoted as 1.9432% EUR per 1 USD.*

such a question never comes up, as an option on a stock is always paid in cash, rather than paid in shares of stock. In Foreign Exchange, both currencies are cash, and it is perfectly reasonable to pay for a currency option in either DOM or FOR currency. To make this clear, let us consider EUR-USD. In any financial markets model, $v(x)$ denotes the value or premium in USD of an option with 1 EUR notional, if the spot is at x, and the raw delta v_x denotes the number of EUR to buy to delta hedge a short position of this option. If this raw delta is negative, then EUR have to be sold (silly but hopefully helpful remark for the non-math-freak). Therefore, xv_x is the number of USD to sell. If now the premium is paid in EUR rather than in USD, then we already have $\frac{v}{x}$ EUR, and the number of EUR to buy has to be reduced by this amount, i.e. if EUR is the premium currency, we need to buy $v_x - \frac{v}{x}$ EUR for the delta hedge or equivalently sell $xv_x - v$ USD. This is called a *premium-adjusted delta* or delta with premium included.

The same result can be derived by looking at the risk management of a portfolio, whose accounting currency is EUR and risky currency is USD. In this case spot is $\frac{1}{x}$ rather than x. The value of the option – or in fact more generally of a portfolio of derivatives – is then $v\left(\frac{1}{x}\right)$ in USD, and $v\left(\frac{1}{x}\right)\frac{1}{x}$ in EUR, and the change of the portfolio value in EUR as the price of the USD measured in EUR is

$$
\begin{aligned}
\frac{\partial}{\partial \frac{1}{x}} \frac{v\left(\frac{1}{x}\right)}{x} &= \frac{\partial}{\partial x} \frac{v\left(\frac{1}{x}\right)}{x} \frac{\partial x}{\partial \frac{1}{x}} \\
&= \frac{v_x\left(\frac{1}{x}\right)x - v\left(\frac{1}{x}\right)}{x^2} \left(\frac{\partial \frac{1}{x}}{\partial x}\right)^{-1} \\
&= \frac{x v_x - v}{x^2} \left(-\frac{1}{x^2}\right)^{-1} \\
&= -[x v_x - v]. \qquad (53)
\end{aligned}
$$

We observe that both the trader's approach deriving delta from the premium as well as the risk manager's approach deriving delta from the portfolio risk arrive at the same number. Not really a surprise, isn't it?

The premium-adjusted delta for a vanilla option in the Black-Scholes model becomes

$$
\begin{aligned}
-[x v_x - v] &= -\left[\phi x e^{-r_f \tau} \mathcal{N}(\phi d_+) - \phi[x e^{-r_f \tau} \mathcal{N}(\phi d_+) - K e^{-r_d \tau} \mathcal{N}(\phi d_-)]\right] \\
&= -\phi K e^{-r_d \tau} \mathcal{N}(\phi d_-) \qquad (54)
\end{aligned}
$$

in USD, or $-\phi e^{-r_d \tau} \frac{K}{x} \mathcal{N}(\phi d_-)$ in EUR. If we sell USD, instead of buying EUR, and if we assume a notional of 1 USD rather than 1 EUR (= K USD) for the option, the premium-adjusted delta becomes just

$$
\phi e^{-r_d \tau} \mathcal{N}(\phi d_-). \qquad (55)
$$

If you ever wondered, why delta uses $\mathcal{N}(d_+)$, and not $\mathcal{N}(d_-)$, which is really not fair, you now have an answer: both these terms are deltas, and only the FX market can really explain what's going on:

- $\phi e^{-r_f \tau} \mathcal{N}(\phi d_+)$ is the delta if the premium is paid in USD,
- $\phi e^{-r_d \tau} \mathcal{N}(\phi d_-)$ is the delta if the premium is paid in EUR.

In FX options markets there is no preference between the two, as a premium can always (well normally always) be paid in either currency. The premium-adjusted delta is therefore also related to the dual delta (31).

Default Premium Currency Quotations in FX require some patience, because we need to first sort out which currency is domestic, which is foreign, what is the notional currency of the option, and what is the premium currency. Unfortunately this is not symmetric, since the counterpart might have another notion of domestic currency for a given currency pair. Hence in the professional inter bank market there is a generic notion of delta per currency pair. Table 1.3 provides a short overview. Details on all currency pairs can be found in your risk management system (well, if you have a good one). Essentially there are only four currency pairs with a premium paid in domestic currency by default. All other pairs use premium-adjustment.

premium-unadjusted	premium-adjusted
EUR/USD	USD/CAD
GBP/USD	EUR/GBP
AUD/USD	USD/JPY
NZD/USD	EUR/JPY
	USD/BRL
	USD/CHF
	EUR/CHF
	USD/ILS
	USD/SGD
	EUR/TRY

Table 1.3: *Default premium currency for a small selection of currency pairs. LHS currency pairs assume premium paid in USD (domestic currency), RHS assume premium paid in foreign currency.*

Example of Delta Quotations

We consider two examples in Table 1.4 and Table 1.5 to compare the various versions of deltas that are used in practice.

delta ccy	prem ccy	FENICS	formula	delta
% EUR	EUR	lhs	$\delta_{raw} - P$	44.72
% EUR	USD	rhs	δ_{raw}	49.15
% USD	EUR	rhs [flip F4]	$-(\delta_{raw} - P)S/K$	-44.72
% USD	USD	lhs [flip F4]	$-(\delta_{raw})S/K$	-49.15

Table 1.4: *1y EUR call USD put strike $K = 0.9090$ for a EUR–based bank. Market data: spot $S = 0.9090$, volatility $\sigma = 12\%$, EUR rate $r_f = 3.96\%$, USD rate $r_d = 3.57\%$. The raw delta is 49.15%EUR and the value is 4.427%EUR.*

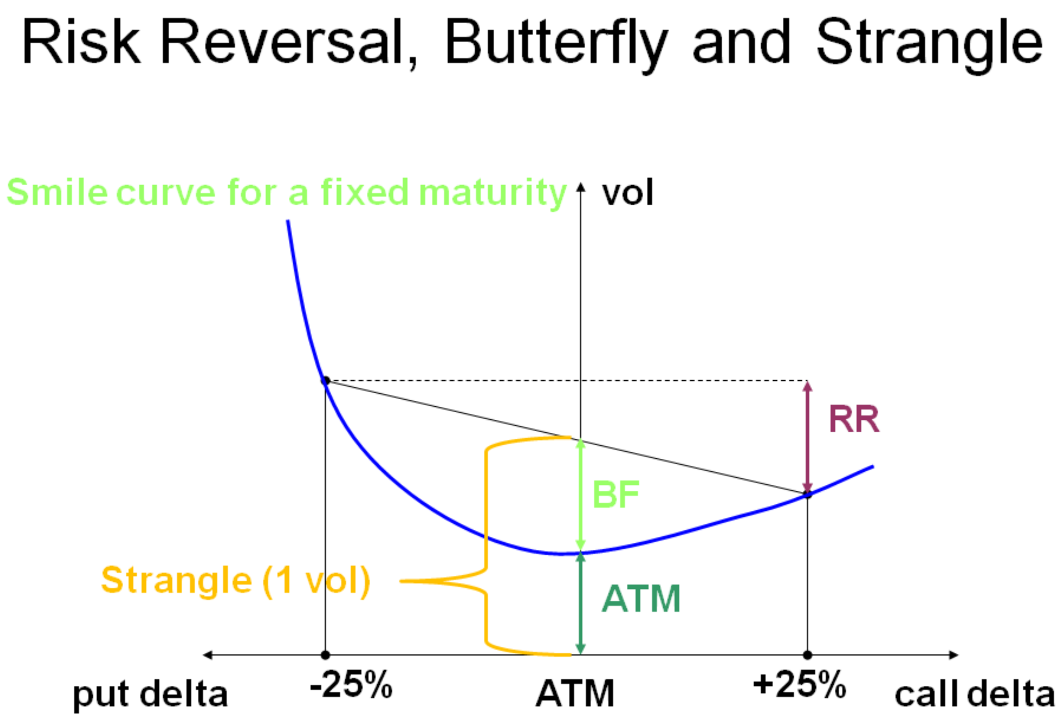

Figure 1.10: *Relationship of risk reversal, butterfly, and market strangle volatility (the 1-vol-strangle).*

1.5.4 At-The-Money Volatility Interpolation

We denote by σ the volatility, which is the *spot volatility* for the time interval from horizon to expiry. The corresponding spot variance is denoted by $\sigma^2\tau$, where τ is the time difference between expiry and horizon. Corresponding forward volatilities σ_f and forward variances apply to a time interval starting later than horizon. The interpolation of at-the-money volatilities takes into account the effect of reduced volatility on weekends and on days closed in the global main trading centers London or New York and the local market, e.g. Tokyo for JPY–trades. The change is done for the one–day forward volatility. You may apply a reduction in the one–day forward variance of 25% for each London and New York closed day. For local market holidays you may use a reduction of 25%, where local holidays for EUR are ignored. Weekends can be accounted for by a reduction to 15% variance. The variance on trading days is adjusted to match the volatility on the pillars of the ATM–volatility curve exactly. Obviously, the reduction percentages are arbitrary and different traders may have different opinions about these.

The procedure starts from the two pillars t_1, t_2 surrounding the date t_r in question. The ATM forward volatility for the period is calculated based on the consistency condition

Spot reference	1.1500 EUR-USD
Company buys	EUR call USD put with lower strike
Company sells	EUR call USD put with higher strike
Maturity	1 year
Notional of both Call options	EUR 1,000,000
Strike of the long Call option	1.1400 EUR-USD
Strike of the short Call option	1.1800 EUR-USD
Premium	USD 20,000.00
Premium of the long EUR call only	USD 63,000.00

Table 1.12: *Example of a Call spread*

Critical Assessment

The Call Spread lowers the cost of the protection against a rising EUR for the treasurer, but fails to protect the extreme risk. Viewed as an insurance it covers small losses but fails to cover potential big losses. One can use it, but we would want to be sure the treasurer understands the consequences. The situation is different for a different client type: the investor, i.e. the market participant *without* the underlying cash flow. The investor buying a Call Spread merely waives participation in an extreme rise of the underlying market. This is why Call Spreads are commonly used in private banking and retail banking.

Ratio Call Spread

A variation of a Call Spread is a *Ratio Call Spread*. The treasurer/investor sells more than one Call with the higher strike, e.g. two Calls. The Call Spread becomes *leveraged* with a leverage of 2. The incentive is to generate a strategy that's even cheaper than the Call Spread, ideally in fact zero cost. One can achieve this by lowering the higher strike or by increasing the leverage. The problem will then be that the position of a long Ratio Call Spread can become negative if spot increases substantially. Let me tell you the story of a Turkish trader who set up a speculative position in USD-TRY in 2008 using a highly leveraged ratio call spread on a margined account following a trade idea that the Turkish Lira would depreciate over the coming months, but not become weaker than 1.6000 USD-TRY, a level the trader had chosen for the higher strike, as illustrated in Figure 1.21.

It turned out that the trader had underestimated the Gamma/Vega exposure with rising spot, which consequently led to a high VaR. The trader received repeated margin calls , could eventually not meet them and the bank had force-closed his position. The trader filed a claim

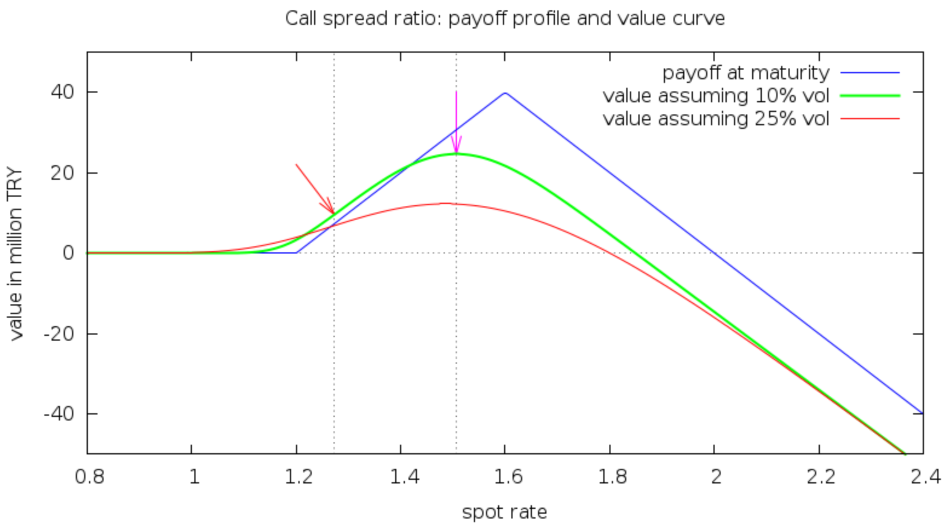

Figure 1.21: *Position of a Ratio Call Spread reflecting the view of a rising and sharp landing of USD-TRY at about 1.6000 in 2008*

against the bank stating that the close-out was unfair, and that eventually the Lira did what he had predicted.

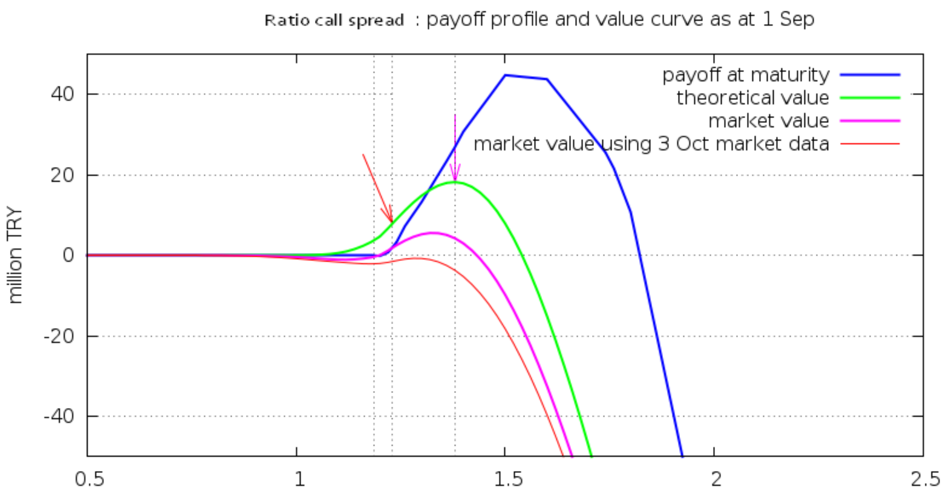

Figure 1.22: *Smile effect in a Ratio Call Spread in USD-TRY*

The claimant being an experienced SuperDerivatives user alleged he had never heard of smile. The effect of smile on the valuation is illustrated in Figure 1.22. What went wrong? At inception of the trade, the value of the position was close to zero, because the leverage of the Ratio Call Spread was designed like that. Furthermore, since the vega and gamma of the long Call and the vega and gamma of the short Call neutralized each other, the trader was deceived by spotting a zero-cost and zero-risk strategy in his portfolio. Then, ignoring the smile effect and underestimating the change of the vega/gamma position with a rising USD-TRY spot led to unexpected continued margin calls. It was impossible to navigate this position through the October 2008 financial crisis with a limited amount of cash.

Loss Calculation In order to calculated the loss of the position, which was delta-hedged on top of all for a reason I failed to understand, the claimant's dodgy expert calculated loss amount by constructing a "zero gamma - long theta" portfolio.

- Consider the change of value of the option book V:

$$V(S+\delta S, t+\delta t) = V(S,t) + \Delta\,\delta S + \tfrac{1}{2}\Gamma\,\delta S^2 + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2)$$

- Then the change of the delta-hedged option portfolio $P(S,t) \stackrel{\Delta}{=} V(S,t) + h\,S$ becomes

$$P(S+\delta S, t+\delta t) = P(S,t) + (h+\Delta)\delta S + \tfrac{1}{2}\Gamma\,\delta S^2 + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2).$$

- The P&L at time $t+\delta t$ is

$$\begin{aligned} & P(S+\delta S, t+\delta t) - P(S,t) \\ =& (h_t+\Delta)\delta S + \tfrac{1}{2}\Gamma\,\delta S^2 + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2) \end{aligned} \tag{163}$$

- The standard delta hedge $h_t := -\Delta$ yields

$$\text{P\&L at time } (t+\delta t) = \tfrac{1}{2}\Gamma\,\delta S^2 + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2)$$

The dodgy expert had argued that the spot reference for the delta hedge is unknown, because the delta hedge had been executed during the day, and that therefore, one must use the average of the previous day's end of day spot and the current day's end of day spot as a proxy for the spot reference for the intra-day delta hedge. While this may sound reasonable to an outsider, it is obvious that this average approach uses the information of the future spot, and is therefore a crystal ball approach. Some basic calculations make this very clear. Using the dodgy expert's "average hedge"

$$h_t = -\frac{\Delta_{t+\delta t} + \Delta_t}{2} = -\frac{V'(S+\delta S, t+\delta t) + V'(S,t)}{2},$$

the fact that $\Delta_t = V'(S,t)$, and Equation (163) we obtain the P&L at time $t+\delta t$ as

$$\begin{aligned} & \delta S(h_t + \Delta_t) + \tfrac{1}{2}\delta S^2\,\Gamma + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2) \\ =& \delta S \frac{-\Delta_{t+\delta t} - \Delta_t + 2\Delta_t}{2} + \tfrac{1}{2}\delta S^2\,\Gamma + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2) \\ =& -\delta S \frac{\Delta_{t+\delta t} - \Delta_t}{2} + \tfrac{1}{2}\delta S^2\,\Gamma + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2) \\ =& -\delta S \tfrac{1}{2} V''(S,t)\delta S + \tfrac{1}{2}\delta S^2\,\Gamma + \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2), \\ =& \Theta\,\delta t + \mathrm{o}(\delta t, \delta S^2). \end{aligned}$$

The dodgy expert introduces a systematic error in the P&L by artificially removing the entire gamma risk. The claim was based on artificially generated money. The mistakes were spotted by the court rather quickly and the Turkish trader withdrew his claim. As a conclusion I would like to reiterate that even simple vanilla structures can cause surprises and losses, which are unpleasant for all parties involved. Issues like leverage and short options should always be carefully discussed with the buy-side to avoid such surprises and losses.

1.6.2 Risk Reversal

Very often corporates seek so-called zero-cost strategies to hedge their international cash-flows. Since buying a call requires a premium, the buyer can sell another option to finance the purchase of the call. A popular liquid product in FX markets is the Risk Reversal or Collar or Range Forward. The term *Cylinder* is also used as a synonym for the Risk Reversal, or more often actually refers to a more general form of a Risk Reversal to distinguish it from the standard case. A risk reversal is a combination of a long call and a short put. It entitles the holder to buy an agreed amount of a currency (say EUR) on a specified date (maturity) at a pre-determined rate (long strike) assuming the exchange rate is above the long strike at maturity. However, if the exchange rate is below the strike of the short put at maturity, the holder is obliged to buy the amount of EUR determined by the short strike. Therefore, buying a risk reversal provides full protection against rising EUR. The holder will typically exercise the option only if the spot is above the long strike at maturity. The risk on the upside is financed by a risk on the downside. Since the risk is reversed, the strategy is named Risk Reversal.

Advantages

- Full protection against stronger EUR/weaker USD
- Can be structured as a zero cost strategy

Disadvantages

- Participation in weaker EUR/stronger USD is limited to the strike of the sold put

For example, a company wants to sell 1 M USD. At maturity T:

1. If $S_T < K_1$, it will be obliged to sell USD at K_1. Compared to the market spot the loss can be large. However, compared to the outright forward rate at inception of the trade, K_1 is usually only marginally worse.
2. If $K_1 < S_T < K_2$, it will not exercise the call option. The company can trade at the prevailing spot level.
3. If $S_T > K_2$, it will exercise the option and sell USD at strike K_2.

around ATM. To clarify: vega attains its maximum if the strike is chosen as the one that makes a straddle delta-neutral. Vega is the first derivative of the option's value as volatility changes. Vanna is the second derivative of the option's value as spot and volatility change. As a mixed second derivative it has two interpretations:

Interpretation 1: Vanna = ∂ vega /∂ spot

Interpretation 2: Vanna = ∂ delta /∂ vol

1. To understand why vega peaks ATM and vanna OTM, plot vanilla vega on the spot space.
2. Calculate vanna for an at-the-money straddle as a formula in the Black-Scholes model.
3. Following interpretation 2, plot a vanilla (call) delta for a small volatility and a big volatility.

Straddle Volga

Calculate volga for an at-the-money straddle.

Butterfly Premium Difference

By how much does the premium of a long butterfly constructed via call options as in Equation (164) differ from the premium of a long butterfly constructed via strangle and straddle?

Short Gamma Long Vega

Find a strategy (in the sense of a linear combination) of vanilla options that is short gamma and long vega (in the Black-Scholes model). Explain why this is not possible for single vanilla option. Hint: revisit Equation (19) for gamma and Equation (24) for vega of a vanilla option in the Black-Scholes model.

1.7 First Generation Exotics

For the sake of example we consider EUR/USD - the most liquidly traded currency pair in the foreign exchange market. Internationally active market participants are always subject to changing foreign exchange rates. To hedge this exposure an immense variety of derivatives transactions are traded worldwide. Besides vanilla (European style put and call) options, the so-called first generation exotics have become standard derivative instruments.

1.7.1 Classification

The term *first generation exotic* does not refer to a clearly defined set of derivatives contracts, especially not in a legal sense. However, it is universally agreed that Foreign Exchange Transactions (spot and forward contracts) and vanilla options are not in the set. It is also universally agreed that flip-flop-kiko-tarns and correlation swaps are not in the set either. We can then classify first generation exotics by:

Time of Introduction: Here we consider the history and the time when certain contracts first traded.

Existence of Standardized Deal Confirmations: We would classify a transaction as first generation exotic if there exists a standardized deal confirmation template, such as the ones provided by ISDA.

Replicability: We would classify a transaction as first generation exotic if it can be statically or semi-statically replicated or approximated by spot, forward and vanilla option contracts.

Trading Volume: We would classify a transaction as first generation exotic, if its trading volume is sufficiently high (and the transaction is not a spot, forward or vanilla option).

There can also be other approaches to classify first generation exotics. I would like to point out that a first generation exotic not necessarily needs to be a currency option. For example, a flexi forward can be considered a first generation exotic in terms of both timing and standardization, but is clearly not an option. A variance swap can be considered a first generation exotic in terms of both standardization and replicability, but is clearly not an option, because there is no right to exercise. Classification by trading volume would change the set of first generation exotics over time and is consequently not suitable for classification purposes. The various classifications would generate overlaps as well as differences. One could certainly argue to label barrier options as first generation exotic, because they would satisfy all of the above: timing, standardization, replicability and volume. For Asian options, the timing criterion would make them first generation as they started trading in Tokyo in 1987, but there is – even in 2016 – no standardized deal confirmation provided by ISDA. Power options satisfy timing and replicability, but not standardization or trading volume. This leads to the effect that the transition between the generations is not strict and can depend on the person you ask and classification the respective person has in mind. A clean approach to classification could be sticking to the standardization, which would classify barrier options and touch products, as well as variance and volatility swaps as first generation exotic, based on the existing ISDA Definitions and their supplements. The question which transaction is standardized can then be viewed in light of ISDA's *Barrier Option Supplement* [78], which appeared in 2005. ISDA has extended the 1998 FX and Currency Option Definitions [77] to the range of touch products and single and double barrier options, including time windows for barriers. These are (a) options that knock in or out if the underlying hits a barrier (or one of two barriers) and (b) all kind of touch products: a one-touch [no-touch] pays a fixed amount of either USD or EUR if the spot ever [never] trades at or beyond the touch-level and zero otherwise. Double one-touch and no-touch contracts work the same way but have two barriers. More on barrier options will be in Section 1.7.3. The ISDA Barrier Option Supplement contains all the relevant definitions required to confirm these transactions by standardized short templates. It is clearly defined what a *barrier event* or a *determination agent* is. However, for purposes of classification, the product range covered by this ISDA supplement is not necessarily viewed as equivalent by all market participants. Moreover, the set of first generation exotics would then change each time ISDA publishes a new supplement. My personal preference is to classify the set of first generation exotics by the time of introduction in the market. This is reflected in this section.

1.7.2 European Digitals and the Windmill Effect

In this section we discuss the digital options along with the questions

1. how can we price digital options with smile?
2. is the implied volatility of a digital option the same as the implied volatility of the corresponding vanilla option?
3. what is the windmill effect?

Digital Options

(European) digital options pay off

$$v(T; S_T) = \mathbb{I}_{\{\phi S_T \geq \phi K\}} \text{ domestic paying,} \tag{165}$$

$$w(T; S_T) = S_T \mathbb{I}_{\{\phi S_T \geq \phi K\}} \text{ foreign paying.} \tag{166}$$

In the domestic paying case the payment of the fixed amount is in domestic currency, whereas in the foreign paying case the payment is in foreign currency. We obtain for the theoretical value functions

$$v(t; x) = e^{-r_d \tau} \mathcal{N}(\phi d_-), \tag{167}$$

$$w(t; x) = x e^{-r_f \tau} \mathcal{N}(\phi d_+), \tag{168}$$

of the digital options paying one unit of domestic and paying one unit of foreign currency respectively.

The question is, how we can use the existing smile for vanilla options to read off a suitable volatility that we can plug into (167) to get a smile-adjusted value for the digital. In particular, can we take the same volatility as for the vanilla with strike K? The answer can be found by looking at the static replication and its associated cost.

Replication of Digital Options

An obvious attempt to replicate a domestic digital is the call spread illustrated in Figure 1.29.

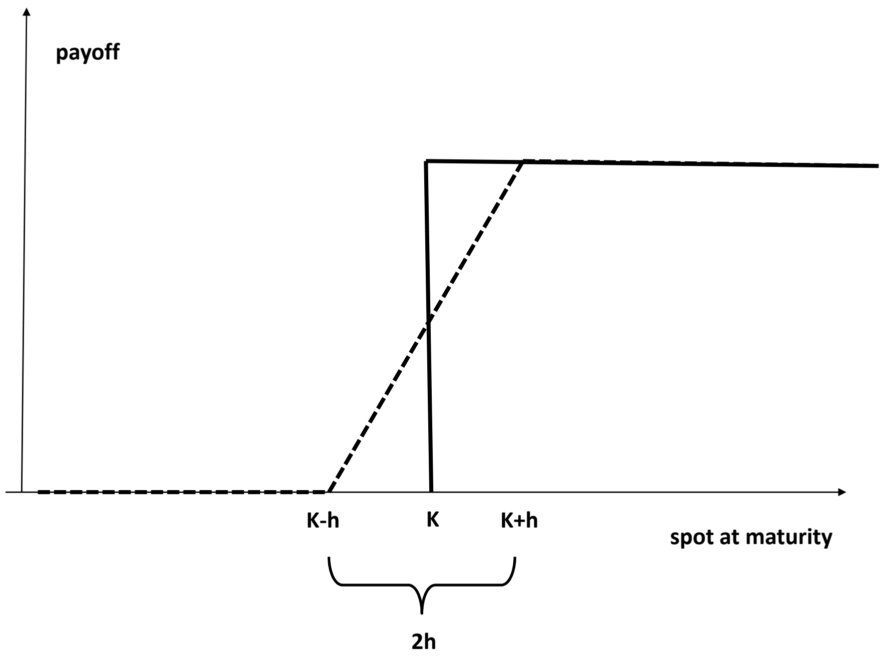

Figure 1.29: *Replicating a digital call with a vanilla call spread*

In the limit we have

$$\text{digital}(K) = \lim_{h \to 0} \frac{\text{vanilla}(K-h) - \text{vanilla}(K+h)}{2h} \tag{169}$$

However, since the number $\frac{1}{2h}$ corresponds to the (foreign!) notional of the vanilla options to trade, there are practical limitations we need to approximate the digital with a call spread with finite notional. To be on the safe side, the replication can be built as a super-replication with the upper strike chosen as the strike of the digital. This is too expensive to be used for pricing. For pricing we go for the symmetric compromise and chose one strike to be lower than the strike of the digital and the other one higher. The practical limitation for the difference of the two strikes is two pips. This is equivalent to a factor of 5000 to compute the notional of the vanilla options. In practice one would mostly take a larger difference or equivalently a smaller notional, say a factor of 50 for the notional multiplier. In this case the two strikes are two big figures apart. Consequently, we need to think very carefully about which volatilities to chose for the pricing. Taking the same volatility for the digital as for the corresponding vanilla would mean that we would price the options in the replicating portfolio with a flat volatility. Since the smile is not constant, this could produce a significant error. We should take the market volatilities for the replication to find a good price for the digital with smile. The mismatch is caused by the windmill effect and is illustrated in Figure 1.30.

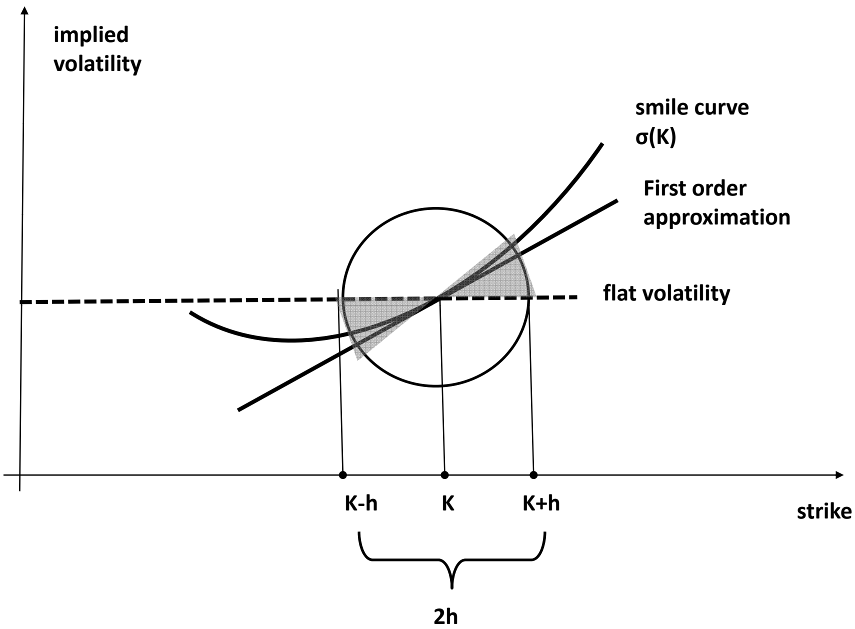

Figure 1.30: *Windmill effect. The shaded gray areas show the miss-pricing. Working with a flat volatility is not sufficient. A first order approximation is attained using the windmill-adjustment and will be considerably better. Working on the smile curve exactly is achieved by the call spread replication.*

We conclude from Equation (169) that the value of the domestic-paying digital call is generally, and independent of any model, given by

$$\text{digital}(K) = -\frac{\partial}{\partial K}\text{vanilla}(K). \tag{170}$$

This helps us to get a much better approximation of the smile-adjusted value of the digital call. Let us abbreviate the vanilla call value function by $v(K,\sigma)$ and notice that, in a valuation with smile, the volatility σ is itself a function of K. The rest is a consequent application of calculus 101:

$$\begin{aligned}\text{digital}(K) &= -\frac{\partial}{\partial K}v(K,\sigma(K)) \\ &= -v_K(K,\sigma(K)) - v_\sigma(K,\sigma(K))\sigma'(K). \end{aligned}\tag{171}$$

The first term is the dual delta of the vanilla, i.e. $e^{-r_d\tau}\mathcal{N}(\phi d_-)$ as in (167). The second term is the windmill-adjustment. It depends on the vanilla vega at the given strike and the slope of the smile on the strike space. The vega impact is maximal for the ATM (delta-neutral) strike and the contribution decreases from that ATM strike in both directions. Vanilla vega is always positive. The slope of the smile on the strike space means that we need to consider the implied volatility as a function of strike, i.e. strike is on the x-axis and implied volatility on the y-axis. This slope can be positive or negative. For a strongly down-skewed market, the slope is negative, whence the windmill-adjustment is positive, and the digital call (put) will be valued higher (lower) when smile is taken into consideration. In short: down-skew leads to marking the digital calls up and the digital puts down; up-skew leads to marking the digital calls down and the digital puts up. This effect is highly important, as the market price of the digital translates to the market price of European knock-out, the one-touch, and hence the market price of the reverse-knock-out and therefore into all structured products that contain these first generation exotics as building blocks. Note that the slope of the implied volatility curve is the local slope around the strike under consideration. Its sign is not necessarily equal to the sign of the 25-delta-risk reversal. We provide an example in Table 1.20. The windmill-adjustment is analytically tractable for a parametric smile interpolation, see Formula (162) in Section 1.5.12.

Digital value without smile	$-v_K(K,\sigma(K)) = e^{-r_d\tau}\mathcal{N}(\phi d_-)$	0.322134
Value of the replication	$\frac{1}{2h}[v(K-h,\sigma(K-h)) - v(K+h,\sigma(K+h))]$	0.358975
Windmill-adjustment	$-v_\sigma(K,\sigma(K))\sigma'(K)$	0.036845
Digital value with smile	$-v_K(K,\sigma(K)) - v_\sigma(K,\sigma(K))\sigma'(K)$	0.358978
Implied volatility		22.005%

Table 1.20: *Windmill-adjustment for a digital call paying one unit of domestic currency. Contract data: Time to maturity* τ *=186/365, strike* K *=1.4500; market data spot* S *=1.4000,* r_d *=2.5%,* r_f *=4.0%, volatilities* $\sigma(K)$ *=15.0000%,* $\sigma(1.4499)$ *=15.0010%,* $\sigma(1.4501)$ *=14.9990%,* h *=0.0001.*

Volatility Implied by Digital Options

With the windmill-adjustment it is obvious, that digital options can't be priced with the same volatility as the corresponding vanilla. Technically, it is possible to retrieve the volatility from a digital option's price. This is equivalent to retrieving the volatility from a given delta. It boils down to a quadratic equation with two solutions, similar as in Equation (62). The volatility implied by the digital call price listed in Table 1.20 is based on this result. Note that the implied volatility of about 22% is very different from the smile volatility 15%, so in terms of volatility, the windmill effect is all the more visible.

1.7.11 Exercises

Foreign Digital Value via Static Replication

Derive Equation (168) using a static replication of the foreign digital by domestic digital and vanilla options.

Foreign Digital Value via Change of Measure

Derive Equation (168) by calculating the expected value of the payoff (166) and a change of measure.

Compound

Consider a EUR-USD market with spot at 1.2500, EUR rate at 2.5%, USD rate at 2.0%, volatility at 10.0% and the situation of a treasurer expecting to receive 1 M USD in one year, that he wishes to change into EUR at the current spot rate of 1.2500. In 6 months he will know if the company gets the definite order. Compute the price of a vanilla EUR call USD put in EUR. Alternatively compute the price of a compound with two thirds of the total premium to be paid at inception and one third to be paid in 6 months. Do the same computations if the sales margin for the vanilla is 1 EUR per 1,000 USD notional and for the compound is 2 EUR per 1,000 USD notional. After six months the company ends up not getting the order and can waive its hedge. How much would it get for the vanilla if the spot is at 1.1500, at 1.2500 and at 1.3500? Would it be better for the treasurer to own the compound and not pay the second premium? How would you split up the premium for the compound to persuade the treasurer to buy the compound rather than the vanilla? (After all there is more margin to earn.)

Perpetual One-Touch Replication

Find the fair price and a semi-static replication of a *perpetual one-touch*, which pays 1 unit of the domestic currency if the barrier $H > S_0$ is ever hit, where S_0 denotes the current exchange rate. How about payment in the foreign currency? How about a *perpetual no-touch*? These thoughts are developed further to a *vanilla-one-touch duality* by Peter Carr [23].

Perpetual Double-One-Touch

Find the value of a *perpetual double-one-touch*, which pays a rebate R_H, if the spot reaches the higher level H before the lower level L, and R_L, if the spot reaches the lower level first. Consider as an example the EUR-USD market with a spot of S_0 at time zero between L and H.

Let the interest rates of both EUR and USD be zero and the volatility be 10%. The specified rebates are paid in USD. There is no finite expiration time, but the rebate is paid whenever one of the levels is reached. How would you replicate a short position (semi-) statically?

Strike-Out Replication and Impact of Jumps

A call (put) option is the right to buy (sell) one unit of foreign currency on a maturity date T at a pre-defined price K, called the strike price. A knock-out call with barrier B is like a call option that becomes worthless, if the underlying ever touches the barrier B at any time between inception of the trade and its expiration time. Let the market parameters be spot $S_0 = 120$, all interest rates be zero, volatility $\sigma = 10\%$. In a liquid and jump-free market, find the value of a one-year *strike-out*, i.e. a down-and-out knock-out call, where $K = B = 100$.

Suppose now, that the spot price movement can have downward jumps, but the forward price is still constant and equal to the spot (since we assume zero interest rates). How do these possible jumps influence the value of the knock-out call?

The solution to this problem is used for the design of *turbo notes*, see Section 2.6.4.

Strike-Out Call Vega

What is the vega profile as a function of spot for a strike-out call? What can you say about the sign of vega?

Double-No-Touch with Notional in Foreign Currency

Given Equation (235), which represents the theoretical value (TV) of a double-no-touch in units of domestic currency, where the payoff currency is also domestic. Let us denote this function by

$$v^d(S, r_d, r_f, \sigma, L, H), \tag{356}$$

where the superscript d indicates that the payoff currency is domestic. Using this formula, prove that the corresponding value in domestic currency of a double-no-touch paying one unit of *foreign* currency is given by

$$v^f(S, r_d, r_f, \sigma, L, H) = Sv^d\left(\frac{1}{S}, r_f, r_d, \sigma, \frac{1}{H}, \frac{1}{L}\right). \tag{357}$$

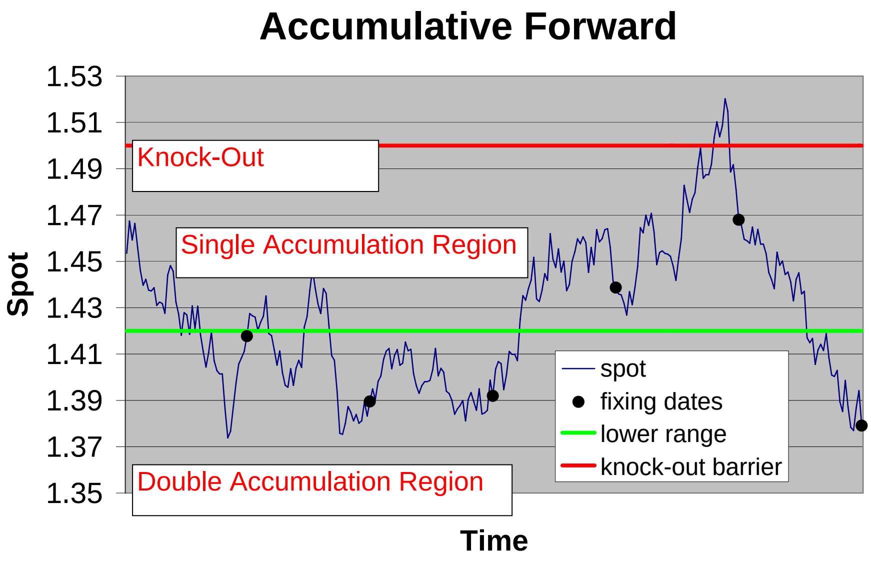

Figure 2.10: *Ranges for an accumulative forward in EUR/GBP in GBP-EUR quotation.*

Notional is computed by

$$\text{Notional} \cdot \left[\frac{\text{RangeDays}}{\text{Days}} + 2\frac{\text{TopDays}}{\text{Days}}\right], \tag{12}$$

Days is the number of all business days in the period.

RangeDays is the number of business days in the period that the reference rate fixes between the pre-agreed exchange rate and the knock-out barrier.

TopDays is the number of business days in the period that the reference rate fixes below the pre-agreed exchange rate.

Reference Rate is the EUR/GBP rate in GBP-EUR quotation by the European Central Bank published on Reuters page ECB37.

Settlement is physical on the delivery date.

Knock-Out Condition: If GBP-EUR trades at or above the knock-out barrier at any time, then the transaction terminates and the client sells 50% of the accumulated EUR amount up to the knock-out date if the knock-out occurs after 14h15 Frankfurt Time. The GBP-EUR daily trading range will be determined by the bank and should a rate query arise, can be cross-referenced with three market making banks.

Period	122 days
Start date	08-Apr-03
End date	24-Sep-03
Delivery date	26-Sep-03
Pre-agreed exchange rate	K = 1.4200
Knock-out barrier	1.5000
Client	sells EUR (buys GBP) at K
Notional	EUR 7,500,000.00
Notional per business day	EUR 61,475.41
Premium	Zero
EUR/GBP spot reference	1.4535 GBP-EUR
12mth outright forward	1.4463 (GBP 3.53%, EUR 2.45%)

Table 2.12: *Terms and conditions of an accumulated forward in GBP-EUR*

Example: If the GBP-EUR reference rate fixing remains in the range 1.4200 and 1.5000 for 110 days of the business days before trading at 1.5000, then the contract would terminate at that point with a notional of EUR 3,381,147.55, which the client sells at 1.4200 for settlement on the delivery date (26-Sep-03).

2.1.12 Boomerang Forward

The following zero cost transaction could serve as alternative for corporates, particularly those who trade accumulators. As an example we consider a treasurer buying EUR 10 M (selling USD) at a spot reference of 1.2000 and 1 year outright forward reference of 1.2000.

- If EUR/USD remains above $B = 1.1349$ for the $T = 1$ year period, the bank will pay $(S(T) - 1.1790) \cdot 10/S(T)$ M EUR to the treasurer (cash settlement). Or in the case of delivery settlement, the treasurer can buy EUR at 1.1790, which is much better than the initial spot and the initial outright forward rate.
- If EUR/USD trades below B at any time during that year period, the treasurer pays $(1.2190 - S(T)) \cdot 10$ M USD to the bank, unless spot is above 1.2190 at maturity, in which case the bank pays $(S(T) - 1.2190) \cdot 10/S(T)$ M EUR to the treasurer. In other words, in case of knock-in at B the treasurer is locked into a forward contract with strike 1.2190. This is also his guaranteed worst case.

The boomerang forward can be structured as follows. The treasurer

1. buys 1Y 1.1790 EUR call USD put KO at 1.1349,
2. sells 1Y 1.2190 EUR put USD call RKI at 1.1349,
3. buys 1Y 1.2190 EUR call USD put KI at 1.1349,

all with a notional of EUR 10 M. The good feature here as compared to accumulators is the existence of a guaranteed worst case. The zero cost strategy only hurts the client if the spot falls significantly, so the barrier is hit and the client is faced with a much higher exchange rate than the prevailing spot. However, to be fair, we must compare the final exchange rate to the outright forward rate at inception, and their difference is less than 2 big figures.

2.1.13 Amortizing Forward

A treasurer can enhance a company's effective foreign exchange rate at zero cost by taking risk, but still participate in the exchange rate moving in its favor. One of the possible solutions can be an amortizing forward, which provides the treasurer with a worst case exchange rate, but if the spot rate moves against him the notional of the forward contract decreases following a pre-specified amortization schedule exhibited in Figure 2.11.

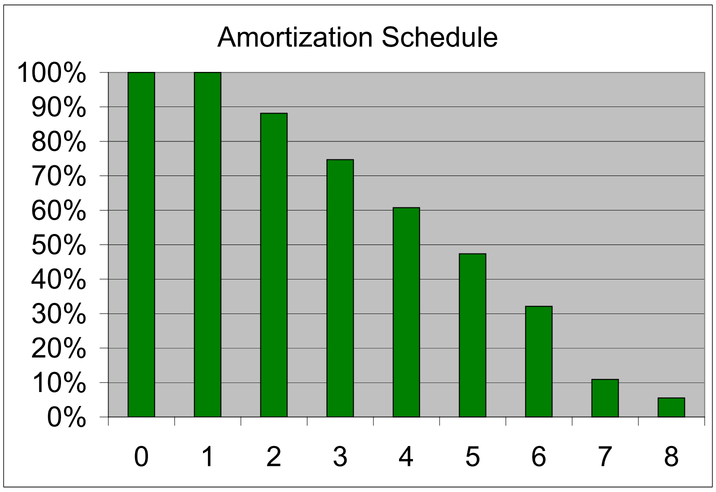

Figure 2.11: *Amortization schedule over a time span of eight quarters of an amortizing forward contract for a EUR buyer/USD seller*

For contract parameters maturity in years T, strike K and knock-out barrier B, notional of the underlying N, payment schedule $0 = t_0 < t_1 < t_2 \ldots, t_n = T$, the payoff is

$$F(S, K, B, t_i, N) \stackrel{\Delta}{=} N \sum_{m=1}^{n} (S_{t_m} - K) \prod_{i=1}^{m-1} \left[\min \left[1, \left(\frac{S_{t_i} - B}{K - B} \right)^+ \right] \right]. \quad (13)$$

independently of the number of strike resets, the client is guaranteed to have 12 cashflows. This may raise the initial strike to about 1.4000.

Participation Another example for the greedy or the payoff-language freak is a *Market Following TPF*: Here the client is concerned about the lack of participation in any favorable market development, i.e. if EUR/USD depreciates creating losses on the TPF. In a market following TPF, each time the market fixes out-of-the-money the client's strike improves by a pre-determined offset amount. For instance, if at any expiry EUR/USD spot is below the strike, the strike is reduced by 0.5 big figures. This may raise the initial strike to about 1.4100.

Disclaimer

Who reads disclaimers? Or instruction manuals? Well, actually, it is quite entertaining. Look at this extract taken from a bank:

> The transaction(s) or products(s) mentioned herein may not be appropriate for all investors and before entering into any transaction you should take steps to ensure that you fully understand the transaction and have made an independent assessment of the appropriateness of the transaction in the light of your own objectives and circumstances, including the possible risks and benefits of entering into such transaction. You should also consider seeking advice from your own advisers in making this assessment. If you decide to enter into a transaction, you do so in reliance on your own judgment. The information contained in this document is based on material we believe to be reliable; however, we do not represent that it is accurate, current, complete, or error free. Assumptions, estimates and opinions contained in this document constitute our judgment as of the date of the document and are subject to change without notice. Any projections are based on a number of assumptions as to market conditions and there can be no guarantee that any projected results will be achieved.

One can't read this often enough. In my opinion it is a good exercise to calculate the worst case scenario in each product and define an exit strategy at the beginning of the transaction and strictly follow it, especially if the tarf is traded as an investment product on a margined account.

2.2.4 Pivot Target Forward (PTF)

A pivot target forward belongs to the class of target forwards. Again we set up a sequence of forward FX transactions based on a fixing schedule, count the profits and have the trade stop once the profit reaches a pre-specified target. Losses do not offset the profits in the accumulated profit counting scheme. This product is a bet on the spot staying in a range near initial spot. If the spot leave this range, and does not come back, then potential losses are unlimited, where as potential profits are always limited to the target.

The idea is to generate a zero cost product by paying a small profit with a high probability and causing a catastrophic loss with a small probability. On average it is worth zero, or more precisely, worth a negative amount to the client that corresponds to the sales margin for the bank and cost of the trade.

Terms of a Traded Pivot Target Forward in USD-CAD

Let us look at an example of a transaction where a bank traded a pivot target forward with a HNWI on 22 July 2008. The Term Sheet is displayed in Table 2.35.

Individual Terms: Each of the FX Transactions to which this Confirmation relates have the following individual terms:

In the confirmation there is normally a list of each fixing day. This will be too boring to fill this book. Basically, it starts with the first fixing day of 23 July 2008 and ends with the 22 July 2009, with daily fixings except weekends.

Representations: Each party represents to the other party as of the date that it enters into this Transaction that (absent a written agreement between the parties that expressly imposes affirmative obligations to the contrary for this Transaction):

(i) Non-Reliance. It is acting for its own account, and it has made its own independent decisions to enter into this Transaction and as to whether the Transaction is appropriate or proper for it based upon its own judgment and upon advice from such advisers as it has deemed necessary. It is not relying on any communication (written or oral) of the other party as investment advice or as a recommendation to enter into this Transaction, it being understood that information and explanations related to the terms and conditions of this Transaction shall not be considered to be investment advice or a recommendation to enter into the Transaction. No communication (written or oral) received from the other party shall be deemed to be an assurance or guarantee as to the expected results of this Transaction.

(ii) Assessment and Understanding. It is capable of assessing the merits of and understanding (on its own behalf or through independent professional advice), and understands and accepts the terms and conditions and risks of this Transaction. It is also capable of assuming, and assumes, the risks of the Transaction.

(iii) Status of Parties. The other party is not acting as a fiduciary for or adviser to it in respect of this Transaction.

(iv) Early Termination Provisions: this Transaction shall early terminate and cancel in whole on such Fixing Date (an 'Early Termination Event'). Following such Early Termination Event, the parties shall be relieved of all further payment obligations under the Transaction described herein, except for

1. the obligation of the bank to pay Counterparty on the Coupon Payment Settlement Date (a) the positive Coupon Payment Amounts as determined on the Fixing Dates preceding the Early Termination Event and (b) the Adjusted Coupon Payment Amount. (For the avoidance of doubt, this net payment shall be a payment of CAD 1,500,000) and

2. the obligation of Counterparty to pay the bank on the Coupon Payment Settlement Date the absolute amount of the negative Coupon Payment Amounts as determined on the Fixing Dates preceding the Early Termination Event.

(v) Definitions. 'Fixing Exchange Rate' shall mean the mid CAD/USD[1] rate, expressed as the amount of Canadian Dollar per one United States Dollar for settlement in one Business Day, as displayed at approximately 4:00 p.m. London Time for that Fixing Date by WM Company on the applicable Reuters page, or such other symbol or page that may replace such symbol or page for the purpose of displaying such exchange rate; provided, however, that if such pages are no longer published or is not published as of the designated time and date, and no replacement symbol or page is designated, the Calculation Agent shall determine such affected Fixing Exchange Rate in good faith and in a commercially reasonable manner.

(vi) Business Days. Business Days applicable to each applicable Fixing Date: shall mean all days on which The WM Company, through it's currency market data services, publishes spot rates for the relevant currency pair (the dates on which such services will not be provided may be found on its internet website page, http://www.wmcompany.com/). Business Days applicable to Coupon Payment Settlement Date: Toronto

Critical Judgment

This pivot target forward traded at zero cost and subsequently caused a substantial loss during the financial crisis in October 2008. The client type was an investor (without the underlying cash flow). The way a transaction like this sells is based on human psychology: There is a small gain with a high probability and a disastrous loss with a low probability. The buyer hopes that the small probability is zero. The transaction becomes like selling a lottery ticket. Many humans have problems with interpreting probabilities. Suppose you board a plane and you are told that you will get 50% off the price of your ticket if you fly in the plane which has been classified as crashing with 0.1% probability. Would you fly? What about if you get 90% discount if the plane crashes with 0.01% probability? In reality what one should consider is not only the probability of the disaster, but also whether the disaster is an acceptable worst

[1] If you wonder why the bank did not write USD/CAD then, so do I.

case. We list sample scenarios of a pivot target forward in Figure 2.12. Human psychology furthermore does not easily capture the change of the probability distribution over time. In Figure 2.13 we illustrate the density of the short term, which indicates that a profit occurs almost surely, and compare it with the long term, which shows that as time passes the loss scenarios will become more and more likely.

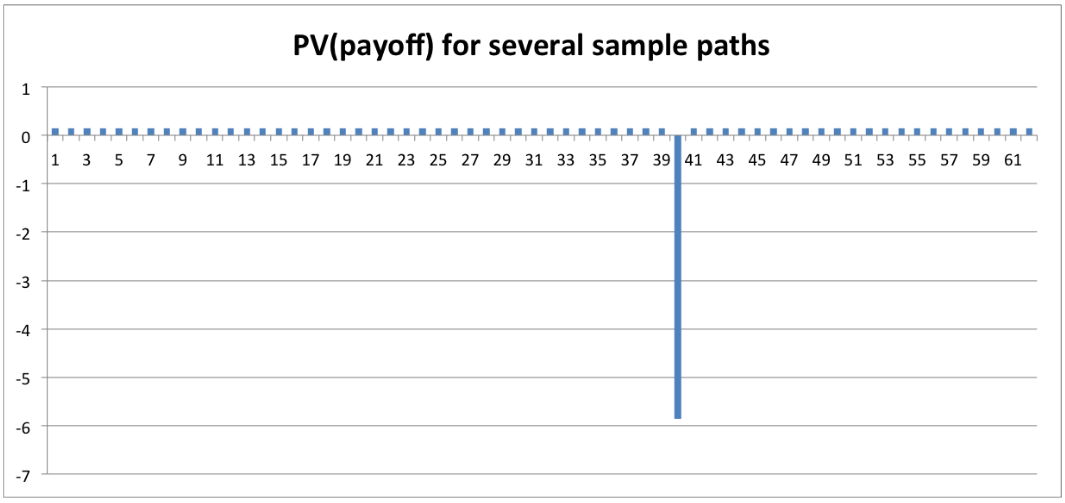

Figure 2.12: *P&L Scenarios Pivot Target Forward*

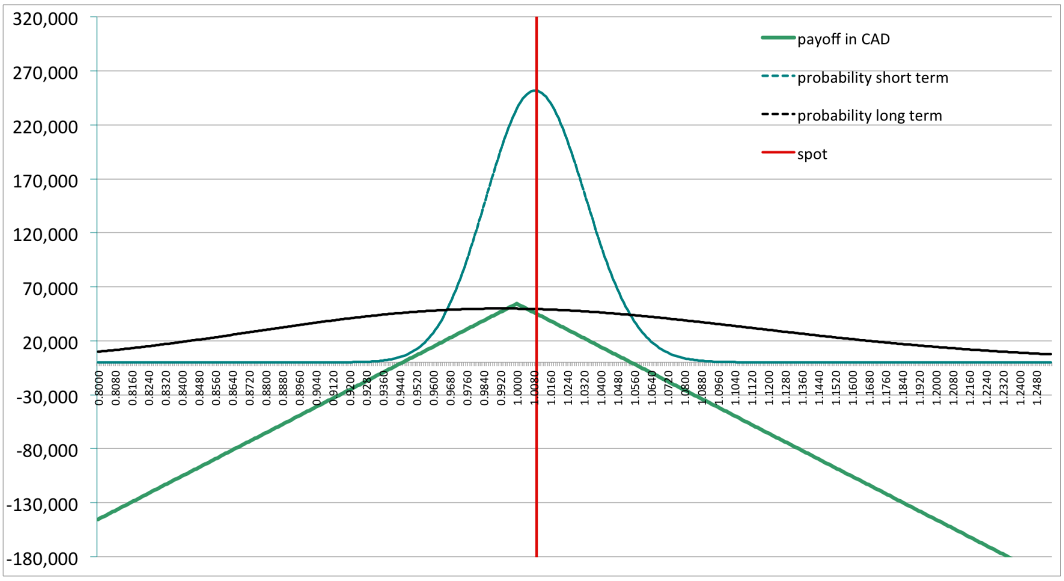

Figure 2.13: *Pivot Target Forward Payoff and Psychology*

Notional Amount	USD 1,000,000
High Strike	1.0545
Pivot	1.0000
Low Strike	0.9455
Observation Dates	As defined in Schedule
Settlement Dates	23 July, 2009
CAD Settlement Amount	If FX Rate ≤ Pivot:
	USD Notional Amount · (FX Rate - Low Strike);
	if FX Rate > Pivot:
	USD Notional Amount · (High Strike - FX Rate);
	if Settlement Amount is positive, Party B will
	deliver the CAD Settlement Amount to Party A;
	if the Settlement Amount is negative, Party A will
	deliver the CAD Settlement Amount to party B.
Positive Intrinsic Value	With respect to each Observation Date,
	Day 1: $\max(0,$ CAD Settlement Amount$)$;
	Day 2 to expiry: Positive Intrinsic Value of previous day
	$+\max(0,$ CAD Settlement Amount$)$
Target Cap Level	CAD 1,500,000
Target Knock Out Event	If at an Observation Date (the Knock out observation date),
	the positive intrinsic value at the Observation Date
	is equal to or exceeds the Target Cap Level,
	this transaction shall be automatically terminated and thereafter no
	future CAD Settlement Amounts shall be calculated.
	The settlement arising at the Knock Out
	Observation Date will be adjusted as follows:
Adjusted Settlement Amount	Target Cap Level-Positive Intrinsic Value of Previous Day
	Party B will pay Party 1 Adjusted Settlement Amount
FX Rate	The Mid WMR USD/CAD spot exchange on each applicable
	observation date evidenced at 4:00 p.m. London Time on Reuters Page
	WMRSpot35 expressed as the number of units of
	CAD per unit of USD as determined by the calculation agent at its sole discretion.
Day Adjustment	Modified Following
Calculation Agent	Party B

Table 2.35: *Term sheet of a pivot target forward in USD-CAD.*

2.2.5 KIKO Tarn

Generally, there is no end to the variants of tarns. One can easily introduce all kind of barriers: European and American barriers, knock-in and knock-out barriers. In this section we consider a case study of an actual trade of an AUD-JPY knock-in-knock-out (kiko) tarn between an American investment bank and an Indonesian HNWI on 27 June 2008, the summer before the Lehman bankruptcy. Normally, tarns are zero premium strategies. Unusually, in this case, the investor receives an upfront premium of AUD 400,000.

KIKO Tarn Product Description

This FX tarn obliges the investor to purchase some amount of AUD and pay in JPY at the fixed rate of exchange of 97.50 JPY per AUD (the 'strike price'), once a fortnight for fifty-two weeks. The strike price is set below the market price prevailing at the trade date which was 101.99. A 'knock-in level' is additionally defined, fixed at 89.50. The amount of AUD that the investor is obliged to purchase each week is either 2,000,000 if the exchange rate is above the strike, or 4,000,000 if the exchange rate is below the knock-in. However, if the exchange rate is above 103.40 (the 'knock-out level'), then the strategy is terminated and no further purchases take place. The knock-out condition applies to all fifty-two weeks, and doesn't affect past deliveries.

An additional feature in the tarn is a cap on gains. For each purchase of AUD 2,000,000 (when the exchange rate is above the strike), the gain is realized immediately and paid in JPY. Should these gains exceed JPY 68,000,000 in aggregate, then the strategy is terminated and no further purchases take place. We outline the key features as follows.

Objective: To profit from a stable AUD/JPY exchange rate

View: Range-bound (market stays between strike and knockout until maturity)

Risk: Exposed to higher volatility. If the exchange rate rises above the knockout, no further purchases happen and the profit to date is locked in, thereby limiting the profit potential. If the exchange rate falls below the knock-in, the size of weekly purchases doubles and at a rate that is then far above the market, i.e., underwater. Either of these conditions is a negative for the investor.

Potential Gain: Maximum gain is JPY 68,000,000 or approximately AUD 657,640.

Potential Loss: Maximum loss is theoretically AUD 104,000,000 in the event that the exchange rate falls to zero without ever rising above the knockout or triggering the cap on gains. Unlike equity, for a major currency pair this is not a feasible scenario. The all-time low of 55.00 would represent a loss of AUD 80,363,636. This is not far from the low of 57.12 for 2007-2008.

Leveraged: Yes: ratio of options sold to options bought is $2\times$.

Margined: Yes: amplifies gains and losses, and reduces the ability of the investor to hold to maturity.

Generally a kiko tarn is a derivative investment on one underlying similar to an accumulator. The essential difference is that the profits are capped by a maximum amount called the *target*. The investor receives profits if the spot goes up, and makes a loss if the spot goes down. Cash flows are specified and occur usually on a sequence of currency fixing dates. Typically, fixings are chosen daily, weekly, fortnightly or monthly. The product terminates if either a knock-out barrier is reached or if the accumulated profit of the client reaches a pre-specified *target*. There are many different variations traded in the market.

I would like to continue with our example of such an investment. The product starts on June 27 2008 and ends on July 6 2009. Let $K = 97.50$ denote the strike, $B = 103.40$ the knock-out barrier and $L = 89.50$ the additional lower knock-in barrier. There are 26 fixing dates during this one-year contract in a fortnightly sequence, starting from July 11 2008. Let S_i denote the fixing of the AUD-JPY exchange rate on date i. The client receives the amount

$$\text{REC} = \begin{cases} 2 \times (S_i - K) & \text{if } S_i \geq K \\ 4 \times S_i & \text{if } S_i \leq L \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

and pays the amount

$$\text{PAY} = \begin{cases} 4 \times K & \text{if } S_i \leq L \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

in Million of JPY. This is illustrated in Figure 2.14.

The fortnightly payments terminate automatically,

1. if a currency fixing is at or above B,
2. or if the accumulated profit reaches the target of 68 M JPY.

The accumulated profit is the sum of the fortnightly profits

$$\text{PROFIT} = 2 \times \max(S_i - K, 0) \tag{24}$$

in million of JPY. This means twice the difference of the fixing S_i and the strike K, provided that this difference is positive.

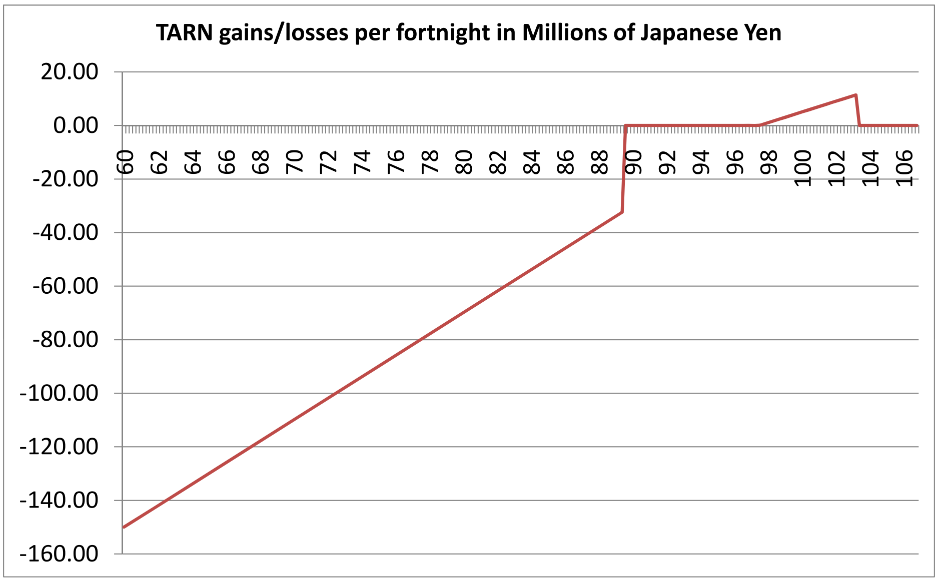

Figure 2.14: *Illustration of profits and losses in a long kiko TARN (Target Redemption Range Accrual Note). A range of possible AUD-JPY fixings is plotted on the x-axis. The y-axis denotes the profit and loss per fixing in million of JPY.*

Scenario Analysis

The best case each fortnight happens, when the spot is just before the barrier B, which would be

$$\begin{aligned} \text{MAXPROFIT} &= 2 \times \max(B - K, 0) \\ &= 2 \times \max(103.40 - 97.50, 0) \\ &= 2 \times \max(5.90, 0) \\ &= 2 \times 5.90 \\ &= 11.8 \text{ M JPY}. \end{aligned}$$

If there was no target, then the maximum profit for all the 26 fortnights would be 11.8 $\times$ 26=306.8 M JPY. However, because of the target the total profit is limited to 68 M JPY. The figures shows clearly the *limited* upside potential and the *unlimited* downside. For example, should the AUD-JPY spot go down to 80.00, then the the investor would make a loss of 70 M JPY or equivalently 875,000 AUD each fortnight. In this case the total loss would be AUD 875,000 $\times$ 26 $=$ AUD 22,750,000.

Hedging the Moving Strike Turbo Spot Unlimited

In a backwardation scenario of the forward curve, the issuer does not need to worry, as earning sales margin comes for free, because the hedge of the down-and-out call using the forward is cheaper than the down-and-out call even in the case of $r = 0$. The issuer just pockets the cost of carry. However, in a contango situation the use of r becomes crucial as the issuer would otherwise pay the cost of carry. It has been interesting to observe how much time it took for various banks in 2003/2004, when the EUR-USD forward curve switched from backwardation to contango, to discover the reasons for their losses in dealing with the turbo notes.

2.7 Hybrid FX Products

A real hybrid FX product is a transaction whose terms and conditions depend on more than one asset class where the components of the asset classes can not be separated. We have seen that forwards, deposits and swaps can be enhanced by starting with a worst case and then buying FX options or series of these to participate in certain FX market movements. These structures have an FX component that is separable from the basic product. This is like lego. You take the building blocks, possibly from different asset classes, and build a product of these lego blocks. For hybrids it is different. Examples for real hybrid products where FX is one of the asset classes include but are not limited to

Long term FX options. The interest rate risk of long term FX options is so prominent that we can no longer work with the Black-Scholes or any one factor model assuming deterministic interest rates. We need to rather model the future interest rates in the two currencies as a stochastic process. Modeling both rates along with the exchange rates requires at least a three-factor model.

Options with deferred delivery. Usually the delivery date of options is two business days after the maturity date. However, it can happen that we need to settle the cash flows of an exercised option at a much later date. For example, consider a client buying a 6-month double-no-touch, whose payoff is supposed to enhance the interest rates of a 5 year swap with semi-annual cash flows. In such a case the delivery date may be four and a half years after the maturity of the double-no-touch. This cash on hold is subject to interest rates in the future, whence modeling the value of such a deferred delivery double-no-touch no longer just depends on the exchange rate. Here we can't separate the interest rate risk of the FX-derived payoff, because we don't know at inception whether there is going to be any cash flow. The asset classes IR (interest rates) and FX (foreign exchange) are not separable.

Interest rate products with a knock-in or knock-out barrier in FX.

Equity products with a knock-in or knock-out barrier in FX.

Credit products with a knock-in or knock-out barrier in FX.

Derivatives in a non-FX market quantoed into another currency.

The most prominent types of FX hybrids are FX vanilla options with a long tenor and so-called *Power Reverse Dual Currency Bonds (PRDC)*. There are also a number of quanto products and other hybrids. We will now cover these in the next sections in some more detail.

2.7.1 Long-Term FX Options

Long-term options are hidden hybrids. In fact, the way their prices are quoted in the market is just like short-dated ones: implied volatility assuming a Black-Scholes model. This insinuates that FX spot volatility is the driving source of risk. Well guess what: it isn't.

2.7.2 Power Reverse Dual Currency Bonds

A PRDC is a generic name for an entire class of transactions (bonds and swaps), whose coupons depend on an exchange rate or are quantoed into other currencies, and/or whose notionals may be converted into other currencies. A detailed overview can be found, e.g. in Baum [11]. The reason PRDCs came up was the low interest environment in Japan along with the pension funds in Japan that had promised high yields to their investors. Now how to boost the coupons? We have learned that generating returns is normally done via carry trades or selling options or selling option-like features (callability). Essentially, all of these approaches tend to increase the risk of the investor. The risk has been amplified further, because practically all pension funds ended up having the same positions.

Trade Features

We start with a very common example of a long dated (typically 20Y to 30Y) dual currency trade in USD and JPY, in the format of a swap:

1. Principal exchange of USD 1 M against JPY 115 M at inception
2. Financing (funding) coupons depending on LIBOR, e.g. 6M JPY-LIBOR
3. Power (structured) coupon depending on FX rate, e.g. USD/JPY
4. Reverse principal exchange of JPY 115 M against USD 1 M at maturity

The *power coupon* depends on the FX rate. As a simple example we consider a call spread

$$\text{power coupon} = N \cdot \min\left(\max\left(F\frac{S_T}{S_0} + M, \text{Floor}\right), \text{Cap}\right)\tau, \tag{54}$$

where N denotes the notional, F the fixed rate, M the margin, T the coupon fixing date, S_0 the initial spot rate, S_T the spot rate at the coupon fixing date, and $\tau = T - t$ the tenor. For example, with an initial spot of $S_0 = 120$ in USD-JPY, the payoff

$$\text{power coupon} = \min\left(\max\left(24\%\frac{S_T}{120.00} - 18\%, 0\%\right), 6\%\right) \quad (55)$$

represents a call spread with coupon of 0% for USD-JPY spot below 90.00, 6% for USD-JPY spot above 120.00 and linear interpolation in between, see Figure 2.35.

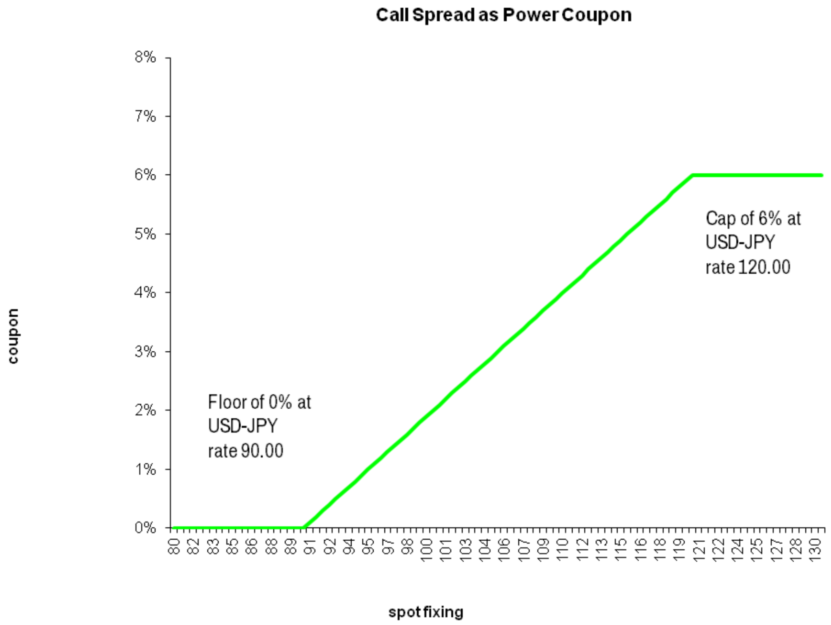

Figure 2.35: *PRDC power coupon via USD-JPY call spread.*

The main reasons why a PRDC with a power coupon is considered attractive by the buy-side are that the first coupon(s) is (are) typically guaranteed, and that the buy-side receives high coupons if USD-JPY does not decrease, i.e., if JPY does not strengthen. In the first decade of 2000 USD interest rates were higher than JPY interest rates, whence the USD-JPY forward curve is in backwardation. The buy-side taking a view in USD-JPY *not* going backwards is essentially a carry trade, 90% of all trade ideas. Furthermore, the buy-side investor receives USD coupons, for which she would receive more JPY as the forward curve indicates. The principle of the cash flows of a Power Reverse Dual are illustrated in Figure 2.36.

Soon after the principal idea of a power coupon had spread the greed-driven path to even higher coupons took its natural course. One way is to make the Power Reverse Dual Callable by the issuing bank, typically Bermudan style with a right to call on the coupon dates. However, since it is very difficult to predict for the buy-side when the bank is likely to call the bond, a more transparent early termination based on an FX spot hitting a pre-specified barrier as introduced. This feature is referred to as *auto-callable*. Needless to mention that multiple coupons, auto-callability, and Bermudan callability can and have been combined. To pick up an even higher carry effect, one can also structure a PRCD with USD funding and power

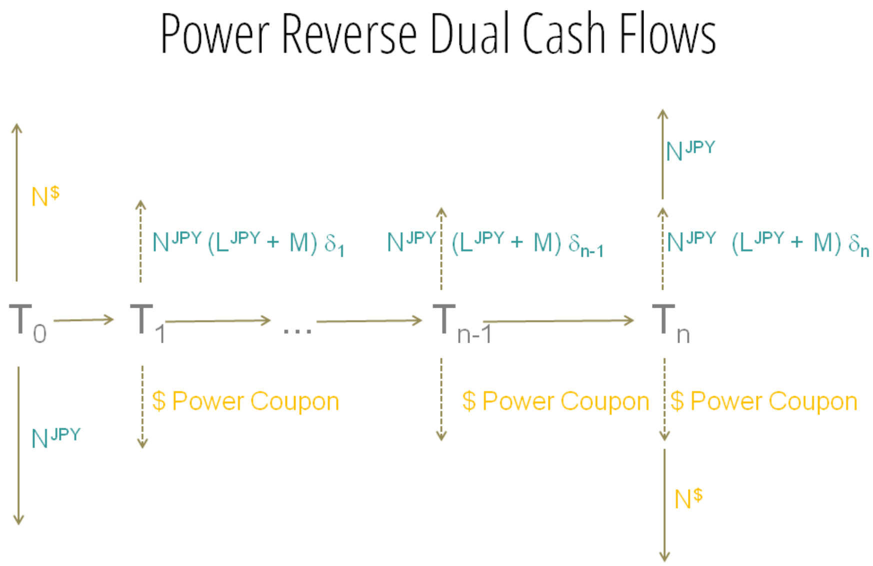

Figure 2.36: *PRDC power coupon via USD-JPY call spread.*

coupons depending on AUD/JPY. AUD/JPY is known to be *the* carry trade currency Japanese housewives have applied, commonly and generally referred to as Mrs. Watanabe.

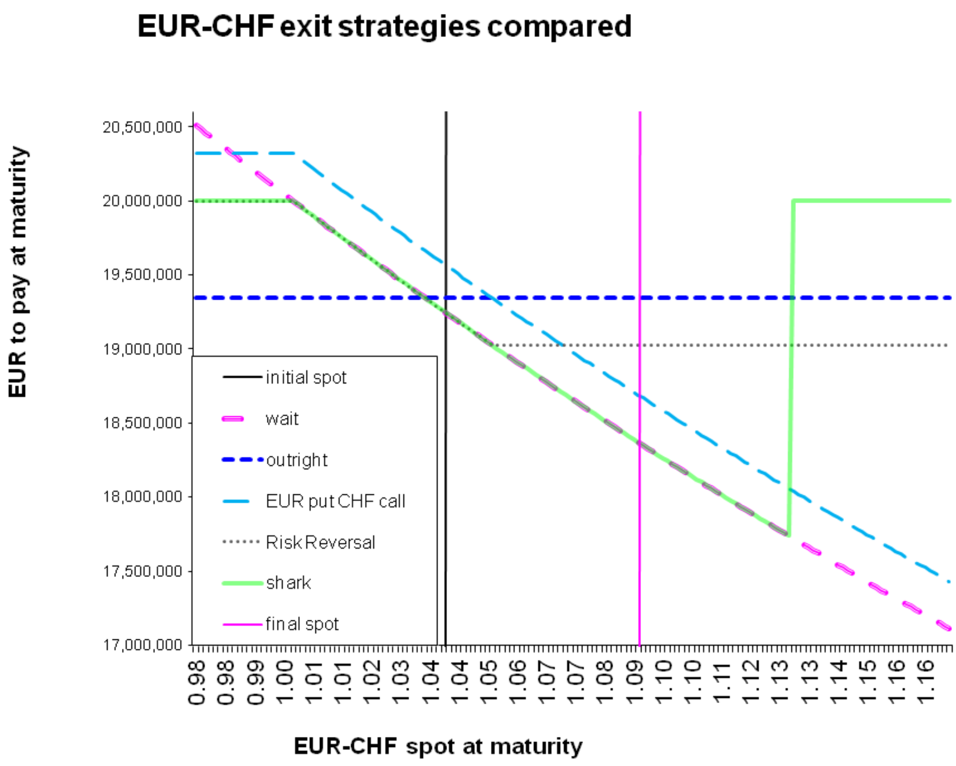

Figure 2.40: *Comparison of exit strategies of a sick floan in EUR-CHF. The treasurer needs to buy CHF 20 M for EUR in 6 months to pay back his floan. His goal is to minimize the EUR amount required to buy CHF. Initial spot 1.0400 and terminal spot 1.0900 shown in vertical lines.*

EUR-CHF trades at 1.0800 in 6 months, then the treasurer needs only 18.5 M EUR to buy back the 20 M CHF, so she would save a noticeable amount of EUR 842,360 compared to the outright in strategy 1. Subtracting the initial cost of the option the treasurer would still be able to save more than EUR 500,000, compared to strategy 1. However, the initial cost of the strategy puts many treasurers off, or the funds for the premium are just not available or it is impossible to get a sign-up majority in the responsible committees. From a markets' point of view, we notice that the risk reversal in EUR-CHF is at -2.80% favoring the CHF calls, which is extremely high and makes all CHF calls much more expensive. This is because the majority of the market participants are afraid of a further drop of EUR-CHF in the next 6 months. Therefore, we consider two more strategies.

Strategy 3 - Risk Reversal

The treasurer trades a zero-cost cylinder/collar/risk reversal, which means she buys the CHF call EUR put at strike 1.0000 and sells the EUR call CHF put strike 1.0510. The effect is that now she doesn't need to pay any premium, is protected at a level of 1.0000 and can

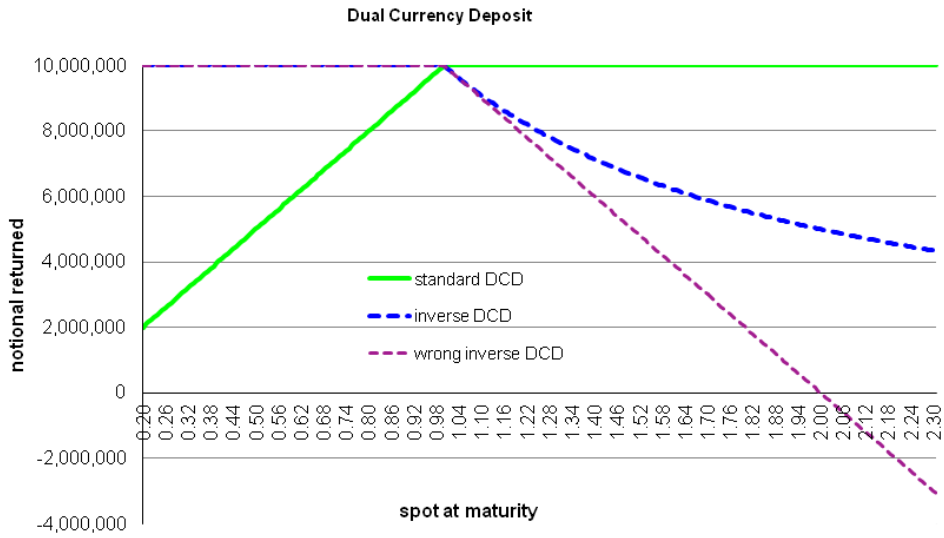

Figure 2.41: *USD Notional returned for a DCD. In the standard case the investor sells a USD Call EUR Put. In the inverse case selling USD Put EUR Call would potentially lead to a negative value. One way to correct this would be the self-quanto USD Put EUR call.*

Summary

The inverse DCD requires a non-vanilla self-quanto option to structure it in a way that is analogous to the standard DCD. This is not a liquid flow product, and this is why many banks don't offer the inverse DCD. An alternative way to structure the inverse DCD using only vanilla options can be done as an exercise.

2.8.5 Exercises

Inverse DCD

Structure the inverse DCD for a EUR investor using only vanilla options, i.e. assume the investor deposits N EUR, receives a coupon above market, and participates in a stable or rising EUR-USD exchange rate and loses parts of his EUR notional if EUR-USD ends up below a level K at maturity. Ensure that the investment has a capital guarantee of 0%, i.e. the notional to be returned does not go negative.

Risk Reversal Case Study in EUR-USD

Consider a treasurer who needs to buy USD 10 M in 6 months. The underlying market is EUR-USD on Jan 12 2017: Spot ref 1.0500, USD money market rate 1.112%, EUR money

This issue and possible methods for measurement of the effectiveness were discussed in theory and in examples. In the case study, the question as to whether the prospective and retrospective hedge effectiveness exists is tested in the framework of a cash flow hedge for a foreign exchange forecast transaction that is hedged with a Shark Forward Plus.

As an overall result, the hedge effectiveness for the example could be confirmed for both, the prospective and the retrospective hedge effectiveness. One critical issue is the choice of an appropriate method to test for the effectiveness. The VRM as well as the *Regression Analysis* deliver stable results for both tests. The *Dollar-Offset Method* is likely to fail the test, especially when using the period-by-period method instead of the cumulative method.

Summing up, hedge accounting is possible for the Shark Forward Plus. However, the choice of the method to test for effectiveness is crucial for the result. For the entity it is also a question of the effort that is required to set up the more complicated methods like the VRM or *Regression Analysis*, compared to the simpler *Dollar-Offset Method*. Within this context the effort is often rewarded with a better result for the desired hedge effectiveness.

3.1.8 Relevant Original Sources for Accounting Standards

1. International Accounting Standards Board,
   International Accounting Standard 1, Financial Instruments:
   Disclosure and Presentation, (1997), as at July 1997

2. International Accounting Standards Board,
   International Accounting Standard 32, Financial Instruments:
   Disclosure and Presentation, (2004), as at 31 March 2004

3. International Accounting Standards Board,
   International Accounting Standard 39, Financial Instruments:
   Recognition and Measurement, (2004), as at 31 March 2004

4. International Accounting Standards Board,
   Guidance of Implementing International Accounting Standard 39,
   Financial Instruments: Recognition and Measurement, (2004), as of 31 March 2004

3.2 Hedge Accounting under IFRS 9

In this section we will provide an overview about hedge accounting under IFRS 9 and then test the effectiveness of a Forward Plus in a case study. This is based on Kazmaier [85]. New hedge accounting principles have been published by *International Financial Reporting Standards* in IFRS 9 on *Financial Instruments* [1]. The *International Accounting Standards*

Board (IASB) has determined that IFRS 9 "shall be applied by all entities to all types of financial instruments."

3.2.1 Hedge Effectiveness

IFRS 9 defines hedge (in)effectiveness as follows:

> Hedge effectiveness is the extent to which changes in the fair value or the cash flows of the hedging instrument offset changes in the fair value or the cash flows of the hedged item (for example, when the hedged item is a risk component, the relevant change in fair value or cash flows of an item is the one that is attributable to the hedged risk). Hedge ineffectiveness is the extent to which the changes in the fair value or the cash flows of the hedging instrument are greater or less than those on the hedged item.[3]

The major difference and advantage of IFRS 9 in comparison with IAS 39 is that the effective component can be accounted for in OCI, whereas the ineffective part is booked directly into P&L. IAS 39 did not allow a split into OCI and P&L in the case of ineffectiveness. In addition, as a result of the regression analysis, the 80-125% range, was not supposed to be exceeded. In cases where values were below 80% or above 125%, the complete amount would be qualified as ineffective. The next section explores hedge effectiveness in more detail.

3.2.2 Documentation and Qualifying Criteria

The new hedge accounting requirements, including the hedge effectiveness assessment, must be documented. A summary of all the documentation and qualification requirements for IFRS 9 for documentation is outlined below in direct quotations as this is the central requirement of the standard. As auditors have the last word on the hedge effectiveness assessment, this shall help identify the minimum analysis requirements for hedge effectiveness assessment and which requirements need to be verified by the auditor. A hedging relationship can qualify for hedge accounting only if all of the following criteria of IFRS 9.6.4.1 are met[4]:

(a) the hedging relationship consists only of eligible hedging instruments and eligible hedged items.

(b) at the inception of the hedging relationship there is formal designation and documentation of the hedging relationship and the entity's risk management objective and strategy for undertaking the hedge. That documentation shall include identification of the hedging instrument, the hedged item, the nature of the risk being hedged and how the entity will assess whether the hedging relationship meets the hedge effectiveness requirements (including its analysis of the sources of hedge ineffectiveness and how it determines the hedge ratio).

[3] cf. IASB, 2015, IFRS 9.B6.4.1
[4] cf. IASB, 2015, IFRS 9.6.4.1

(c) the hedging relationship meets all of the following hedge effectiveness requirements:

  (i) there is an economic relationship between the hedged item and the hedging instrument (see paragraphs B6.4.4-B6.4.6);

  (ii) the effect of credit risk does not dominate the value changes that result from that economic relationship (see paragraphs B6.4.7-B6.4.8); and

  (iii) the hedge ratio of the hedging relationship is the same as that resulting from the quantity of the hedged item that the entity actually hedges and the quantity of the hedging instrument that the entity actually uses to hedge that quantity of hedged item. However, that designation shall not reflect an imbalance between the weightings of the hedged item and the hedging instrument that would create hedge ineffectiveness (irrespective of whether recognized or not) that could result in an accounting outcome that would be inconsistent with the purpose of hedge accounting (see paragraphs B6.4.9-B6.4.11)[5].

Furthermore, IFRS 9.B6.5.21 states: "When re-balancing a hedging relationship, an entity shall update its analysis of the sources of hedge ineffectiveness that are expected to affect the hedging relationship during its (remaining) term (see paragraph B6.4.2). The documentation of the hedging relationship shall be updated accordingly."

3.2.3 Case Study: Shark Forward

We consider an example where party A is exposed to an appreciating USD relative to EUR. Although the entity wants to hedge against an increasing EUR/USD rate, it wishes to be flexible enough to participate in favorable exchange rates. The entity is willing to accept an exchange rate below the outright forward rate as the worst case scenario. To hedge this FX exposure, A may enter into a shark forward plus (see Section 2.1.6) with the terms shown in Table 3.4.

Generally, the shark forward plus (also called forward plus, forward extra, enhanced forward or forward with profit potential) is suitable for entities that want to fix a forward price while they can still benefit from a spot movement in which they take a view. This type of instrument provides some potential for limiting possible losses with the level near the forward rate. On the maturity day, party A sells USD 100 M to bank B. The exchange rate applied depends on the path of the spot during the time till maturity. The following scenarios are possible at maturity:

1. If the spot at maturity is above 1.0800 EUR-USD or if the trigger 1.0500 EUR-USD has been breached during the lifetime of the contract , party A sells the notional at the worst case of 1.0800 EUR-USD.

2. If the spot at maturity is between the trigger rate of 1.0500 EUR-USD and 1.0800 EUR-USD, and the trigger of 1.0500 EUR-USD has not been breached during the lifetime of

[5] cf. IASB, 2015, IFRS 9.6.4.1

4.2 Bid–Ask Spreads

Bid-ask or bid-offer spreads are the price quotes for sellers and buyers of financial assets and derivatives respectively. The spread indicates the sales margin a trading desk earns: the wider the spread, the higher the risk, and/or margin of the product. Wide spreads can also indicate a lack of liquidity or increased risk. Different markets have different spreads. The inter bank market has the tightest spreads, because the banking community normally knows very well, how much financial products should cost. Spreads turn to be slightly wider for corporate and institutional clients and very wide for retail clients. There is no fixed rule on how bid-ask spreads should be set up.

In an e-commerce FX options trading environment it is important to set some rules how to compute spreads automatically. One starts with simple and liquid products like the vanilla and one-touch contracts and sets up some rules to derive spreads for other exotics from these basic spreads. For example, it can be done as follows.

4.2.1 Vanilla Spreads

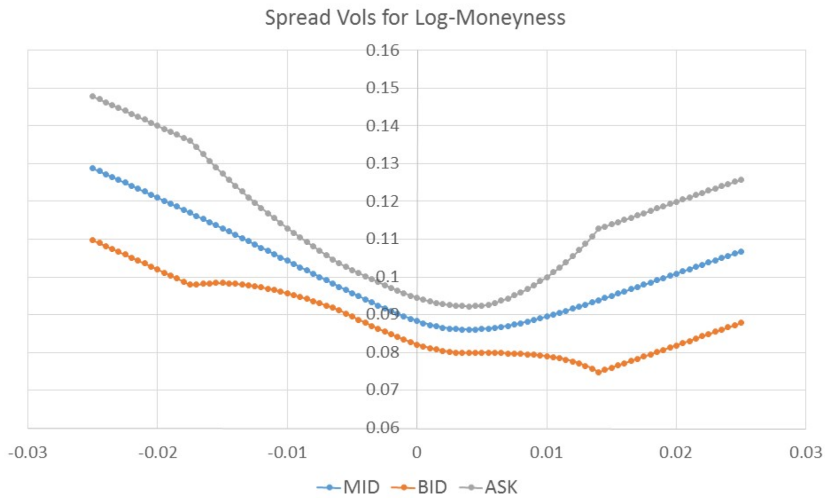

Figure 4.8: *Vanilla bid-ask spreads on log-moneyness space in implied volatilities*

The spreads for vanilla options are usually specified as an ATM *volatility spreads* (spread in terms of implied volatility); they vary over tenor and currency pair. The vanilla *price spreads* are calculated for the maturity pillars $\{$1w, 2w, 1m, 2m, 3m, 6m, 9m, 1y, 18m, 2y$\}$ using the ATM volatility and ATM spread, the spot and the respective rates. A corresponding spread matrix expresses half the vanilla spread in terms of volatility for the maturity pillar. In order to

5. A median trade bid and a median trade offer are calculated separately. From these results, a mid-rate is defined.
6. A standard spread will be applied to this mid-rate to compute a new bid and a new offer rate. Those rates will be published as the benchmark rate if they fulfill a specific "tolerance check threshold" that might trigger a request for review by a staff member.

The description is outlined in Reuters' FX benchmark manual [112], and on their website financial.thomsonreuters.com/benchmarks.

Bloomberg Fixing Cleaning and selecting inputs using the BGN methodology is the first step of the process. The main product used in the industry is the spot reference rate. BFIX takes a snapshot of those BGN ticks with an interval of 30 minutes. This procedure is repeated the whole day and the rates are published 15 seconds after the fixing time.

The methodology used to compute the BFIX is a Time-Weighted Average Price (TWAP) of the BGN mid rates. This average is taking into account data *before and after* the fixing time. Every BFIX rate is divided in multiple slices of one second. The inputs for every slice are the geometric mid rates of the BGN bid and ask ticks. The TWAP is then computed based on those data points, using a triangular formula, as exhibited in Figure 4.9. The peak of the triangle is the fixing time and carries a weight of 10%. Around this peak, weight of every slice is decreasing linearly. As a result, the pre-fix time weight approximately 88.52% and the post-fix time 1.48%.

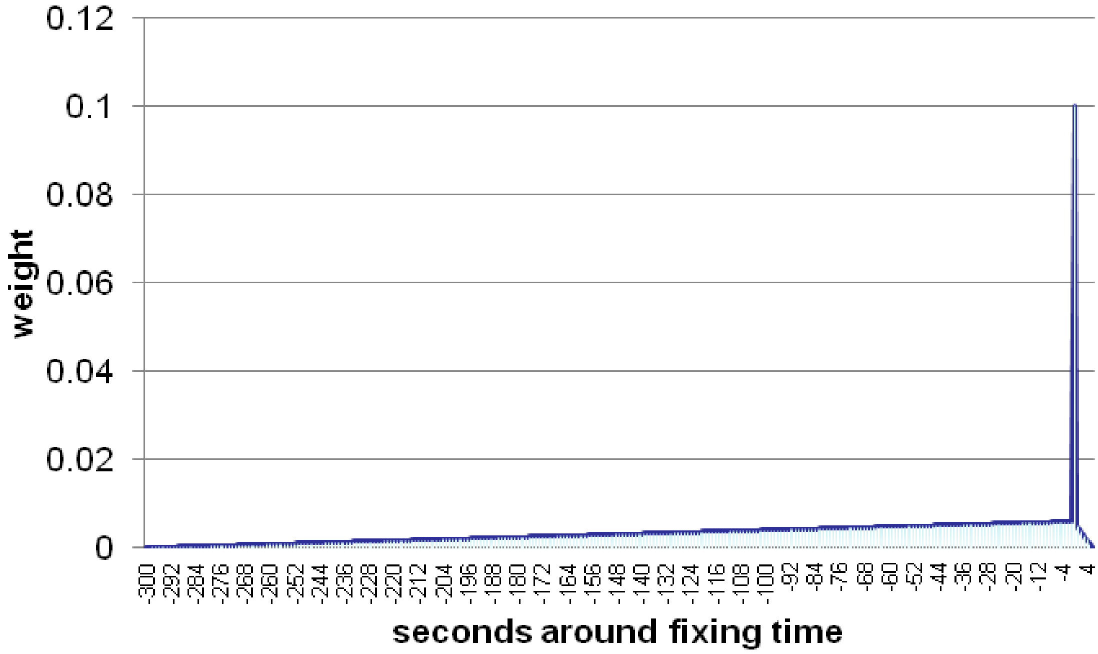

Figure 4.9: *BFIX TWAP weights assigned to the 306 snapshots*

This common rule varies according to the time window assigned to the TWAP. This time

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