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Stochastic Risk Premiums, Stochastic Skewness in Currency Options, and Stochastic Discount Factors in International Economies*

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Abstract

We develop models of stochastic discount factors in international economies that produce stochastic risk premiums and stochastic skewness in currency options. We estimate the models using time-series returns and option prices on three currency pairs that form a triangular relation. Estimation shows that the average risk premium in Japan is larger than that in the US or the UK, the global risk premium is more persistent and volatile than the country-specific risk premiums, and investors respond differently to different shocks. We also identify high-frequency jumps in each economy, but find that only downside jumps are priced. Finally, our analysis shows that the risk premiums are economically compatible with movements in stock and bond market fundamentals.

JEL Classification: G12; G13; F31; C52.

Keywords: Stochastic discount factors; International economy; Stochastic risk premium; Stochastic skewness; Currency options; Foreign exchange rate dynamics; Time-changed Lévy processes; Unscented Kalman filter.

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1. Introduction

At the core of financial economics is to infer the dynamic structure of stochastic discount factors, which determines how investors price various sources of risks differently. In particular, since the ratio of the stochastic discount factors in two economies governs the exchange rate between them, the exchange rate market offers a direct information source for assessing the relative risk-taking behavior of investors in international economies. Exploiting this link, Brandt and Santa-Clara (2002) gauge the degree of market incompleteness and estimate the risk premium dynamics using the time series of a currency pair and its short-term at-the-money option implied volatility. Brandt, Cochrane, and Santa-Clara (2006) compare the stock portfolio return variance to the variance of the exchange rate to analyze the degree of international risk sharing between two economies. They find that compared to the large return variance on stock portfolios, the currency return variance is rather small, which could be an indication of a high degree of international risk sharing or an anomaly by itself.

In this paper, we propose to identify the multi-dimensional structure of stochastic discount factors in international economies using the time-series of currency returns and option prices. Specifically, using three currency pairs that form a triangular relation, i.e., dollar-yen, dollar-pound, and yen-pound, we study the dynamic behaviors of the stochastic discount factors and stochastic risk premiums in the three economies: the US, Japan, and the UK.

Compared to the extant literature, we make contributions in several dimensions. First, instead of trying to identify the stochastic discount factors in two economies using one currency pair, we identify the stochastic discount factors in three economies using three currency pairs that form a triangular relation. Exploiting the currency triangle facilitates identification of the stochastic discount factors and enables us to draw a sharper distinction between the risk premium dynamics on global versus country-specific risks. Second, we make full use of currency options data across all available strikes and maturities underlying all three currency pairs through an option pricing model that is internally consistent with our stochastic discount factor specification across the three economies. Third, our stochastic discount factor specification incorporates a realistic jump structure that not only allows differential pricing for upside and downside jumps, but also accommodates a wide variety of jump behaviors, ranging from the compound Poisson jumps used in traditional studies (e.g., Merton (1976)) to infinite-activity jumps that can arrive an infinite number of times within any finite time interval. Fourth, our model accommodates stochastic risk premiums from both the global and the country-specific risk components in each economy, and generates stochastic skewness in the currency return distribution, both

of which are salient features of the currency and currency options market.[1]

Given our stochastic discount factor specification, we derive currency return dynamics and price options on the three currency pairs analytically. By casting the theoretical model into a state-space form, we estimate the model parameters and extract the global and country-specific risk premium rates from the time series of currency returns and option prices. Through model estimation, we empirically study how the risk premiums of an economy react differently to shocks on different types of risks.

Our estimation reveals several results about the structure of risk premiums in the three economies. First, during our sample period, the average risk premium in Japan is significantly higher than the average risk premium in the US or the UK. Second, risk premiums on the global risk component and the country-specific risk components show distinct dynamics. The risk premium rate on the global risk factor is both more persistent and more volatile than the risk premium rate on the country-specific risk factors. Third, investors respond to global and country-specific shocks differently. Investors increase their risk premium when the country-specific risk receives a negative shock. In contrast, the risk premium declines when the global risk component receives a negative shock.

Estimation also shows that, to capture the currency return dynamics and to generate realistic currency option pricing behaviors, it is crucial to incorporate a high-frequency jump component in the stochastic discount factor of each economy. The origin of these jumps can be tied to the way in which markets respond to information (Andersen, Bollerslev, Diebold, and Vega (2003), Beber and Brandt (2005), Piazzesi (2005), and Pasquariello and Vega (2006)). Furthermore, although an economy can receive both negative and positive shocks, investors only price downside jumps as a potential source of risk. This finding explains why financial markets react more strongly to negative economic news than to positive news (Andersen, Bollerslev, Diebold, and Vega (2005)). More broadly, our empirical analysis shows that including high frequency jumps and allowing stochastic risk premiums in our specification are both instrumental to enhancing model performance.

The estimated risk premium dynamics on the global and country-specific risk components suggest that the stochastic discount factors share a large global risk component, and that shocks on the global risk premium rate have more long-lasting impacts than shocks on country-specific risk premiums. Furthermore, our estimated

[1] A long list of studies have documented strongly time-varying currency risk premiums, e.g., Fama (1984), Bekaert and Hodrick (1992), McCurdy and Morgan (1992), Dumas and Solnik (1995), Saá-Requejo (1995), Engel (1996), Bansal (1997), Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002), Brandt, Cochrane, and Santa-Clara (2006), and Brennan and Xia (2006). Most recently, Carr and Wu (2006) find that the risk-neutral currency return distribution inferred from currency options shows strongly time-varying skewness.

stochastic discount factors generate high values for the international risk sharing index defined in Brandt, Cochrane, and Santa-Clara (2006), suggesting that the currency options market embeds a high degree of international integration among the three economies.

Finally, we study how the extracted risk premiums co-move with economic fundamentals in the bond and stock market in the three economies. The analysis shows that a reduction in the short-term interest rate and a steepening of the yield curve have the effect of raising country-specific risk premiums (Campbell and Shiller (1991) and Fama and Bliss (1987)). We also find that country-specific risk premiums increase with interest-rate cap and stock index option volatilities in the corresponding economy. Overall, the risk premiums that we extract from currency options markets are economically compatible with the movements in the bond and stock market fundamentals in the three economies.

Traditional literature often studies the behavior of risk premiums through various types of expectation hypothesis regressions. Under the null hypothesis of zero or constant risk premium, the slope coefficients of these regressions should be unity. Hence, the point estimates on the regression slopes reveal whether the risk premium is constant or time-varying. Recently, researchers have recognized the rich information content of option markets and started to infer the risk premium behavior from a joint analysis of options and the underlying assets. The focus of this strand of literature is on stock index and stock index options in a single economy, mainly the US.[2] In this setting, the estimated stochastic discount factors are typically one-dimensional projections on the single stock index. The pricing of risks that are orthogonal to the stock index is largely missed by this projection. Furthermore, it is difficult to use a one-dimensional projection to study the multi-dimensional nature of the stochastic discount factors in international economies. In contrast, the currency and its options market provide a more direct information source for assessing the multi-dimensional dynamic behaviors of stochastic discount factors in international economies. Moreover, when the market is not completed by domestic securities such as bonds and stocks, currency and currency options can help complete the market.

The paper is organized as follows. Section 2 articulates the idea of inferring stochastic discount factors in international economies from options on currencies that form a triangular relation. Section 3 proposes models of stochastic discount factors that include both a global risk factor and country-specific risk factors and allow the risk premiums on the two types of risks to follow separate dynamics. We analyze what minimal structures

[2]Examples include Jackwerth and Rubinstein (1996), Bakshi, Cao, and Chen (2000), Bates (2000), Pan (2002), Engle and Rosenberg (2002), Bakshi and Kapadia (2003), Jones (2003, 2006), Eraker (2004), Bliss and Panigirtzoglou (2004), and Broadie, Chernov, and Johannes (2006). Recently, Driessen and Maenhout (2004) and Mo and Wu (2006) investigate the nature of jump and volatility risks using stock index options from three countries.

are necessary to capture the stylized evidence in currency returns and currency options, and derive tractable solutions for option pricing and for the characteristic function of the currency returns. Section 4 describes the currency and currency options data set for the triangle of dollar-yen, dollar-pound, and pound-yen exchange rates, as well as the estimation procedure. Section 5 discusses the estimation results and Section 6 concludes.

2. Inferring stochastic discount factors from options on a currency triangle

We describe a set of N economies by fixing a filtered probability space $\{\Omega, \mathcal{F}, \mathcal{P}, (\mathcal{F}_t)_{0\leq t\leq \mathcal{T}}\}$, with some fixed horizon $\mathcal{T}$. We assume no arbitrage in each economy. Therefore, for each economy, we can identify at least one strictly positive process, ${\mathcal{M}_t}^h$ ($h = 1,\ldots,N$), which we call the state-price deflator, such that the deflated gains process associated with any admissible trading strategy is a martingale (Cochrane (2004), Duffie (1992), and Harrison and Kreps (1979)). We further assume that ${\mathcal{M}_t}^h$ itself is a semimartingale. The ratio of ${\mathcal{M}_t}^h$ at two time horizons is referred to as the stochastic discount factor, or the pricing kernel.

We use X^h to summarize the uncertainty in economy h and represent the state-price deflator via the following multiplicative decomposition (with ${\mathcal{M}_0}^h = 1$):

$$ {\mathcal{M}_t}^h \quad = \quad \exp\left(-\int_0^t r_s^h\, ds\right) \mathcal{E}\left(-\int_0^t \gamma_s^h\, dX_s^h\right), \quad h = 1,2,\ldots,N, \tag{1} $$

where r_t^h denotes the instantaneous interest rate in economy h, γ_t^h denotes the market price of risk in economy h, and $\mathcal{E}(\cdot)$ denotes the stochastic exponential martingale operator (Jacod and Shiryaev (1987) and Rogers and Williams (1987)), which defines the Radon-Nikodým derivative that transforms the statistical measure $\mathcal{P}$ to the economy-h risk-neutral measure ${\mathcal{Q}}^h$:

$$ \left.\frac{d{\mathcal{Q}}^h}{d\mathcal{P}}\right|_t \equiv \mathcal{E}\left(-\int_0^t \gamma_s^h\, dX_s^h\right). \tag{2} $$

In equation (1), both r_t and γ_t can be stochastic. The shocks X^h can be multi-dimensional, in which case $\gamma_t^h dX_t^h$ denotes an inner product. In a Lucas (1982)-type exchange economy, the stochastic discount factor can be interpreted as the ratio of the marginal utilities of aggregate wealth over two time horizons, and X^h can be interpreted as return shocks to aggregate wealth in the economy.

No arbitrage dictates that the ratio of the stochastic discount factors between two economies determines the exchange rate dynamics between them (Dumas (1992), Saá-Requejo (1995), Bakshi and Chen (1997),

Basak and Gallmeyer (1999), Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002), Brandt, Cochrane, and Santa-Clara (2006), and Pavlova and Rigobon (2007)). Let S_t^{fh} denote the time-t currency-h price of currency f, with h being the home economy, we have,

$$\frac{S_{t+\tau}^{fh}}{S_t^{fh}} = \frac{\mathcal{M}_{t+\tau}^{\ f}/\mathcal{M}_t^{\ f}}{\mathcal{M}_{t+\tau}^{\ h}/\mathcal{M}_t^{\ h}}, \qquad h, f = 1, 2, \ldots, N. \tag{3}$$

Equation (3) defines the formal link between the stochastic discount factors in any two economies and the exchange rate movements between them. In complete markets, the stochastic discount factor for each economy is unique. Hence, the ratio of two stochastic discount factors uniquely determines the exchange rate dynamics between the two economies. When markets are incomplete with primary domestic securities such as bonds and stocks, there may exist multiple stochastic discount factors that are consistent with the prices of these securities. In this case, exchange rates and currency options help complete the markets by requiring equation (3) to hold between any viable stochastic discount factors in the two economies (Rogers (1997) and Brandt and Santa-Clara (2002)).

The extant literature often uses bond prices or stock indices in a single economy to study the stand-alone behavior of the stochastic discount factor in that economy. In this paper, we advocate the use of currency and its options in studying the joint dynamics of stochastic discount factors in international economies, a direction also explored in Brandt and Santa-Clara (2002) and Brandt, Cochrane, and Santa-Clara (2006). Based on a generic orthogonal decomposition of the stochastic discount factor, Constantinides (1992), Rogers (1997), Leippold and Wu (2002), and Brandt and Santa-Clara (2002) show that there are risk dimensions that do not affect bond and stock pricing in a single economy, but can influence the pricing of currency claims in international economies.

To illustrate this point, consider the following heuristic orthogonal decomposition of the stochastic discount factor in an economy h,

$$\mathcal{M}_t^{\ h} = \mathcal{N}_x^{\ h}[X_t]\,\mathcal{N}_y^{\ h}[Y_t]\,\mathcal{N}_u^{\ h}[U_t], \tag{4}$$

where X, Y, and U denote three sets of mutually independent Markovian state vectors that define the risk and pricing of the economy, with the martingale assumption: $\mathbb{E}^{\mathcal{P}}\left(\mathcal{N}_y^{\ h}[Y_t]\right) = \mathbb{E}^{\mathcal{P}}\left(\mathcal{N}_u^{\ h}[U_t]\right) = 1$, where $\mathbb{E}^{\mathcal{P}}(\cdot)$ denotes the expectation operator under measure $\mathcal{P}$.

In this motivational setting, the time-0 value of a zero-coupon bond with maturity t becomes,

$$B^h(0,t) = \mathbb{E}^{\mathcal{P}}\left(\mathcal{M}_t^{\,h}\right) = \mathbb{E}^{\mathcal{P}}\left(\mathcal{N}_x^{\,h}[X_t]\right), \tag{5}$$

which is only a function of the state vector X. The risk and pricing about the other two dimensions of the economy Y and U do not show up in bond pricing and hence cannot possibly be identified from the term structure of interest rates. Furthermore, the risk factors Y and U affect stock valuation when they are correlated with future cash flows to the stock. For example, if we assume that stock cash flow, $D[.]$, is only a function of Y, the time-0 stock value, H_0^h, will reveal the dynamics of X and Y, but not U:

$$H_0^h \;=\; \mathbb{E}^{\mathcal{P}}\left(\int_0^\infty \mathcal{M}_t^{\,h} D[Y_s]\, ds\right) = \mathbb{E}^{\mathcal{P}}\left(\int_0^\infty \mathcal{N}_x^{\,h}[X_s] ds\right) \mathbb{E}^{\mathcal{P}}\left(\int_0^\infty \mathcal{N}_y^{\,h}[Y_s]\, D[Y_s] ds\right). \tag{6}$$

Therefore, under this setting, we will not be able to fully identify the true stochastic discount factor using bond and stock prices alone. In contrast, since the exchange rate relates to the ratio of the two stochastic discount factors in the home and foreign economies,

$$\frac{S_t^{fh}}{S_0^{fh}} = \frac{\mathcal{N}_x^{\,f}[X_t]\,\mathcal{N}_y^{\,f}[Y_t]\,\mathcal{N}_u^{\,f}[U_t]}{\mathcal{N}_x^{\,h}[X_t]\,\mathcal{N}_y^{\,h}[Y_t]\,\mathcal{N}_u^{\,h}[U_t]}, \tag{7}$$

the risk factors X, Y, and U will all influence currency return and currency option dynamics as long as the two economies are not fully symmetric. Therefore, exploiting the currency dynamics information is crucial not only for understanding the multi-dimensional structure of risk and pricing in international economies, but also for revealing risk dimensions not spanned by bonds and stocks.

Based on similar arguments, Brandt and Santa-Clara (2002) propose to use currency returns and options to gauge the degree of market incompleteness. They call a security market incomplete if the risks in that economy cannot be fully spanned by domestic securities such as bonds and stocks. According to this definition, the economy defined by the stochastic discount factor in (4) is incomplete as domestic bonds only span risk X and domestic stocks only span risk Y, with the risk U left unspanned. Both $\mathcal{N}_x^{\,h}[X_t]\,\mathcal{N}_y^{\,h}[Y_t]$ and $\mathcal{N}_x^{\,h}[X_t]\,\mathcal{N}_y^{\,h}[Y_t]\,\mathcal{N}_u^{\,h}[U_t]$ with arbitrary values of U are admissible stochastic discount factors that are consistent with domestic bond and stock prices. However, only $\mathcal{N}_x^{\,h}[X_t]\,\mathcal{N}_y^{\,h}[Y_t]\,\mathcal{N}_u^{\,h}[U_t]$ with the appropriate U dynamics can match the exchange rate dynamics according to (7). Brandt and Santa-Clara use currency market information to identify the U risk and use the relative magnitude of the identified U risk to measure the degree of market incompleteness. Along the same direction, we propose to use time-series returns and option prices on

a triangle of currency pairs to identify the stochastic discount factors in the three underlying economies.

3. Modeling stochastic risk premiums and stochastic skewness

We propose a class of models for the stochastic discount factors that are flexible enough to generate stochastic risk premiums and stochastic skewness in currency returns. Our model parameterization provides the foundation for extracting the evolution of risk premiums from currency option prices and currency returns. Formally, we have,

$$\mathcal{M}_t{}^h = \exp\left(-r^h t\right) \exp\left(-W^g_{\Pi^h_t} - \frac{1}{2}\Pi^h_t\right) \exp\left(-\left(W^h_{\Lambda^h_t} + J^h_{\Lambda^h_t}\right) - \left(\frac{1}{2} + k_{J^h}\left[-1\right]\right)\Lambda^h_t\right), \tag{8}$$

which decomposes the stochastic discount factor into three orthogonal components. The first component captures the contribution from interest rates. Since a large portion of currency return movements is independent of interest rate movements (Backus, Foresi, and Telmer (2001) and Brandt and Santa-Clara (2002)) and stochastic interest rates have little impact on short-term currency option prices (Bates (1996)), we assume deterministic interest rates for simplicity and use r^h to denote the spot interest rate of the relevant time and maturity.

The second component incorporates a global diffusion risk factor $W^g_{\Pi^h_t}$, where W^g denotes a standard Brownian motion and $\Pi^h_t \equiv \int_0^t \gamma^h_s ds$ defines a stochastic time change that captures the stochastic risk premium on this global risk factor. The stochastic time-changed Brownian motion notation $W^g_{\Pi^h_t}$ is equivalent in probability to the classical representation $\int_0^t \sqrt{\gamma^h_s} dW^g_s$, with γ^h_s being the instantaneous variance rate (see page 173 of Revuz and Yor (1991)). Consequently, Π^h_t captures the integrated variance over time $[0,t]$. Based on this connection and equation (1), we label γ^h_t as the risk premium rate (per unit time) and use the superscript h on γ_t to indicate that different economies can price the same source of risk differently. The term $\frac{1}{2}\Pi^h_t$ is the convexity adjustment that makes $\exp\left(-W^g_{\Pi^h_t} - \frac{1}{2}\Pi^h_t\right)$ an exponential martingale. The mathematical treatment of time-changed Lévy processes can be found in Carr and Wu (2004).

The third component describes a country-specific jump-diffusion risk factor $\left(W^h_{\Lambda^h_t} + J^h_{\Lambda^h_t}\right)$, where W^h denotes another standard Brownian motion independent of the global risk component W^g, and J^h denotes a pure jump Lévy component. We apply a separate stochastic time change to this country-specific jump-diffusion risk factor $\Lambda^h_t \equiv \int_0^t \nu^h_s ds$ to capture the stochastic risk premium on country-specific risks, with ν^h_t being the risk premium rate on the country-specific risk factor. Applying the time change to the Brownian motion $W^h_{\Lambda^h_t}$

implies that the risk premium rate ν_t^h captures the instantaneous variance rate of the Brownian motion W^h. Likewise, applying the time change to the jump component $J^h_{\Lambda_t^h}$ indicates that ν_t^h is also proportional to the jump arrival rate. Again, the term $\left(\frac{1}{2}+k_{J^h}[-1]\right)\Lambda_t^h$ represents the convexity adjustment for $\left(W^h_{\Lambda_t^h}+J^h_{\Lambda_t^h}\right)$ so that the third component in (8) is also an exponential martingale. The term $k_{J^h}[s]$ denotes the cumulant exponent of the Lévy jump component J^h, defined as[3]

$$k_{J^h}[u] \equiv \frac{1}{t}\ln \mathbb{E}^{\mathcal{P}}\left(e^{uJ_t^h}\right), \quad u \in \mathcal{D} \subseteq \mathcal{C}. \tag{9}$$

Economically, incorporating the jump component is important in capturing large discontinuous movements in economic fundamentals and financial security prices as shown in Almeida, Goodhart, and Payne (1998), Andersen, Bollerslev, Diebold, and Vega (2003), Andersen, Bollerslev, Diebold, and Vega (2005), Beber and Brandt (2005), and Pasquariello and Vega (2006). Statistically, it also helps to generate currency return non-normality and realistic currency option behaviors at short horizons. Furthermore, through stochastic time changes, we also capture the intensity variation in the information flow and generate stochastic volatility and stochastic risk premium for each risk component. Statistically, stochastic volatility also helps in generating currency return non-normality at intermediate to long horizons. Our model incorporates both jumps and stochastic volatility to describe distinct aspects of the international economy.

In principle, we can also allow a jump component in the global risk factor, but experimental estimation shows that the jump in the global risk factor is not significant. Hence, we choose a pure diffusion specification for the global factor to maintain parsimony.

To appreciate how the key ideas fit together, one can appeal to the Lucas (1982) economy where X^h in equation (1) can generically be interpreted as return shocks to aggregate wealth. Accordingly, our model of stochastic discount factors in (8) can be viewed as decomposing return shocks to the aggregate wealth into a global component and a country-specific component, each with a separate and stochastic risk premium. Through model estimation, we study how investors respond to different types of risks in international economies.

Based on our formulation of the stochastic discount factors in (8), we can also investigate the degree of international risk sharing by estimating the relative proportion of variation in the stochastic discount factor that is driven by the global risk component versus the country-specific risk component. In this regard, we

[3] A cumulant exponent is normally defined on the positive real line, but it is convenient for option pricing to extend the definition to the subset of the complex plane ($u \in \mathcal{D} \subseteq \mathcal{C}$) where the exponent is well-defined.

can achieve similar objectives as in Brandt, Cochrane, and Santa-Clara (2006), but with different financial instruments. Brandt, Cochrane, and Santa-Clara analyze the degree of international risk sharing by comparing the currency return variance to the sum of the stock portfolio return variance in the two economies. In this paper, we identify the global and country-specific risk components and their risk premium rates using time-series returns and option prices on a triangle of currency pairs underlying three economies.

3.1. Specification of jumps and risk premium rate dynamics

A parsimonious way to capture asymmetry across economies is to use a vector of scaling coefficients $\xi = \left\{\xi^h\right\}_{h=1}^N$ to model the average difference in risk premium in different economies. Asymmetries arise when the economies have different risk magnitudes and/or when investors have different risk preferences. For identification, we normalize the scaling coefficient for the US economy to unity: $\xi^{US} = 1$. Then, deviations of the scaling coefficients from unity for other economies capture their average differences in risk premium from the US economy.

With the scaling coefficients, we assume that the jump component J^h in each economy is i.i.d. and that the Lévy density ($\pi^h[x]$) of each jump component obeys an exponentially dampened power law:

$$\pi^h[x] = \begin{cases} \lambda e^{-\beta_+ x} x^{-\alpha-1} & x > 0 \\ \lambda e^{-\beta_- |x|} |x|^{-\alpha-1} & x < 0 \end{cases}, h = 1, 2, \ldots, N, \tag{10}$$

with $\alpha \in (-1, 2)$ and $\lambda, \beta_+, \beta_- > 0$. We adopt this specification from Carr, Geman, Madan, and Yor (2002) and Wu (2006) over the classic Merton (1976) compound Poisson jump model for several reasons. First, setting $\alpha < 0$ in (10) generates compound Poisson jumps that are similar in behavior to the Merton model. Furthermore, even within the compound Poisson jump class, our separate parameterization of upside and downside jumps with different scaling coefficients (β_+, β_-) allows us to investigate the differential pricing of upside versus downside risks in an economy, a task that cannot be achieved with the normal jump size distribution assumption in the Merton model. Finally, allowing the power coefficient α to take on different values can generate different types of jump behaviors from finite-activity compound Poisson jumps ($\alpha < 0$) to infinite-activity jumps with finite variation ($0 \leq \alpha < 1$), and to even higher-frequency jumps with infinite variation ($1 \leq \alpha < 2$). Instead of restricting the jump specification to one specific type, we choose an encompassing specification and let the data decide which jump type is the most appropriate in capturing the economic behaviors.

Under the Lévy density specification in (10) and when $\alpha \neq 0$ and $\alpha \neq 1$, the cumulant exponent is,

$$k_J[u] = \Gamma[-\alpha]\,\lambda\left((\beta_+ - u)^\alpha - (\beta_+)^\alpha + (\beta_- + u)^\alpha - (\beta_-)^\alpha\right) + uC[\delta], \tag{11}$$

where $\Gamma[-\alpha]$ denotes the Gamma function and $C[\delta]$ is an immaterial drift term that depends on the exact form of the truncation function used in computing the cumulant exponent (Jacod and Shiryaev (1987)). We can henceforth safely ignore this term in our analysis and drop this term in our representations. The Lévy density has singularities at $\alpha = 0$ and $\alpha = 1$, in which cases the cumulant exponent takes on different forms:

$$\begin{aligned} k_J[u] &= -\lambda \ln(1 - u/\beta_+) - \lambda \ln(1 + u/\beta_-) && \text{when} \quad \alpha = 0, \\ k_J[u] &= \lambda(\beta_+ - u)\ln(1 - u/\beta_+) + \lambda(\beta_- + u)\ln(1 + u/\beta_-) && \text{when} \quad \alpha = 1. \end{aligned} \tag{12}$$

For the country-specific risk component, we accommodate the average difference in the risk premium rates across different economies by applying the constant scaling coefficients to an otherwise independent and identical risk premium rate dynamics:

$$\Lambda_t^h = \xi^h \int_0^t Y_s^h\, ds, \tag{13}$$

where Y_t^h can be regarded as the country-specific risk premium rate factor. We model its dynamics using the square-root process of Cox, Ingersoll, and Ross (1985),

$$dY_t^h = \kappa_Y\left(\theta_Y - Y_t^h\right)dt + \omega_Y\sqrt{Y_t^h}\,dW_t^{Yh}, \quad h = 1, 2, \cdots, N, \tag{14}$$

where $\rho_Y = \mathbb{E}(dW^{Yh}dW^h)/dt$ captures the correlation between shocks of the country-specific diffusion risk and its risk premium rate. It is important to note that the dynamics specification in (14) governs N independent processes, one for each economy.

For the global risk factor, we apply the same set of scaling coefficients to a global risk premium rate factor to preserve parsimony:

$$\Pi_t^h = \xi^h \Pi_t, \quad \text{with} \quad \Pi_t = \int_0^t Z_s\, ds, \tag{15}$$

where the global risk premium rate factor Z_t is also assumed to follow a square-root process,

$$dZ_t = \kappa_Z(\theta_Z - Z_t)\,dt + \omega_Z\sqrt{Z_t}\,dW_t^Z, \tag{16}$$

with $\rho_Z = \mathbb{E}(dW^Z dW^g)/dt$. By design, the global risk and the country-specific risk, as well as the corresponding risk premium rates, are orthogonal to each other: $\mathbb{E}(dW^g dW^h) = 0, \mathbb{E}(dW^Z dW^{Yh}) = 0$ for all $h = 1, 2, \ldots, N$.

We identify the model using currency options on dollar-yen, dollar-pound, and pound-yen exchange rates, and the time-series returns on the respective currencies. For the three economies, the model has one global diffusion risk component and three country-specific jump-diffusion risk components. The risk premium rate on each of the four risk components is stochastic. Thus, our estimation on the three economies identifies four risk premium rates: one global risk premium rate factor Z_t and three country-specific risk premium rates $(Y_t^{USD}, Y_t^{JPY}, Y_t^{GBP})$. The model has 14 parameters for the three economies:

$$\Theta \equiv [\xi^{JPY}, \xi^{GBP}, \kappa_Z, \theta_Z, \omega_Z, \rho_Z, \kappa_Y, \theta_Y, \omega_Y, \rho_Y, \lambda, \beta_+, \beta_-, \alpha]. \tag{17}$$

Within each model, we consider three special cases for the jump specification with α fixed at -1, 0, and 1, respectively. The three different α's generate finite-activity, infinite-activity with finite variation, and infinite-variation jumps, respectively.

We also estimate models with strict symmetry: $\xi^h = 1$ for all h. Reality aside, this special class highlights the issue of stochastic discount factor identification using exchange rates. A key implication of strict symmetry is that the contribution of the global risk factor in the two economies cancels. Thus, from currency returns and currency options, we can no longer identify the global risk component. Accordingly, we can only estimate the eight parameters that control the country-specific risk components of the three economies: $\Theta \equiv [\kappa_Y, \theta_Y, \omega_Y, \rho_Y, \lambda, \beta_+, \beta_-, \alpha]$.

3.2. Stochastic risk premiums, stochastic skewness, and currency return dynamics

To highlight our contributions relative to traditional approaches, we emphasize two themes in this subsection: (1) What are the sources of stochastic skewness in currency returns? and (2) what minimal structures are necessary to reconcile the observed patterns from the triangle of currency returns and options?

Under our model specification, the log currency return over horizon $[0,t]$ is,

$$\begin{aligned}\ln S_t^{fh}/S_0^{fh} &= \left(r^h - r^f\right)t + \left(\sqrt{\xi^h} - \sqrt{\xi^f}\right)W^g_{\Pi_t} + \frac{1}{2}\Pi_t\left(\xi^h - \xi^f\right) + \\ &\left(W^h_{\Lambda^h_t} + J^h_{\Lambda^h_t} + \left(\frac{1}{2} + k_J[-1]\right)\Lambda^h_t\right) - \left(W^f_{\Lambda^f_t} + J^f_{\Lambda^f_t} + \left(\frac{1}{2} + k_J[-1]\right)\Lambda^f_t\right), \qquad (18)\end{aligned}$$

where the exchange rate dynamics between the two economies (h and f) are governed by (i) one diffusion global risk component (W^g_t), (ii) two jump-diffusion country-specific risk components ($W^h_t + J^h_t, W^f_t + J^f_t$), and (iii) three risk premium rates (Z_t, Y^h_t, Y^f_t) that define the three stochastic time changes ($\Pi_t, \Lambda^h_t, \Lambda^f_t$).

To see how such a structure is necessary to generate stochastic risk premiums and stochastic skewness in currency options underlying the three economies, we start with the special case where the risk premium rates are constant: $Z_t = \theta_Z$ and $Y^h_t = Y^f_t = \theta_Y$. The currency risk premium per unit time in country h becomes:

$$\mathsf{RP}^h \equiv \mathbb{E}^{\mathcal{P}}\left(S^{fh}_1/S^{fh}_0\right) - (r^h - r^f) = \left(\xi^h - \sqrt{\xi^h\xi^f}\right)\theta_Z + \xi^h\left(1 + k_J[1] + k_J[-1]\right)\theta_Y, \qquad (19)$$

where the first term captures the contribution from the global risk factor and the second term captures the contribution from the country-specific risk factor in country h. Under this special case, the risk premium RP^h is a constant. We introduce stochastic currency risk premium via the stochastic time changes Π_t, Λ^h_t, and Λ^f_t, or equivalently the stochastic risk premium rates Z_t, Y^h_t, and Y^f_t.

In the absence of stochastic risk premiums, the currency return is governed by three Brownian motions with constant volatilities and two jump components with constant arrival rates. The two jump components can generate distributional non-normality (skewness and kurtosis) for the currency return. By taking successive partial derivatives of the cumulant exponent, we can show that the variance (c_2) and the third (c_3) and fourth cumulants (c_4) for the currency return are,

$$\begin{aligned} c_2 &= \lambda\left(\xi^h + \xi^f\right)\theta_Y\,\Gamma[2-\alpha]\left((\beta_+)^{\alpha-2} + (\beta_-)^{\alpha-2}\right) + V_d, \\ c_3 &= \lambda\left(\xi^h - \xi^f\right)\theta_Y\,\Gamma[3-\alpha]\left((\beta_+)^{\alpha-3} - (\beta_-)^{\alpha-3}\right), \qquad (20) \\ c_4 &= \lambda\left(\xi^h + \xi^f\right)\theta_Y\,\Gamma[4-\alpha]\left((\beta_+)^{\alpha-4} + (\beta_-)^{\alpha-4}\right), \end{aligned}$$

where $V_d \equiv (\sqrt{\xi^h} - \sqrt{\xi^f})^2\theta_Z + \left(\xi^h + \xi^f\right)\theta_Y$ captures the variance contribution from the diffusion components. The diffusion components have zero contribution to higher-order cumulants. The currency return shows nonzero skewness or non-zero third cumulant c_3 when (1) the jump component in the log stochastic

discount factor is asymmetric: $\beta_+ \neq \beta_-$, and (2) the two economies are asymmetric in the average magnitudes of risk premiums: $\xi^h \neq \xi^f$. In fact, these two conditions are necessary for the existence of any non-zero odd-order cumulants beyond three. In contrast, the fourth cumulant (c_4) or the excess kurtosis for the currency return is strictly positive as long as the jump component is not degenerating ($\lambda \neq 0$). Nevertheless, since all the cumulants in (20) are constant, a model with constant risk premiums cannot capture the evidence from currency option markets that the currency return skewness is stochastic (Carr and Wu (2006)). Stochastic skewness in currency return distribution warrants stochastic risk premium.

When the risk premium rates are allowed to be stochastic as in currency dynamics (18), currency return skewness can also arise from three additional sources: (1) correlation (ρ_Z) between W_t^g and Z_t, (2) correlation (ρ_Y^h) between W_t^h and Y_t^h, and (3) correlation (ρ_Y^f) between W_t^f and Y_t^f. Allowing the three risk premium rates (Z_t, Y_t^h, Y_t^f) to be stochastic produces both stochastic volatility and stochastic skewness in currency returns.

3.3. Relating risk premium rates to currency option prices

To price currency options, we first derive the generalized Fourier transform of the currency return under the home-currency risk-neutral measure Q^h, $\phi_s^Q \equiv \mathbb{E}^Q\left(e^{iu\ln S_t^{fh}/S_0^{fh}}\right)$. Then, we compute option prices numerically via fast Fourier inversion (Carr and Madan (1999)).

Under our model specification, we can derive the generalized Fourier transform in analytical form,

$$\phi_s^Q = \exp\left(iu(r^h - r^f)t - b_g(t)Z_0 - c_g(t) - b_h(t)Y_0^h - c_h(t) - b_f(t)Y_0^f - c_f(t)\right), \tag{21}$$

where $\left(Z_0, Y_0^h, Y_0^f\right)$ are the time-0 realized levels of the three risk premium rates and the coefficients $[b(t), c(t)]$ on each risk premium rate take the same functional forms,

$$\begin{aligned} b_c(t) &= \frac{2\psi^c\left(1-e^{-\eta^c t}\right)}{2\eta^c - \left(\eta^c - \kappa^{\mathcal{N}c}\right)\left(1-e^{-\eta^h t}\right)}, \\ c_c(t) &= \frac{\kappa_c\theta_c}{\omega_c^2}\left(2\ln\left(1 - \frac{\eta^c - \kappa_c^{\mathcal{N}}}{2\eta^c}\left(1-e^{-\eta^c t}\right)\right) + (\eta^c - \kappa_c^{\mathcal{N}})t\right), \end{aligned} \tag{22}$$

with $\eta^c = \sqrt{\left(\kappa_c^{\mathcal{N}}\right)^2 + 2\omega_c^2\psi^c}$ for $c = g,h,f$, $\kappa_g = \kappa_Z$, $\kappa_h = \kappa_f = \kappa_Y$, $\omega_g = \omega_Z$, $\omega_h = \omega_f = \omega_Y$, and

$$\begin{aligned}
\kappa_g^{\mathcal{N}} &= \kappa_Z - iu\left(\sqrt{\xi^h} - \sqrt{\xi^f}\right)\omega_Z\rho_Z + \sqrt{\xi^h}\omega_Z\rho_Z, & \psi^g &= \tfrac{1}{2}\left(\sqrt{\xi^h} - \sqrt{\xi^f}\right)^2\left(iu + u^2\right), \\
\kappa_h^{\mathcal{N}} &= \kappa_Y + (1-iu)\sqrt{\xi^h}\omega_Y\rho_Y, & \psi^h &= iu\left(\tfrac{1}{2} + k_J^Q[1]\right) + \tfrac{1}{2}u^2 - k_J^Q[iu], \\
\kappa_f^{\mathcal{N}} &= \kappa_Y + iu\sqrt{\xi^f}\omega_Y\rho_Y, & \psi^f &= iu\left(\tfrac{1}{2} + k_J[-1]\right) + \tfrac{1}{2}u^2 - k_J[-iu].
\end{aligned} \tag{23}$$

According to (18) and (21), the stochastic evolution in the risk premium rates (Z_t, Y_t^h, Y_t^f) impacts the currency return dynamics and currency option prices by stochastically altering return volatility and skewness. It is this analytical link that allows us to identify the evolution of risk premium rates from currency option prices. In a related study, Brandt and Santa-Clara (2002) use short-term at-the-money currency option volatility to approximate the instantaneous variance of the currency return, and specify the market price of risk as a linear function of the instantaneous variance, which becomes an observable quantity under their approximation. Here, through the linkage built in (21), we exploit the information in currency options quotes across all available maturities and strikes underlying three currency pairs to identify the stochastic discount factors in the three economies.

Our stochastic discount factor modeling also has direct implications for empirical models of currency returns and currency options. Compared to the extant literature on currency option pricing, e.g., Bates (1996), Bollen, Gray, and Whaley (2000), and Dupoyet (2006), our modeling framework distinguishes itself in several key dimensions. First, whereas all traditional models generate little time-variation in the skewness of the currency return distribution, our model is consistent with the stochastic skewness feature, leading to more realistic currency return distributions. Second, compared to one-factor volatility dynamics in earlier studies, our model incorporates richer stochastic volatility dynamics. The return volatility on each currency pair is driven by three stochastic risk premium rates for one global risk factor and two country-specific risk factors, respectively. These stochastic risk premium rates generate both stochastic volatility and stochastic skewness from multiple sources. Third, the Lévy density in (10) allows not only finite-activity jumps used in earlier studies (when $\alpha < 0$), but also infinite-activity jumps that generate an infinite number of jumps within any finite interval. Finally, the currency option pricing literature often starts by specifying a dynamic process for an underlying currency pair (say, dollar-yen), and then analyzes its implications for options on this currency pair. Its inherent links to other currency pairs (say, dollar-pound, yen-pound) and their options are largely ignored as options on each currency pair are analyzed on a stand-alone basis. In this paper, we specify the

stochastic discount factors for the US, Japan, and the UK, and price options on dollar-yen, dollar-pound, and yen-pound within one consistent framework. Maintaining this internal consistency is important not only for precluding cross-currency arbitrages, but also for determining how the risk and pricing of different economies are related to one another.

3.4. Conditional likelihoods of currency returns

For estimation, we also need to develop the log likelihood function for the currency return time series. We first derive the characteristic function of currency returns under the statistical measure $\mathcal{P}$ and then obtain the density of the currency return via fast Fourier inversion. Based on the $\mathcal{P}$-dynamics for the currency return in (18), the characteristic function, $\phi_s^{\mathcal{P}} \equiv \mathbb{E}^{\mathcal{P}}\left(e^{iu\ln S_t^{fh}/S_0^{fh}}\right)$, can be derived as,

$$\phi_s^{\mathcal{P}} \quad = \quad \exp\left(iu(r^h - r^f)t - b_g(t)Z_0 - c_g(t) - b_h(t)Y_0^h - c_h(t) - b_f(t)Y_0^f - c_f(t)\right), \tag{24}$$

where the coefficients $[b_c(t), c_c(t)]$ for $c = h, f, g$ are given by (22) with,

$$\begin{array}{lll} \kappa_g^{\mathcal{N}} = \kappa_Z - iu(\sqrt{\xi^h} - \sqrt{\xi^f})\omega_Z\rho_Z, & \psi^g[u] = -\frac{1}{2}iu\left(\xi^h - \xi^f\right) + \frac{1}{2}(\sqrt{\xi^h} - \sqrt{\xi^f})^2u^2, \\ \kappa_h^{\mathcal{N}} = \kappa_Y - iu\sqrt{\xi^h}\omega_Y\rho_Y, & \psi^h[u] = -iu\left(\frac{1}{2} + k_J[-1]\right) + \frac{1}{2}u^2 - k_J[iu], \\ \kappa_f^{\mathcal{N}} = \kappa_Y + iu\sqrt{\xi^f}\omega_Y\rho_Y, & \psi^f[u] = iu\left(\frac{1}{2} + k_J[-1]\right) + \frac{1}{2}u^2 - k_J[-iu]. \end{array} \tag{25}$$

By the triangular arbitrage relation, the time-t yen-pound cross exchange rate is completely determined by the other two primary currency pairs: dollar-yen and dollar-pound. A separate quote on the cross rate is redundant. However, it is important to realize that the two marginal distributions for dollar-yen and dollar-pound (whether under $\mathcal{P}$ or $\mathcal{Q}$) are insufficient to determine the distribution of the yen-pound cross rate. For this reason, the cross-currency option quotes are not redundant, but offer incremental information about the risk-neutral marginal distribution of the yen-pound cross rate. Applying fast Fourier inversion to the characteristic function in (24) yields the marginal likelihoods of the dollar-yen, dollar-pound, and yen-pound returns. Including the marginal likelihood of all three currency pair is useful for identifying the stochastic discount factors in the US, Japan, and the UK, even though the log cross rate $\ln(S_t^{GBPJPY})$ is a linear combination of the two log primary rates $\ln(S_t^{JPYUSD})$ and $\ln(S_t^{GBPUSD})$.

4. Data and estimation

4.1. Data description

We obtain over-the-counter quotes on currency options and spot exchange rates for three currency pairs that form a triangular relation: JPYUSD (the dollar price of one yen), GBPUSD (the dollar price of one pound), and GBPJPY (the yen price of one pound), over the sample period of November 7, 2001 to January 28, 2004. The data are sampled weekly. Options quotes are available at seven fixed time-to-maturities: one week, one, two, three, six, nine, and 12 months. At each maturity, quotes are available at five fixed moneyness. There are a total of 12,285 option quotes. The five options at each maturity are quoted in the following forms:

- **Delta-neutral straddle implied volatility (SIV)**: A straddle is a sum of a call option and a put option with the same strike. The SIV market quote corresponds to a strike that makes the Black-Scholes delta of the straddle zero: $\Delta_S^c + \Delta_S^p = 0$, where $\Delta_S^c = e^{-r^f \tau} N[d_1]$ and $\Delta_S^p = -e^{-r^f \tau} N[-d_1]$ are the Black-Scholes delta of the call and put options in the straddle, respectively. $N[\cdot]$ denotes the cumulative normal function, and $d_1 = \frac{\ln(S_t/K)+(r^h-r^f)\tau}{IV\sqrt{\tau}} + \frac{1}{2} IV\sqrt{\tau}$, with IV being the implied volatility input, τ being the option time-to-maturity, and K being the strike price of the straddle. Since the delta-neutral restriction implies $d_1 = 0$, the implicit strike is close to the spot price.

- **Ten-delta risk-reversal, RR[10], and 25-delta risk-reversal, RR[25]**: The RR[10] measures the difference in Black-Scholes implied volatilities between a ten-delta out-of-the-money call option and a ten-delta out-of-the-money put option: $RR[10] = IV^c[10] - IV^p[10]$. RR[25] is analogously defined on 25-delta call and put options. Option traders use risk-reversal quotes to quantify the asymmetry of the implied volatility smile, which reflects the skewness of the risk-neutral currency return distribution.

- **Ten-delta butterfly spread, BF[10], and 25-delta butterfly spread, BF[25]**: Butterfly spreads are defined as the average difference between out-of-the-money implied volatilities and the delta-neutral straddle implied volatility: $BF[10] = (IV^c[10] + IV^p[10])/2 - \text{SIV}$ and $BF[25] = (IV^c[25] + IV^p[25])/2 - \text{SIV}$. Butterfly spread quotes capture the curvature of the implied volatility smile, which reflects the kurtosis of the risk-neutral currency return distribution.

Based on the above definitions, we recover the underlying implied volatilities as: (i) $IV^c[25] = \text{BF}[25] + \text{SIV} + \text{RR}[25]/2$, (ii) $IV^p[25] = \text{BF}[25] + \text{SIV} - \text{RR}[25]/2$, (iii) $IV^c[10] = \text{BF}[10] + \text{SIV} + \text{RR}[10]/2$, and (iv) $IV^p[10] = \text{BF}[10] + \text{SIV} - \text{RR}[10]/2$. For model estimation, the volatility quotes are converted into out-of-the-money option prices. In this calculation, the maturity-matched domestic and foreign interest rates are

constructed using LIBOR and swap rates taken from Bloomberg.

Table 1 reports the mean, the standard deviation, and the t-statistics on the significance of the sample mean for risk-reversal and butterfly spread series, all in percentages of the corresponding delta-neutral straddle implied volatility. The t-statistics are adjusted for serial dependence according to Newey and West (1987), with the number of lags optimally chosen according to Andrews (1991) based on an AR(1) specification.

Average butterfly spreads are uniformly positive and highly significant across all maturities, implying that out-of-the-money option implied volatilities on average are significantly higher than the at-the-money implied volatility. The lowest t-statistic is 10.98. Regardless of the currency pair, the butterfly spread quotes are strongly supportive of excess kurtosis in the risk-neutral currency return distribution.

The sign and magnitudes of risk-reversals are informative about the asymmetry of the conditional return distribution. For JPYUSD, the sample averages of the risk-reversals are positive, implying that out-of-the-money calls are on average more expensive than out-of-the-money puts during our sample period. This evidence suggests that, on average, the JPYUSD risk-neutral conditional return distribution is positively skewed. The average risk-reversals for GBPUSD are also positive, albeit to a lesser degree. In contrast, the average magnitudes of risk-reversals are negative for GBPJPY, implying the presence of negative risk-neutral return skewness.

Figure 1 plots the time-series of ten-delta risk-reversals in the left-panels and ten-delta butterfly spreads in the right-panels, with maturities fixed at one month (solid lines) and three months (dashed lines). Over the sample period, there is significant variation in both risk-reversals and butterfly spreads, more so for risk-reversals. Indeed, the risk-reversals vary so much that the sign switches. The ten-delta risk-reversals on JPYUSD have varied from -20% to over 50% of the at-the-money implied volatility, the risk-reversals on GBPUSD have varied from -10 to 20%, and the risk-reversals on GBPJPY have varied from -35 to 15%.

[Fig. 1 about here.]

4.2. Maximum likelihood estimation

We estimate the models using the time-series of both currency returns and currency option prices on JPYUSD, GBPUSD, and GBPJPY. Since the risk premium rates are not directly observable, we cast the models into a state-space form and infer the risk premium rates at each date using a filtering technique. We estimate the model parameters by maximizing the aggregate likelihoods of options and currency returns.

In the state-space form, we regard the risk premium rates in the three economies as unobservable states. For the general asymmetric models, we use $V_t \equiv [Y_t^{USD}, Y_t^{JPY}, Y_t^{GBP}, Z_t]^\top$ to denote the (4×1) state vector. For the symmetric models, we drop the global risk premium rate Z_t from the state vector since it is no longer identifiable. We specify the state propagation equation using an Euler approximation of the risk premium rates dynamics:

$$V_t = A + \Phi V_{t-1} + \sqrt{\mathcal{G}_t}\,\varepsilon_t, \quad \nu_t \in \Re^{4+} \tag{26}$$

where ε_t denotes an i.i.d. standard normal innovation vector and

$$\begin{aligned} \Phi &= \exp(-\kappa \Delta t), \quad \kappa = < [\kappa_Y, \kappa_Y, \kappa_Y, \kappa_Z] >, \\ A &= (I - \Phi)\theta, \quad \theta = [\theta_Y, \theta_Y, \theta_Y, \theta_Z]^\top, \quad \mathcal{G}_t = \langle [\omega_Y^2 Y_{t-1}^{USD}, \omega_Y^2 Y_{t-1}^{JPY}, \omega_Y^2 Y_{t-1}^{GBP}, \omega_Z^2 Z_{t-1}] \Delta t \rangle, \end{aligned} \tag{27}$$

where $\Delta t = 7/365$ corresponds to the weekly frequency of the data and $\langle \cdot \rangle$ denotes a diagonal matrix with the diagonal elements given by the vector inside the bracket.

We construct the measurement equations on the observed out-of-the-money option prices, assuming additive normally-distributed measurement errors:

$$y_t = \mathcal{O}[V_t; \Theta] + e_t, \quad \mathbb{E}(e_t e_t^\top) = \mathcal{J}, \quad y_t \in \Re^{105+}, \tag{28}$$

where y_t denotes the 105 observed out-of-the-money option prices scaled by Black-Scholes vega at time t for the three currency pairs (across seven maturities and five moneyness categories), and $\mathcal{O}[V_t; \Theta]$ denotes the corresponding model-implied values as a function of the parameter set Θ and the state vector V_t. We assume that the scaled pricing errors are i.i.d. normal with zero mean and constant variance. Hence, we can write the covariance matrix as $\mathcal{J} = \sigma_r I$, with σ_r being a scalar and I being an identity matrix of the relevant dimension.

When both the state propagation equation and the measurement equations are Gaussian and linear, the Kalman (1960) filter generates efficient forecasts and updates on the conditional mean and covariance of the state vector and the measurement series. In our application, the state propagation equation in (26) is Gaussian and linear, but the measurement equation in (28) is nonlinear. We use the unscented Kalman filter (Wan and van der Merwe (2001)) to handle the nonlinearity. The unscented Kalman filter approximates the posterior state density using a set of deterministically chosen sample points (sigma points). These sample points completely capture the true mean and covariance of the Gaussian state variables, and when propagated through the nonlinear functions in the measurement equations, capture the posterior mean and covariance of

the option prices accurately to the second order for any nonlinearity.

Let $\overline{y}_{t+1}$ and $\overline{A}_{t+1}$ denote the time-t forecasts of time-$(t+1)$ values of the measurement series and the covariance of the measurement series, respectively, obtained from the unscented Kalman filter. Assuming normally distributed forecasting errors, we have the log likelihood for each week's option observations as,

$$l_{t+1}[\Theta]^O = -\frac{1}{2}\log\left|\overline{A}_{t+1}\right| - \frac{1}{2}\left(\left(y_{t+1}-\overline{y}_{t+1}\right)^\top\left(\overline{A}_{t+1}\right)^{-1}\left(y_{t+1}-\overline{y}_{t+1}\right)\right). \tag{29}$$

Given the risk premium rates extracted from the options data, we apply fast Fourier inversion to the characteristic function in (24) to obtain the statistical density of weekly returns on each of the three currency pair as a function of the risk premium rates. We use $l_{t+1}[\Theta]^s$ to denote the weekly log likelihood of the currency returns on the three currency pairs.

We choose model parameters to maximize the summation of the weekly log likelihood values on both options and currency returns,

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}\left[\Theta, \{y_t\}_{t=1}^T\right], \quad \text{with} \quad \mathcal{L}\left[\Theta, \{y_t\}_{t=1}^T\right] = \sum_{t=0}^{T-1}\left(l_{t+1}[\Theta]^O + l_{t+1}[\Theta]^s\right), \tag{30}$$

where $T = 117$ denotes the number of weeks in our sample. In defining the likelihood in (30), we assume conditional independence between the options forecasting errors and the currency returns. We further replace the joint density of the currency returns with the product of the three marginal densities for computational feasibility. Using the product of marginal densities incurs some theoretical information loss, but provides significant gains in computational feasibility.

5. Empirical results on risk and pricing in international economies

Building on established themes, we estimate models with both proportional asymmetry and strict symmetry. For each specification, we consider four different parameterizations of the jump component in (10). Specifically, we allow for unrestricted power coefficient, α, and the nested special cases of $\alpha = -1$, $\alpha = 0$, and $\alpha = 1$. Setting $\alpha = -1$ generates a compound-Poisson jump similar in behavior to the jump in Merton (1976) and Bates (1996). Setting $\alpha = 0$ and 1 generates more frequent jump arrivals. The estimated model parameters, their standard errors (in parenthesis), and the maximized log likelihood values are reported in Table 2 for the four symmetric models and in Table 3 for the four asymmetric models.

5.1. The US, Japan, and the UK economies are asymmetric over our sample period

The maximized likelihood values from the general asymmetric specifications (Table 3) are much larger than the corresponding symmetric specifications (Table 2). Likelihood ratio tests for nested models suggest that the differences are statistically significant beyond any reasonable confidence level. The estimated variance of the pricing errors (σ_r^2) of the symmetric models is almost twice as large as that of the asymmetric models. Therefore, by allowing asymmetry between the stochastic discount factors of the US, Japan, and the UK, the models capture the currency return and currency options behavior much better.

The scaling coefficient on the US economy is normalized to unity: $\xi^{USD} = 1$. Hence, under the asymmetric specifications in Table 3, the deviations from unity for the estimates of ξ^{GBP} and ξ^{JPY} measure the degree of asymmetry between the three economies. The estimates for the scaling coefficient on the UK, ξ^{GBP}, are slightly larger than one, but the estimates for the scaling coefficient on Japan, ξ^{JPY}, are much larger at around 1.5. These estimates suggest that the Japanese economy is markedly different from the US economy and the UK economy. The average risk premium rate in Japan is about 50% higher than that in the US or the UK. This larger risk premium can be attributed either to higher risk in the economy or higher risk aversion for investors in Japan.

The observed asymmetry between the three economies represents the average behavior during our three-year sample period. Therefore, our result does not exclude the possibility of unconditional symmetry over the very long run, or other forms of asymmetry during other sample periods. Nevertheless, the average asymmetry during our sample period is crucial in identifying the dynamics of the global risk premium rate.

5.2. Risk premium rates on the global risk factor are more persistent and more volatile

The estimates of the parameters that control the risk premium dynamics are relatively stable across different parameterizations on α. Comparing the estimates for the global risk premium dynamics $(\kappa_Z, \theta_Z, \omega_Z, \rho_Z)$ to those on the country-specific risk premium dynamics $(\kappa_Y, \theta_Y, \omega_Y, \rho_Y)$ in Table 3, we observe that the global risk premium rate is both more persistent and more volatile than the country-specific risk premium rates. The mean-reversion parameter estimates for the global risk premium rate, κ_Z, is not distinguishable from zero, implying near non-stationary behavior. In contrast, the estimates of mean-reversion parameter for the country-specific risk premium rate, κ_Y, range from 3.053 to 5.204, implying a relatively short half-life of two to three months. The different persistence estimates suggest that it is much more difficult to predict changes in

global risk premium rates than to predict changes in country-specific risk premium rates. The difference also implies that shocks on the global risk premium rate last longer over time and have bigger impacts on currency options at longer maturities. By contract, the more transient shocks on country-specific risk premium rates dissipate quickly over time and mainly affect short-term option pricing behaviors.

The volatility coefficient estimates ω_Z for the global risk premium rate are around 0.8, about five times larger than the corresponding volatility coefficients ω_Y for the country-specific risk premium rates, which are between 0.137 to 0.183.

Our findings are consistent with Engle, Ito, and Lin (1990), who use the analogies of meteor showers versus heat waves to describe global versus country-specific shocks, respectively. Using intra-day exchange rate data, they find that volatility clustering in exchange rates is mainly driven by global shocks. Using weekly data on currency returns and currency options, we find that the risk premium rates on the global risk factor are both more persistent and more volatile than the risk premium rates on the country-specific risk factors.

5.3. Risk premium increases when the wealth declines relative *to the global economy*

The correlation parameter ρ_Z captures how the risk premium rate varies with the global shocks while the correlation parameter ρ_Y measures how the risk premium rate varies with the country-specific shocks. The estimates for ρ_Y are strongly negative between -0.702 and -0.999, depending on different α specifications. A negative correlation implies that the risk premium increases when the economy receives a negative country-specific shock. Such a risk premium increase can come from either or both of the two sources: (1) A negative shock is associated with higher economy-wide volatility. (2) Investors become more risk averse after a negative shock and demand higher premium for the same amount of risk.

Intriguingly, we observe that the correlation estimates between the risk premium rate and the global risk factor ρ_Z are positive, ranging from 0.52 to 0.65. Therefore, investors respond quite differently to global shocks than to country-specific shocks. Although investors demand a higher risk premium in the presence of a negative country-specific shock to the economy, they ask for a lower risk premium if the origin of the negative shock is global.

In the context of the Lucas (1982) exchange economy, the stochastic discount factors have the interpretation of marginal utilities of aggregate wealth. In this context, we may generically interpret X^h in equation (1) as return shocks to aggregate wealth in the economy. Then, a possible interpretation for the different responses

is that the risk premium in an economy changes with the *relative* wealth of the economy. Investors demand a higher premium only when the wealth of the economy declines relative to the global economy. When the global risk factor receives a negative shock, the local economy's wealth decreases in absolute terms, but increases relative to the global economy. As a result, the risk premium declines. In contrast, a negative shock to the country-specific risk factor decreases the economy wealth in both absolute and relative terms. The risk premium in this economy increases unambiguously.

When studying how an economy responds to external shocks, it is important to distinguish the different possible sources of the shocks. An analysis that fails to discriminate between country-specific and global shocks can yield misleading conclusions. It is worthwhile to mention that the extant literature often studies the behavior of stochastic discount factors in a single economy using stock index returns and stock index options in that economy. Since the stochastic discount factors estimated from these data are projections on the stock index of a single economy, these studies do not typically distinguish between global shocks versus country-specific shocks. Our joint analysis based on options and time-series returns on a triangle of currency pairs reveals the complex multi-dimensional nature of the stochastic discount factors in international economies and highlights the inadequacy of one-dimensional projections.

5.4. Jumps arrive frequently, but only downside jumps are priced

Our models for the stochastic discount factor incorporate a jump component, the arrival rate of which follows an exponentially dampened power law. Under this specification, the power coefficient α controls the jump type. The model generates finite-activity compound-Poisson jumps as in Merton (1976) when $\alpha < 0$, under which jumps arrive only a finite number of times within any finite interval and hence can be regarded as rare events. On the other hand, when $\alpha \geq 0$, jumps arrive an infinite number of times within any finite interval and can therefore be used to capture more frequent discontinuous movements.

When we estimate the asymmetric model with α as a free parameter, the estimate for α is 0.227. Nevertheless, the estimate has large standard error, suggesting potential identification problems. Thus, we also estimate three special cases with α fixed at -1, 0, and 1, representing three different jump types that encompass both traditional compound Poisson jumps and high-frequency jump specifications. As shown in Table 3 for the asymmetric models, the $\alpha = 1$ model generates the highest likelihood among the three special cases, indicating that jumps in the three economies are not rare events, but arrive frequently. Therefore, replacing the traditional compound Poisson jump with an infinite-activity jump specification generates more promising

currency option pricing results.

Under our jump specification, the relative asymmetry of jumps is controlled by the two exponential dampening coefficients β_+ and β_-. A larger dampening coefficient β_+ implies a smaller arrival rate for positive jumps and vice versa. Table 3 shows that the estimates for β_+ are substantially larger than those for β_-, more so when α is larger and hence when more frequent jumps are allowed. The large estimates for β_+ suggest that the negative of the log stochastic discount factors rarely experience positive jumps. In fact, the standard errors for β_+ estimates are also large, suggesting that we cannot accurately identify the parameter that controls the positive jumps. Therefore, we can safely assume a one-sided jump structure for the log stochastic discount factor by setting the arrival rate of positive jumps to zero: $\pi[x] = 0$ for $x > 0$.

To pursue this angle, Table 4 reports the parameter estimates and maximized log likelihood values under this one-sided jump assumption. The estimates for most of the parameters are close to those reported in Table 3 under the two-sided jump parameterization. The likelihood values are also about the same. The main difference is that with the one-sided jump assumption in Table 4, the standard errors of some parameters decline, showing better identification with the more parsimonious one-sided specification. Therefore, our results support the lack of a significant pricing component for positive jumps in the stochastic discount factor.

The origin of jumps in stochastic discount factors can be tied to the way in which markets respond to information (Andersen, Bollerslev, Diebold, and Vega (2003), Andersen, Bollerslev, Diebold, and Vega (2005), Balduzzi, Elton, and Green (2001), Beber and Brandt (2005), Beber and Brandt (2006), Fleming and Remolona (1999), Hau and Rey (2006), Pasquariello and Vega (2006), and Piazzesi (2005)). Shocks in an economy can jump both up and down. The fact that we can only detect downside jumps in the stochastic discount factor suggests that investors are only concerned with downside jumps in the economy while ignoring upside jumps for pricing. This finding explains why financial markets react more strongly to bad macroeconomic announcement surprises than to good surprises (Andersen, Bollerslev, Diebold, and Vega (2005)).

The presence of priced frequent downside jumps in the stochastic discount factors also provides justification for the prevailing evidence from the stock index option market. Although the statistical return distribution for stock indexes is relatively symmetric, the risk-neutral distributions computed from option prices are highly negatively skewed (Jackwerth and Rubinstein (1996), Bates (2000), Foresi and Wu (2005), Jones (2006), Pan (2002), and Bakshi, Kapadia, and Madan (2003)). Carr and Wu (2003) show that a one-sided α-stable law, without exponential dampening, captures the S&P 500 index options price behavior well. When measure changes are applied using exponential martingales, α-stable laws become exponentially dampened power

laws. Hence, the dampened power law specification subsumes the α-stable specification.

Regarding the relative contribution of stochastic risk premiums versus jumps, we note that they capture different aspects of the stochastic discount factor and that both features are crucial for our empirical results. Economically, the jump component captures the discontinuous movements in both macroeconomic fundamentals and financial securities, and the stochastic risk premium specification captures the intensity variation of the information flow.

5.5. High global risk premium rates lead to high international risk sharing index

One yardstick to assess the plausibility of the estimated risk premiums and stochastic discount factors is to compute the risk sharing index developed by Brandt, Cochrane, and Santa-Clara (2006):

$$\text{RSI} \equiv 1 - \frac{\text{Var}\left(\ln \mathcal{M}_t{}^f - \ln \mathcal{M}_t{}^h\right)}{\text{Var}(\ln \mathcal{M}_t{}^f) + \text{Var}(\ln \mathcal{M}_t{}^h)}. \tag{31}$$

According to our stochastic discount factor specification in (8) and replacing the risk premium rates by their respective long-run means, we can derive the unconditional risk sharing index analytically as,

$$\text{RSI} \quad \equiv \quad 1 - \frac{\left(\sqrt{\xi^h} - \sqrt{\xi^f}\right)^2 \theta_Z + \theta_Y \left(\xi^h + \xi^f\right) \left(1 + \lambda \Gamma \left[2 - \alpha\right] \left(\beta_+^{\alpha-2} + \beta_-^{\alpha-2}\right)\right)}{\left(\xi^h + \xi^f\right) \theta_Z + \theta_Y \left(\xi^h + \xi^f\right) \left(1 + \lambda \Gamma \left[2 - \alpha\right] \left(\beta_+^{\alpha-2} + \beta_-^{\alpha-2}\right)\right)}. \tag{32}$$

Equation (32) shows that the risk sharing index is high when the global risk premium rate is high relative to the country-specific risk premium rate ($\theta_Z > \theta_Y$), and when the two economies are relatively symmetric ($\xi^h \approx \xi^f$). When the two economies are asymmetric, the risk sharing index declines irrespective of the relative proportion of global versus country-specific risk premium. For two highly asymmetric economies, RSI is close to zero even if the two economies move perfectly together. Therefore, the risk sharing index measures both co-movement and asymmetry between two economies.

In Table 5, we report the risk sharing index computed based on the parameter estimates in Table 3 for the asymmetric models. Our estimates for the risk sharing index are high, ranging from 0.9625 to 0.9891. The estimates are stable across different power coefficients α, indicating that the results are robust with respect to different jump specifications.

Our high estimates of the risk sharing index are in line with the results in Brandt, Cochrane, and Santa-Clara (2006). Combining stock portfolio returns of two economies with the currency return, Brandt, Cochrane,

and Santa-Clara attribute the high risk sharing index to two possible explanations: (i) The variability of currency returns is too low, and/or (ii) international risk sharing is high. In this paper, we identify the dynamics of stochastic discount factors in three economies using time-series returns and options on a triangle of exchange rates, and decompose each stochastic discount factor into a global and a country-specific risk component. Our estimation attributes the high risk sharing index to a predominant global risk premium component, and therefore supports their second explanation.

5.6. The risk premium rates co-move with economic fundamentals

A natural question that arises is how the extracted risk premiums are related to observed economic fundamentals. First, to address the inherent link between the risk premium rates and the observed currency option implied volatilities, and to understand the source of identification for our model estimation, we follow Brandt and Santa-Clara (2002) in using squared short-term at-the-money currency option implied volatility (SIV) to approximate the instantaneous variance of the currency return. Under this approximation, our stochastic discount factor model implies the following relation,

$$\left(\mathrm{SIV}_t^{fh}\right)^2 \approx \left(\sqrt{\xi^h}-\sqrt{\xi^f}\right)^2 Z_t + \left(1+\lambda\Gamma\left[2-\alpha\right]\left(\beta_+^{\alpha-2}+\beta_-^{\alpha-2}\right)\right)\left(\xi^h Y_t^h + \xi^f Y_t^f\right). \tag{33}$$

From equation (33), it is clear that the global and country-specific risk premium rates are directly linked to the variance of the currency return and, under the approximation, to currency option implied volatilities. It is based on this linkage that we can identify the risk premium rates from the currency options quotes.

To verify this relation, we regress squared one-month at-the-money currency option implied volatilities on the corresponding risk premium rates, all in weekly changes,

$$\Delta(\mathrm{SIV}_t^{hf})^2 = b_0 + b_1\Delta Z_t + b_2\Delta Y_t^h + b_3\Delta Y_t^f + e_t,$$

where Δ denotes weekly changes and $[Y_t^h, Y_t^f, Z_t]$ are the country-specific and global risk premium rates extracted from the estimated asymmetric model with $\alpha = 1$. We estimate the relation using generalized methods of moment (GMM), where the weighting matrix is computed according to Newey and West (1987) with four lags. Table 6 reports the GMM coefficient estimates and t-statistics. Consistent with the theory behind (33), the intercept estimates are not significantly different from zero and the slope coefficient estimates are close to that inferred from the maximum likelihood parameter estimates in Table 3. Take JPYUSD as an example.

The parameter estimates of $\xi^{JPY} = 1.531$ and $\xi^{USD} = 1$ in Table 3 ($\alpha = 1$) imply a slope coefficient on Z_t of $(\sqrt{\xi^{JPY}} - \sqrt{\xi^{USD}})^2 = 0.056$, which is what we recover from the regression in Table 6. The coefficient estimates for other currency pairs also match closely with the structural parameters in Table 3. Therefore, by construction, the extracted risk premium rates reflect variations in the currency options market.

With this caveat in mind, we investigate whether and how the risk premium rates that we extract from the currency options market co-move with bond and stock market fundamentals in the three economies. For this analysis, we collect, from Bloomberg, four sets of economic fundamentals for each of the three economies:[4]

- *Short-term nominal interest rate:* We capture the level of the short-term interest rate using one-week LIBOR rate in each economy.
- *Slope of the interest-rate term structure:* For each economy, the slope of the term structure is defined as the difference between the 10-year swap rate and the one-week LIBOR rate.
- *Interest-rate cap volatility:* We proxy interest-rate volatility using the at-the-money implied volatility underlying the one-year interest-rate cap contract in each economy.
- *Stock index option volatility:* The stock market volatility is taken to be one-month at-the-money option implied volatility on a major stock index in each economy: the S&P 500 Index (SPX) for the US, the Nikkei-225 Stock Average (NKY) for Japan, and the FTSE 100 Index (UKX) for the UK.

With the four sets of economic fundamentals, we first regress the risk premium rate (Y_t^h) in an economy h on each of the four economic fundamentals ($F_t^{j,h}$) in the same economy,

$$Y_t^h = \vartheta_0 + \vartheta_j F_t^{j,h} + e_t, \quad \Delta Y_t^h = \vartheta_0 + \vartheta_j \Delta F_t^{j,h} + e_t, \qquad h = \text{USD, JPY, GBP}, \quad j = 1,2,3,4, \tag{34}$$

where the regression is performed on both levels and weekly differences. The slope coefficient, ϑ_j, measures how the country-specific risk premium rate co-moves with the j-th economic fundamental variable in that economy. Table 7 reports the GMM estimates and t-statistics of the slope coefficients on each of the four economic variables in each of the three economies in panels A (on levels) and B (on weekly differences). In computing the weighting matrix for the GMM estimation, we follow Newey and West (1987) with 12 lags for the level regressions and four lags for regressions on weekly changes.

[4]Similar economic variables have been used in, for example, Andersen, Bollerslev, Diebold, and Vega (2003), Ang and Piazzesi (2003), Campbell and Shiller (1991), Campbell and Ammer (1993), Cochrane (1991), Cochrane and Piazzesi (2006), Collin-Dufresne and Goldstein (2002), Collin-Dufresne, Goldstein, and Martin (2001), Dumas and Solnik (1995), Fama and Bliss (1987), and Ilmanen (1995).

The estimates share several common features among the three economies. First, a rise in the short-term interest rate is associated with a fall in the country-specific risk premium rate. The coefficient estimates are negative for all six regressions, and significantly so for both Japan and the UK in both level and weekly change regressions. On the other hand, the coefficient estimates on the slope of the interest-rate term structure are mostly positive, significantly so for the UK. Therefore, the overall responses of the country risk premium rates to the level and the slope of the term structure are consistent with economic intuition (Ang and Piazzesi (2003), Campbell and Shiller (1991), Fama and Bliss (1987) and Ilmanen (1995)). The country risk premium increases when the short-term interest rate drops and the yield curve steepens.

The country-specific risk premium rate increases with volatilities in both the interest-rate and the stock markets in that economy. In the level regressions, the coefficient estimates are all positive on interest-rate cap volatilities and significantly so for the US economy. The coefficient estimate on the US stock market volatility is also positive and statistically significant. In weekly difference regressions, the coefficient estimates are positive for both volatility variables and in all three economies, and the estimates are statistically significant for the stock market volatilities in Japan and the UK. These positive coefficient estimates are economically sensible: With fixed market price of risk, we expect the country-specific risk premium rate to increase with the risk level in that economy.

To explain the global risk premium rate, we first compute an average across the three economies on each set of economic fundamentals to create a "global" fundamental (e.g., Fama and French (1998) and Griffin (2002)):

$$F_t^{j,global} \equiv 0.65F_t^{j,USD} + 0.25F_t^{j,JPY} + 0.10F_t^{j,GBP}, \quad j = 1,2,3,4, \tag{35}$$

where the weighting corresponds roughly to the relative GNP of each economy. We have also experimented with alternative weighting schemes and obtained similar results. We regress the global risk premium rate on each of the four global fundamentals, again on both levels and weekly differences:

$$Z_t = \vartheta_0 + \vartheta_j F_t^{j,global} + e_t, \qquad \Delta Z_t = \vartheta_0 + \vartheta_j \Delta F_t^{j,global} + e_t, \quad j = 1,2,3,4. \tag{36}$$

The estimation results are reported in the last two columns of Table 7. The coefficient estimates on the global risk premium rates often take on different signs from the corresponding estimates on the country-specific risk premium rates, suggesting that investors respond to global and country-specific shocks differently. In particularly, although the country-specific risk premium rate increases with the financial market volatility in

the same economy, the global risk premium rate declines with the average volatility of the three economies.[5]

For robustness check, we also regress the risk premium rates on the four sets of economic fundamentals in one multivariate regression. Panels C and D in Table 7 show that the coefficient estimates are largely consistent with those from the univariate regressions. The adjusted-R^2 goodness-of-fit statistics range between 16.03% and 46.82% when the estimation is performed on levels, and between 1.07% and 7.12% when the estimation is performed on weekly differences. As expected, it is far more difficult to explain changes in the risk premium than the risk premium levels. Overall, the variations of the risk premium rates that we extract from the currency options market appear consistent with movements in bond and stock markets in the three economies.

6. Conclusions

In this paper, we propose to infer the multi-dimensional dynamic behaviors of the stochastic discount factors in international economies from the time-series of returns and options on three currency pairs that form a triangular relation. We develop a class of models for stochastic discount factors that are sufficiently flexible to capture the observed behaviors of currency returns and currency options. Through model estimation, we investigate whether investors show a differential response to country-specific risks versus global risks, and to upside jumps versus downside jumps.

Our estimation results show that the average risk premium in Japan is about 50% larger than the average risk premium in the US or the UK. The asymmetry between the three economies enables us to identify both the global risk factor and the country-specific risk factors and their associated risk premium dynamics. We also find that the risk premium rate on the global risk factor is both more persistent and more volatile than the risk premium rates on the country-specific risks, suggesting a high degree of international risk sharing among the three economies. Furthermore, investors react differently to shocks to the global risk factor and the country-specific risk factors. Investors demand a higher risk premium when the economy receives a negative shock that is country-specific, but demand a lower premium when the negative shock is global. Hence, the risk premium in an economy increases only when the wealth of the economy declines relative to the global economy.

[5]The different responses of global and country-specific risk premiums to economic fundamentals have potentially important implications for currency return predictability (Bekaert and Hodrick (1992), Mark (1995), Evans and Lyons (2002), and Engel and West (2005)). Specifically, if we regress currency excess returns on economic fundamentals without differentiating the global from the country-specific component, the slope estimates are likely to be insignificant as the sensitivities of global and country-specific risk components cancel. A possible direction to improve currency return predictability is to separate the global and country-specific components in the estimations.

Our estimation shows that jumps in each economy are not rare events, but arrive very frequently. However, investors only price downside jumps while ignoring upside jumps. Finally, the risk premiums that we extract from the currency and its options market are economically compatible with movements in economic fundamentals in the bond and stock market. Nevertheless, global and country-specific risk premium rates often respond differently to economic shocks, highlighting the importance of separating global from country-specific shocks in predicting currency risk premiums.

Overall, currency returns and currency options prove to be important information sources for identifying the multi-dimensional behaviors of the stochastic discount factors in international economies. Our analysis also shows that it is important to differentiate between global and country-specific risks and to distinguish between upside versus downside jumps in understanding investor behaviors and predicting risk premium variations.

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Table 1
Risk reversals and butterfly spreads

Each maturity has four set of volatility quotes in the form of ten-delta risk-reversal (denoted RR[10]), 25-delta risk-reversal (denoted RR[25]), ten-delta butterfly spread (denoted BF[10]), and 25-delta butterfly spread (denoted BF[25]), all as percentages of the corresponding at-the-money implied volatility (SIV). Each row represents a single maturity. The first column denotes the option maturity, with 'w' denoting weeks and 'm' denoting months. Reported are the mean, the standard deviation, and the *t*-statistics on the significance of the sample mean for each risk-reversal and butterfly spread series. The *t*-statistics adjust serial dependence according to Newey and West (1987), with the number of lags optimally chosen according to Andrews (1991) based on an AR(1) specification. Data are weekly from November 7, 2001 to January 28, 2004.

Mat.	RR[10]			RR[25]			BF[10]			BF[25]		
						JPYUSD						
1w	11.63	13.81	3.34	6.45	7.59	3.35	13.65	3.84	11.77	3.40	0.74	15.50
1m	12.53	13.64	3.20	6.94	7.56	3.20	13.90	3.40	12.75	3.57	0.62	18.42
2m	13.91	14.89	2.85	7.55	8.08	2.83	14.49	2.93	14.81	3.70	0.53	21.80
3m	14.47	15.78	2.59	7.86	8.61	2.58	14.91	2.56	17.18	3.79	0.47	25.42
6m	15.30	17.98	2.21	8.23	9.74	2.20	15.43	2.20	19.71	4.02	0.38	31.52
9m	15.79	19.41	2.08	8.45	10.36	2.08	16.23	2.04	21.75	4.13	0.40	29.23
12m	16.19	20.47	2.00	8.63	10.94	2.00	16.55	2.03	21.78	4.18	0.43	27.25
						GBPUSD						
1w	5.86	8.07	2.93	3.26	4.42	2.98	9.74	2.65	11.11	2.82	0.59	15.90
1m	5.73	7.08	2.79	3.21	3.93	2.86	9.79	2.39	10.98	2.83	0.55	14.79
2m	5.51	6.32	2.81	3.19	3.60	2.94	9.55	2.12	11.56	2.76	0.48	15.91
3m	5.30	5.81	2.79	3.01	3.25	2.90	9.64	1.68	15.46	2.71	0.42	17.74
6m	4.87	5.40	2.25	2.75	2.97	2.32	9.53	1.15	25.83	2.47	0.46	13.75
9m	4.80	5.27	2.16	2.72	2.91	2.19	9.49	0.99	29.88	2.46	0.42	13.89
12m	4.68	5.30	2.01	2.67	2.89	2.09	9.37	0.91	32.86	2.42	0.41	15.14
						GBPJPY						
1w	-5.85	12.08	-1.73	-3.18	6.58	-1.72	11.09	2.56	17.06	2.95	0.80	14.38
1m	-6.42	12.32	-1.70	-3.51	6.69	-1.71	11.51	2.16	20.36	3.17	0.48	26.87
2m	-6.32	12.48	-1.62	-3.41	6.68	-1.62	12.02	2.12	19.55	3.31	0.45	28.28
3m	-6.02	12.57	-1.52	-3.28	6.74	-1.54	12.44	2.13	18.19	3.44	0.43	28.59
6m	-5.76	12.62	-1.43	-3.12	6.80	-1.43	13.07	2.00	18.26	3.54	0.49	21.35
9m	-5.72	12.75	-1.40	-3.08	6.86	-1.39	13.47	2.16	16.51	3.65	0.60	16.67
12m	-5.70	13.01	-1.35	-3.06	6.98	-1.35	13.64	2.11	16.83	3.69	0.63	15.74

Table 2
Risk and pricing in strictly symmetric economies

Entries report the maximum likelihood estimates of the structural parameters and their standard errors (in parentheses) for the models admitting stochastic currency risk premium and stochastic skewness under strict symmetry. Four separate models are estimated that respectively allow the power coefficient, α, in the dampened power law specification for the jump component to take values of $\alpha = -1$, $\alpha = 0$, $\alpha = 1$, and α unrestricted. Estimation is based on weekly currency return and currency options data from November 7, 2001 to January 28, 2004 (117 weekly observations for each series). The last row reports the maximized log likelihood value. σ_r^2 represents the variance of the measurement error.

Θ	$\alpha = -1$		$\alpha = 0$		$\alpha = 1$		Free α	
	Country-specific risk premium rate dynamics:							
κ_Y	2.149	(0.108)	1.912	(0.096)	1.531	(0.053)	1.210	(0.081)
θ_Y	0.003	(0.000)	0.003	(0.000)	0.004	(0.000)	0.001	(0.014)
ω_Y	0.149	(0.010)	0.150	(0.009)	0.148	(0.008)	0.081	(0.486)
ρ_Y	-0.252	(0.054)	-0.321	(0.048)	-0.412	(0.046)	-0.898	(5.433)
	Country-specific jump risk structure:							
λ	17.684	(1.589)	5.255	(0.500)	1.184	(0.392)	0.747	(9.170)
β_-	4.623	(0.117)	4.146	(0.078)	3.835	(1.032)	4.420	(4.146)
β_+	43.513	(6.9e2)	58.234	(4.4e2)	97.645	(3.7e2)	3.1e4	(4.5e6)
α	-1	—	0	—	1	—	1.810	(0.403)
	Performance metrics:							
σ_r^2	0.336	(0.004)	0.334	(0.004)	0.329	(0.004)	0.324	(0.005)
$\mathcal{L}/T$	1.62		1.58		1.67		1.93	

Table 3
Risk and pricing in proportionally asymmetric economies

Entries report the maximum likelihood estimates of the structural parameters and their standard errors (in parentheses) for the models admitting stochastic currency risk premium and stochastic skewness under proportional asymmetry. Four separate models are estimated that respectively allow the power coefficient, α, in the dampened power law specification for the jump component to take values of $\alpha = -1$, $\alpha = 0$, $\alpha = 1$, and α unrestricted. Estimation is based on weekly currency return and currency options data from November 7, 2001 to January 28, 2004. The last row reports the maximized average daily log likelihood value. σ_r^2 represents the variance of the measurement error.

Θ	$\alpha = -1$		$\alpha = 0$		$\alpha = 1$		Free α	
	Average risk premiums:							
ξ^{JPY}	1.507	(0.027)	1.508	(0.028)	1.531	(0.035)	1.531	(0.034)
ξ^{GBP}	1.017	(0.005)	1.016	(0.006)	1.007	(0.006)	1.007	(0.005)
	Global risk premium rate dynamics:							
κ_Z	0.000	(0.006)	0.000	(0.006)	0.000	(0.006)	0.000	(0.005)
θ_Z	0.230	(0.069)	0.231	(0.065)	0.356	(0.220)	0.357	(0.223)
ω_Z	0.807	(0.069)	0.797	(0.069)	0.815	(0.053)	0.813	(0.050)
ρ_Z	0.650	(0.059)	0.626	(0.059)	0.521	(0.034)	0.524	(0.035)
	Country-specific risk premium rate dynamics:							
κ_Y	5.204	(0.190)	4.921	(0.210)	3.061	(0.059)	3.053	(0.061)
θ_Y	0.003	(0.000)	0.003	(0.000)	0.003	(0.006)	0.003	(0.001)
ω_Y	0.183	(0.006)	0.174	(0.006)	0.137	(0.163)	0.138	(0.016)
ρ_Y	-0.702	(0.046)	-0.713	(0.048)	-0.996	(1.185)	-0.999	(0.115)
	Country-specific jump risk structure:							
λ	18.698	(9.146)	5.659	(1.428)	20.489	(54.032)	815.387	(7.8e3)
β_-	5.132	(0.936)	4.523	(0.686)	36.767	(9.842)	63.069	(59.324)
β_+	1.2e2	(4.3e4)	1.4e2	(7.8e3)	2.5e3	(6.9e5)	4.7e4	(6.9e4)
α	-1	—	0	—	1	—	0.227	(2.205)
	Performance metrics:							
σ_r^2	0.174	(0.002)	0.175	(0.002)	0.167	(0.003)	0.167	(0.002)
$\mathcal{L}/T$	32.97		32.84		33.96		34.10	

Table 4
Risk and pricing in one-sided jump economies

Entries report the maximum likelihood estimates of the structural parameters and their standard errors (in parentheses) for the models admitting stochastic currency risk premium and stochastic skewness under proportional asymmetry and assuming only negative jumps. Four separate models are estimated that respectively allow the power coefficient, α, in the dampened power law specification for the jump component to take values of $\alpha = -1$, $\alpha = 0$, $\alpha = 1$, and α unrestricted. Estimation is based on weekly currency return and currency options data from November 7, 2001 to January 28, 2004. The last row reports the maximized average daily log likelihood value. σ_r^2 represents the variance of the measurement error.

Θ	$\alpha = -1$		$\alpha = 0$		$\alpha = 1$		Free α	
	Average risk premiums:							
ξ^{JPY}	1.507	(0.026)	1.509	(0.027)	1.531	(0.034)	1.530	(0.034)
ξ^{GBP}	1.017	(0.005)	1.016	(0.006)	1.007	(0.005)	1.008	(0.005)
	Global risk premium rate dynamics:							
κ_Z	0.000	(0.005)	0.000	(0.005)	0.000	(0.006)	0.000	(0.006)
θ_Z	0.230	(0.066)	0.231	(0.060)	0.357	(0.196)	0.348	(0.289)
ω_Z	0.807	(0.069)	0.797	(0.068)	0.814	(0.051)	0.805	(0.051)
ρ_Z	0.650	(0.058)	0.626	(0.058)	0.521	(0.034)	0.529	(0.035)
	Country-specific risk premium rate dynamics:							
κ_Y	5.203	(0.185)	4.924	(0.200)	3.053	(0.046)	3.034	(0.065)
θ_Y	0.003	(0.000)	0.003	(0.000)	0.003	(0.000)	0.003	(0.001)
ω_Y	0.183	(0.006)	0.174	(0.006)	0.137	(0.011)	0.138	(0.018)
ρ_Y	-0.702	(0.042)	-0.713	(0.045)	-0.996	(0.094)	-0.999	(0.129)
	Country-specific jump risk structure:							
λ	18.698	(9.137)	5.658	(1.408)	21.199	(10.585)	8.8e2	(9.4e3)
β_-	5.132	(0.935)	4.526	(0.690)	37.329	(9.718)	66.157	(70.052)
α	-1	—	0	—	1	—	0.240	(2.428)
	Performance metrics:							
σ_r^2	0.174	(0.002)	0.175	(0.002)	0.167	(0.003)	0.167	(0.002)
$\mathcal{L}/T$	32.97		32.84		33.96		34.10	

Table 5
Unconditional risk sharing index estimates

Entries report the risk sharing index computed according to the following representation:

$$\text{RSI} \quad = \quad 1-\frac{\left(\sqrt{\xi^h}-\sqrt{\xi^f}\right)^2\theta_Z+\theta_Y\left(\xi^h+\xi^f\right)\left(1+\lambda\Gamma\left[2-\alpha\right]\left(\beta_+^{\alpha-2}+\beta_-^{\alpha-2}\right)\right)}{\left(\xi^h+\xi^f\right)\theta_Z+\theta_Y\left(\xi^h+\xi^f\right)\left(1+\lambda\Gamma\left[-\alpha\right]\left(\beta_+^{\alpha-2}+\beta_-^{\alpha-2}\right)\right)},$$

where the structural parameters estimates are from Table 3 for the four asymmetric models with different jump power coefficients α.

Country pairs	US-Japan	US-UK	Japan-UK
$\alpha=-1$	0.9625	0.9827	0.9641
$\alpha=0$	0.9628	0.9832	0.9643
$\alpha=1$	0.9670	0.9890	0.9677
$\alpha=0.227$	0.9671	0.9891	0.9678

Table 6
Linking risk premium rates to currency option implied volatilities

Entries report the parameter estimates and t-statistics (in brackets) of the following relation,

$$\Delta(\mathrm{SIV}_t^{hf})^2 \quad = \quad b_0 + b_1 \Delta Z_t + b_2 \Delta Y_t^h + b_3 \Delta Y_t^f + e_t,$$

where Δ denotes weekly changes, SIV_t^{hf} denotes the one-month delta-neutral straddle implied volatility on a currency pair, and (Z_t, Y_t^h, Y_t^f) denote the global, home, and foreign risk premium rates, respectively. All risk premiums are extracted from the estimated asymmetric model with $\alpha = 1$. The relation is estimated with generalized methods of moments, where the weighting matrix is calculated according to Newey and West (1987) with four lags.

Currency	b_0		b_1		b_2		b_3		R^2
JPYUSD	0.000	[0.30]	0.056	[26.17]	1.593	[8.97]	2.146	[16.26]	95.43%
GBPUSD	0.000	[0.47]	0.000	[0.00]	1.396	[20.39]	1.347	[21.53]	93.74%
GBPJPY	0.000	[0.56]	0.054	[25.25]	1.815	[23.05]	1.574	[13.25]	95.27%

Table 7
Risk premiums and economic fundamentals

Entries report the coefficient estimates and t-statistics (in brackets) of the following regressions:

$$\begin{aligned}
\text{Panel A:} \quad V_t^h &= \vartheta_0 + \vartheta_j F_t^{j,h} + e_t, \\
\text{Panel B:} \quad \Delta V_t^h &= \vartheta_0 + \vartheta_j \Delta F_t^{j,h} + e_t, \\
\text{Panel C:} \quad V_t^h &= \vartheta_0 + \textstyle\sum_{j=1}^{4} \vartheta_j F_t^{j,h} + e_t, \\
\text{Panel D:} \quad \Delta V_t^h &= \vartheta_0 + \textstyle\sum_{j=1}^{4} \vartheta_j \Delta F_t^{j,h} + e_t,
\end{aligned}$$

where $V_t^h \equiv [Y_t^{USD}, Y_t^{JPY}, Y_t^{GBP}, Z_t]$ denotes the country-specific risk premium rate for each economy and the global risk premium rate, all extracted from the asymmetric model with $\alpha = 1$, and $F_t^{j,h} \equiv [F_t^{j,USD}, Y_t^{j,JPY}, F_t^{j,GBP}, F_t^{j,global}]$ denotes the j-th economic fundamental variable in each economy h, with global fundamental $F_t^{j,global}$ created as a weighted average: $F_t^{j,global} = 0.65F_t^{j,USD} + 0.25F_t^{j,JPY} + 0.10F_t^{j,GBP}$. We estimate each equation using GMM, where the weighting matrix is calculated according to Newey and West (1987) with four lags for regressions on weekly differences and 12 lags for level regressions.

Panel A: Univariate regression in levels

$F^{j,h}$	US		Japan		UK		Global	
Short-term interest rate	-0.006	[-0.13]	-3.710	[-3.38]	-0.121	[-2.76]	10.681	[2.64]
Slope of the term structure	-0.051	[-1.89]	-0.052	[-0.92]	0.067	[1.53]	4.444	[1.92]
Interest-rate cap volatility	0.004	[3.93]	0.001	[1.54]	0.002	[0.29]	-0.195	[-2.38]
Stock index option volatility	0.004	[2.25]	-0.001	[-0.10]	-0.001	[-0.63]	0.005	[0.04]

Panel B: Univariate regression in weekly differences

$F^{j,h}$	US		Japan		UK		Global	
Short-term interest rate	-0.006	[-0.14]	-0.281	[-2.20]	-0.025	[-2.20]	-0.917	[-0.34]
Slope of the term structure	0.010	[0.60]	0.002	[0.06]	0.025	[2.48]	1.001	[0.77]
Interest-rate cap volatility	0.000	[0.79]	0.001	[1.14]	0.004	[1.44]	-0.015	[-0.83]
Stock index option volatility	0.003	[1.71]	0.006	[3.03]	0.002	[2.38]	-0.015	[-0.48]

Panel C: Multivariate regression in levels

	US		Japan		UK		Global	
Intercept	0.000	[0.05]	0.003	[1.64]	0.005	[3.51]	-0.124	[-1.93]
Short-term interest rate	-0.003	[-0.08]	-3.077	[-1.80]	-0.107	[-1.86]	10.003	[2.84]
Slope of the term structure	0.002	[0.10]	-0.027	[-0.52]	0.016	[0.29]	2.216	[1.89]
Interest-rate cap volatility	0.004	[2.47]	0.001	[0.89]	0.008	[1.27]	-0.044	[-1.03]
Stock index option volatility	0.002	[1.24]	0.003	[0.74]	-0.001	[-0.60]	-0.102	[-0.80]
Adjusted-R^2	30.72%		16.03%		16.20%		46.82%	

Panel D: Multivariate regression in weekly differences

	US		Japan		UK		Global	
Intercept	0.000	[0.09]	0.000	[0.74]	0.000	[0.48]	-0.001	[-0.83]
Short-term interest rate	0.018	[0.37]	-0.407	[-2.86]	-0.009	[-0.15]	-0.196	[-0.09]
Slope of the term structure	0.026	[1.60]	0.037	[1.14]	0.016	[0.29]	0.984	[0.81]
Interest-rate cap volatility	0.000	[0.16]	0.001	[1.27]	0.001	[0.27]	-0.009	[-0.33]
Stock index option volatility	0.003	[1.69]	0.007	[3.51]	0.002	[1.95]	0.011	[0.37]
Adjusted-R^2	4.20%		5.57%		7.12%		1.07%	

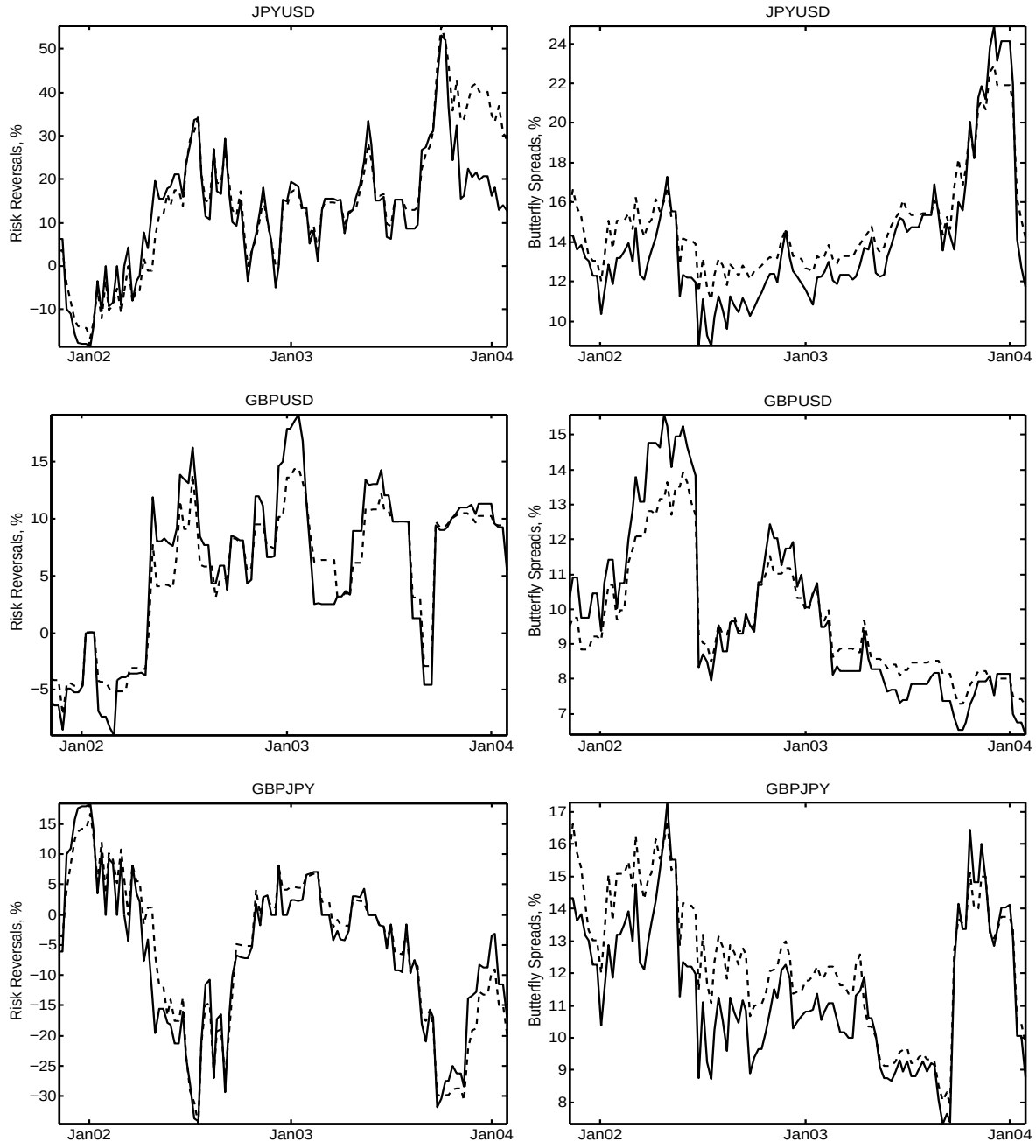

Fig. 1. Time variation in risk reversals and butterfly spreads. Left panels plot the time-series of ten-delta risk-reversals and the right panels plot the time-series of ten-delta butterfly spreads, both as percentages of the corresponding at-the-money implied volatility. The two lines correspond to distinct option maturities of one month (solid line) and three months (dashed line). Data are weekly from November 7, 2001 to January 28, 2004.