Variance Risk Premia

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Abstract

We propose a direct and robust method for quantifying the variance risk premium on financial assets. We show that the risk-neutral expected value of return variance, also known as the variance swap rate, is well approximated by the value of a particular portfolio of options. We propose to use the difference between the realized variance and this synthetic variance swap rate to quantify the variance risk premium. Using a large options data set, we synthesize variance swap rates and investigate the historical behavior of variance risk premia on five stock indexes and 35 individual stocks.

Variance Risk Premia

It has been well-documented that return variance is stochastic. When investing in a security, an investor faces at least two sources of uncertainty, namely the uncertainty about the return as captured by the return variance, and the uncertainty about the return variance itself. It is important to know how investors deal with the uncertainty in return variance to effectively manage risk and allocate assets, to accurately price and hedge derivative securities, and to understand the behavior of financial asset prices in general.

We develop a direct and robust method for quantifying the return variance risk premium on an asset using the market prices of options written on this asset. Our method uses the notion of a variance swap, which is an over-the-counter contract that pays the difference between a standard estimate of the realized variance and the fixed variance swap rate. Since variance swaps cost zero to enter, the variance swap rate represents the risk-neutral expected value of the realized variance. We show that the variance swap rate can be synthesized accurately by a particular linear combination of option prices. We propose to use the difference between the ex post realized variance and this synthetic variance swap rate to quantify the variance risk premium.

Using a large options data set, we synthesize variance swap rates using options data on five stock indexes and 35 individual stocks during the past seven years. We compare the synthetic variance swap rates to realized variance, and study the historical behaviors of variance risk premia on different assets.

We find that the average variance risk premia are strongly negative for the S&P 500 and 100 indexes and for the Dow Jones Industrial Average. The estimates on individual stocks show large cross-sectional variation. We conjecture that there exists a common stochastic variance risk factor in the stock market that asks for a highly negative risk premium. When we use the variance on the S&P 500 index as a proxy for this common variance risk factor and estimate a variance beta for each stock by regressing the stock's return variance on the index variance, we find that the variance risk premia are more negative for stocks with higher variance beta. The negative sign on the variance risk premia indicates that variance buyers are willing to accept a negative average excess return to hedge away upward movements in stock market volatility. In other words, investors regard increases in market volatility as unfavorable shocks to the investment opportunity.

Return variance varies stochastically either due to its correlation with the stock price or return (e.g., the constant elasticity of variance model of Cox (1996) and the local volatility model of Dupire (1994) and Derman and Kani (1994)), or due to its independent variation as a separate source of risk (e.g., the stochastic volatility models of Heston (1993) and Hull and White (1987)), or both. Accordingly, variance risk premia can come from either its correlation with the return risk and return risk premium, or a separate premium on the independent variance variation, or both. We investigate whether the classic capital asset pricing model can explain the negative variance risk premia. We find that the negative correlation between stock index returns and the return variance generates a strongly negative beta, but this negative beta only explains a small portion of the negative variance risk premia. Other risk factors identified by the recent literature, such as size, book-to-market, and momentum, cannot explain the strongly negative variance risk premia, either. Therefore, we conclude that the majority of the market variance risk premium is generated by an independent variance risk factor.

We also analyze the dynamics of the variance risk premia by formulating expectation hypothesis regressions. Under the null hypothesis of constant variance risk premia, a regression of the realized variance on the variance swap rate generates a slope estimate of one. However, the slope estimates from our regressions are significantly lower than one for the S&P and Dow indexes, and also for many of the individual stocks, suggesting that the market variance risk premia are time-varying and correlated with the variance swap rate. Nevertheless, when we regress the log realized variance on the log variance swap rate, the slope estimates are much closer to one, suggesting that although the log variance risk premia are strongly negative, they are not strongly correlated with the logarithm of the variance swap rate.

We check the robustness of our results from several aspects. First, we use numerical analysis to gauge the magnitude of approximation errors in synthesizing the variance swap rates due to jumps and discretization. We find that under commonly used models and model parameters, the approximation errors from the two sources are small. Second, we measure the impacts of the options bid-ask spreads on the variance risk premia estimates, and find that the variance risk premia on S&P and Dow indexes remain strongly negative, regardless of whether we synthesize the variance swap rates using bid, mid, or ask option prices. Third, we evaluate the error-in-variable issue in our expectation hypothesis regressions. We find that measurement errors in the synthetic variance swap rate do bias our slope estimates toward zero, but that our general conclusions remain valid after correcting for the biases: The market variance risk premia are time-varying and correlated with the variance swap rate when defined in dollar terms, but become closer to an independent series when defined in log returns. Finally, we divide our data into two subsample periods, with one corresponding broadly to a bullish market and the other to a bearish market. We find that the variance risk premia on stock indexes are significantly negative under both bullish and bearish market conditions.

In related works, Bakshi and Kapadia (2003a,b) consider the profit and loss arising from delta-hedging a long position in a call option. They argue that this profit and loss is approximately neutral to the directional movement of the underlying asset return, but is sensitive to the movement in the return volatility. Thus, by analyzing the profit and loss from these delta-hedged positions, they can infer useful qualitative properties for the variance risk premia without referring to a specific model. Our approach maintains and enhances the robustness of their model-free approach, as we provide a quantitative measure of the variance risk premia. As a result, we can analyze not only the sign, but also the quantitative properties of the premia.

Bates (1996, 2000, 2003), Eraker (2004), Jones (2003), and Pan (2002) analyze variance risk premia in conjunction with return risk premia by estimating various parametric option pricing models with either Bayesian methods or efficient methods of moments. Most recently, Aït-Sahalia and Kimmel (2007) propose a maximum likelihood method for estimating stochastic volatility dynamics and volatility risk premia based on closed-form approximations (developed in Aït-Sahalia (2002, 2007)) to the true likelihood function of the joint observations on the underlying asset and option prices. Wu (2005) propose to estimate the variance dynamics and variance risk premia without specifying the return dynamics using realized variance estimators from high-frequency return data and variance swap rates synthesized from option prices. Bollerslev, Gibson, and Zhou (2004) construct a risk aversion index using realized variance estimators and the VIX, which approximates the 30-day variance swap rate on the S&P 500 index (Carr and Wu (2006)).

Ang, Hodrick, Xing, and Zhang (2004) form stock portfolios ranked by their sensitivity to volatility risk and analyze the difference among these different stock portfolios. From the analysis, they infer the impact of volatility risk on the expected stock return. Coval and Shumway (2001) study how returns on option investment vary with strike choices and whether the classic capital asset pricing theory can explain the returns. Bondarenko (2004) links the market price of variance risk to hedge fund behavior.

Our empirical analysis of the variance risk premia is based on our theoretical work on synthesizing a variance swap using European options and futures contracts. Carr and Madan (1998), Demeterfi, Derman, Kamal, and Zou (1999a,b), and Britten-Jones and Neuberger (2000) use the same replicating strategy, but under the assumption of continuity in the underlying asset price. Jiang and Tian (2004) extend the result to a jump-diffusion stochastic volatility model. Our derivation is under the most general setting possible. We also quantify the approximation error induced by jumps. Most importantly, we exploit the theoretical developments in synthesizing variance swaps for variance risk premia analysis.

The remainder of the paper is organized as follows. Section 1 lays out the theoretical foundation on how we synthesize the variance swap from vanilla options and how we infer the variance risk premia based on the difference between the synthetic variance swap rate and the realized return variance. Section 2 describes the data and the methodologies that we use to synthesize the variance swap rates and to calculate the realized variance and variance risk premia. Section 3 investigates the historical behavior of the variance risk premia. Section 4 performs robustness analysis. Section 5 concludes.

1. Variance Swap Rates and Variance Risk Premia

A return variance swap has zero net market value at entry. At maturity, the payoff to the long side of the swap is equal to the difference between the realized variance over the life of the contract and a constant called the *variance swap rate*:

$$[RV_{t,T} - SW_{t,T}]\,L, \tag{1}$$

where $RV_{t,T}$ denotes the realized annualized return variance between time t and T, $SW_{t,T}$ denotes the fixed variance swap rate that is determined at time t and paid at time T, and L denotes the notional dollar amount that converts the variance difference into a dollar payoff. No arbitrage dictates that the variance swap rate equals the risk-neutral expected value of the realized variance,

$$SW_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}], \tag{2}$$

where $\mathbb{E}_t^{\mathbb{Q}}[\cdot]$ denotes the time-t conditional expectation operator under some risk-neutral measure $\mathbb{Q}$.

1.1. Synthesizing variance swap rates from options

We use S_t to denote the time-t spot price of an asset, and F_t its time-t futures price of maturity $T > t$. We assume that the futures contract marks to market continuously. No arbitrage dictates that there exists a risk-neutral probability measure $\mathbb{Q}$ defined on a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ such that the futures price F_t solves the following stochastic differential equation:

$$dF_t = F_{t-}\sigma_{t-}dW_t + \int_{\mathbb{R}^0} F_{t-}(e^x - 1)[\mu(dx,dt) - \nu_t(x)dxdt], \tag{3}$$

where W_t is a $\mathbb{Q}$-standard Brownian motion, $\mathbb{R}^0$ denotes the real line excluding zero, F_{t-} denotes the futures price just prior to any jump at time t, and the random counting measure $\mu(dx,dt)$ realizes to a nonzero value for a given x if and only if the futures price jumps from F_{t-} to $F_t = F_{t-}e^x$ at time t. The process $\nu_t(x)$ compensates the jump process so that the last term in equation (3) is the increment of a $\mathbb{Q}$-pure jump martingale. Therefore, equation (3) models the futures price change as the summation of the increments of two orthogonal martingales, a purely continuous martingale and a purely discontinuous (jump) martingale. This decomposition is generic for any continuous-time martingales (Jacod and Shiryaev (1987)). To avoid notational complexity, we assume that the jump process exhibits finite variation: $\int_{\mathbb{R}^0}(|x| \wedge 1)\nu_t(x)dx < \infty$.

The time subscripts on σ_{t-} and $\nu_t(x)$ indicate that both are stochastic and predictable with respect to the filtration $\mathcal{F}_t$. We further restrict σ_{t-} and $\nu_t(x)$ so that the futures price F_t is always positive. Finally, we assume deterministic interest rates so that the futures price and the forward price are identical. So long as futures contracts trade, we need no assumptions on dividends.

Under the specification in equation (3), the quadratic variation on the futures return from time t to T is,

$$V_{t,T} = \int_t^T \sigma_{s_-}^2 ds + \int_t^T \int_{\mathbb{R}^0} x^2 \mu(dx, ds). \tag{4}$$

The annualized return variance is $RV_{t,T} = \frac{1}{T-t} V_{t,T}$. We show that this return quadratic variation can be replicated up to a higher-order error term by a static position in a portfolio of options of the same horizon T and a dynamic position in futures. As the risk-neutral expected value of futures trading is zero, the risk-neutral expected value of the quadratic variation can be approximated by the value of the options in the static portfolio.

Proposition 1 *Under no arbitrage, the time-t risk-neutral expected value of the return quadratic variation of an asset over horizon $[t,T]$ defined in (4) can be approximated by the continuum of European out-of-the-money option prices across all strikes $K > 0$ and at the same maturity date T:*

$$\mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}] = \frac{2}{T-t} \int_0^\infty \frac{\Theta_t(K,T)}{B_t(T) K^2} dK + \varepsilon, \tag{5}$$

where $B_t(T)$ denotes the time-t price of a bond paying one dollar at T, $\Theta_t(K,T)$ denotes the time-t value of an out-of-the-money option with strike price $K > 0$ and maturity $T \geq t$ (a call option when $K > F_t$ and a put option when $K \leq F_t$), and ε denotes the approximation error, which is zero when the futures price process is purely continuous. When the futures price can jump, the approximation error ε is of order $O((\frac{dF_t}{F_{t-}})^3)$ and is determined by the compensator of the discontinuous component,

$$\varepsilon = \frac{-2}{T-t} \mathbb{E}_t^{\mathbb{Q}} \int_t^T \int_{\mathbb{R}^0} \left[e^x - 1 - x - \frac{x^2}{2} \right] \nu_s(x) dx ds. \tag{6}$$

Refer to Appendix A for the derivation. Equation (5) serves our theoretical basis for inferring variance swap rates from vanilla options.

1.2. Quantifying variance risk premia using variance swap rates and realized variance

Using $\mathbb{P}$ to denote the statistical probability measure, we link the variance swap rate to the realized variance through the following valuation equation,

$$SW_{t,T} = \frac{\mathbb{E}_t^{\mathbb{P}}[M_{t,T}RV_{t,T}]}{\mathbb{E}_t^{\mathbb{P}}[M_{t,T}]} = \mathbb{E}_t^{\mathbb{P}}[m_{t,T}RV_{t,T}], \tag{7}$$

where $M_{t,T}$ denotes a pricing kernel and $m_{t,T} = M_{t,T}/\mathbb{E}_t^{\mathbb{P}}[M_{t,T}]$. No arbitrage guarantees the existence of at least one such pricing kernel that prices all traded assets (Duffie (1992)).

Equation (7) can be decomposed into two terms:

$$SW_{t,T} = \mathbb{E}_t^{\mathbb{P}}[m_{t,T}RV_{t,T}] = \mathbb{E}_t^{\mathbb{P}}[RV_{t,T}] + \mathrm{Cov}_t^{\mathbb{P}}(m_{t,T}, RV_{t,T}). \tag{8}$$

The first term $\mathbb{E}_t^{\mathbb{P}}[RV_{t,T}]$ represents the time-series conditional mean of the realized variance. The second term captures the conditional covariance between the normalized pricing kernel and the realized variance. The negative of this covariance defines the *return variance risk premium*. Thus, a direct estimate of the average variance risk premium is the sample average of the difference between the variance swap rate and the realized variance, $RP_{t,T} \equiv RV_{t,T} - SW_{t,T}$. This difference also measures the terminal profit and loss from long a variance swap contract and holding it to maturity.

Dividing both sides of equation (8) by $SW_{t,T}$, we can represent the decomposition in excess returns:

$$1 = \mathbb{E}_t^{\mathbb{P}}\left[m_{t,T}\frac{RV_{t,T}}{SW_{t,T}}\right] = \mathbb{E}_t^{\mathbb{P}}\left[\frac{RV_{t,T}}{SW_{t,T}}\right] + \mathrm{Cov}_t^{\mathbb{P}}(m_{t,T}, \frac{RV_{t,T}}{SW_{t,T}}). \tag{9}$$

If we regard $SW_{t,T}$ as the forward cost of a variance swap investment, $(RV_{t,T}/SW_{t,T} - 1)$ captures the excess return from the investment. The sample average of the excess return represents an estimate of the negative of the covariance term in equation (9), hence the risk premium. To make the distribution closer to normality, we represent the excess return in continuously compounded form and label it as the *log variance risk premium*, $LRP_t \equiv \ln(RV_{t,T}/SW_{t,T})$.

2. Data and Methodologies

The options data are from OptionMetrics, which provides historical prices of options based on closing quotes at the Chicago Board of Options Exchange. Our data sample starts in January 1996 and ends in February 2003. From the data set, we filter out market prices of options on five stock indexes and 35 individual stocks. The list of securities is selected mainly based on quote availability. Specifically, we compute the number of valid option quotes on each security in the data sample, and select the securities with the highest number of valid option quotes. In computing the number of valid quotes, we only retain options that have time-to-maturities within one year, and have strictly positive bid quotes and strictly positive bid-ask spreads. Options on some securities are very actively quoted, but only during a short period of our data sample. In selecting our samples, we further require that the number of active days be greater than 900 for stock indexes and 600 for individual stocks. We apply the following criterion to determine the number of active days: (1) The nearest available maturity must be within 90 days. (2) The actual stock price level must be greater than one dollar. (3) The number of strikes is at least three at each of the two nearest maturities. We compute the synthetic variance swap rates only on the active days defined above.

Table 1 lists the five stock indexes and 35 individual stocks in our sample. For each security, the table lists the company name, the starting and ending dates, the number of active days (N), and the average number of strikes (NK) at the chosen maturities. The index options on the S&P 500 index, the Dow Jones Industrial Index, and the Nasdaq-100 index are European options on the spot indexes. Options on the S&P 100 index and the other 35 individual stocks and the QQQ (the Nasdaq-100 tracking stock) are all American options on the underlying spot. The data set includes closing bid and ask quotes for each option contract and the Black-Scholes implied volatilities based on the mid quote. For the European options, implied volatilities are directly inferred from the Black-Scholes option pricing formula. For the American options, OptionMetrics employs a binomial tree approach that takes account of the early exercise premium. The data set also includes the interest rate curve and the projected dividend yield. Our analysis directly employs the implied volatilities provided by OptionMetrics.

[Insert Table 1 about here.]

We choose a 30-day horizon for the synthetic variance swap rates. At each date for each stock, we choose the two nearest maturities, except when the shortest maturity is within eight days. Then we switch to the next two maturities to avoid potential microstructure effects of very short-dated options.

At each maturity, we first linearly interpolate implied volatilities at different moneyness levels, defined as $k \equiv \ln(K/F)$, to obtain a fine grid of implied volatilities. For moneyness below the lowest available moneyness level in the market, we use the implied volatility at the lowest strike price. For k above the highest available moneyness, we use the implied volatility at the highest strike. Using this interpolation and extrapolation procedure, we generate a fine grid of 2,000 implied volatility points with a strike range of ± 8 standard deviations from at-the-money. The standard deviation is approximated by the average implied volatility. Given the fine grid of implied volatilities, we compute the out-of-the-money option prices using the Black-Scholes formula and replicate the variance swap rate according to a discretization of equation (5).

At each date t, we interpolate the synthetic variance swap rates at the two maturities to obtain the variance swap rate at a fixed 30-day horizon. The interpolation is linear in total variance:

$$SW_{t,T} = \frac{1}{T-t}\left[\frac{SW_{t,T_1}(T_1-t)(T_2-T)+SW_{t,T_2}(T_2-t)(T_1-T)}{T_2-T_1}\right], \tag{10}$$

where T_1 and T_2 denote the two maturity dates, and T denotes the interpolated maturity date such that $T-t$ is 30 days. We have experimented with different interpolation schemes, but found that our main conclusions are not materially affected by the particular choice of the interpolation method.

Corresponding to each 30-day variance swap rate, we also compute the annualized 30-day realized variance,

$$RV_{t,t+30} = \frac{365}{30}\sum_{i=1}^{30}\left(\frac{F_{t+i,t+30}-F_{t+i-1,t+30}}{F_{t+i-1,t+30}}\right)^2, \tag{11}$$

where $F_{t,T}$ denotes the time-t forward price with expiry date at time T (in days). A small difference exists between the return variance defined in equation (11) and the quadratic variation in (4) due to the difference between daily monitoring and continuous monitoring. Since the stock prices in the OptionMetrics data set are not adjusted for stock splits, we manually adjust the stock splits for each stock in calculating the realized variance. We have also downloaded stock prices from Bloomberg to check for robustness. Our definition of the realized variance in equation (11) is similar to the definition in most variance swap contracts in the industry. For robustness, we have also computed alternative realized variance measures based on spot prices and demeaned returns. These variations do not alter our conclusions.

Table 2 reports the summary statistics of the annualized realized variance (RV) and the synthetic variance swap rate (SW). The sample averages of the variance swap rates are higher than the average realized variance for all the five stock indexes and most of the individual stocks. The realized variance series are persistent given the overlapping nature of the estimates. The variance swap rates are also highly persistent, reflecting the persistence of the return variance process. Both variance swap rates and the realized variance show positive skewness and positive excess kurtosis for most stocks and indexes.

[Insert Table 2 about here.]

3. Historical Behavior of Variance Risk Premia

To analyze the historical behavior of variance risk premia, we first establish the existence, sign, and average magnitude of the variance risk premia. Then, we investigate whether the risk premia can be explained by classic risk factors. Finally, we analyze the dynamic properties of the premia using expectation hypothesis regressions.

3.1. Do investors price variance risk?

If investors price variance risk, the sample averages of the realized variance will differ from the average variance swap rates. Table 3 reports the summary statistics of the difference between the realized variance and the variance swap rate, $RP = (RV_{t,T} - SW_{t,T}) \times 100$, in panel A and the log difference $LRP = \ln(RV_{t,T}/SW_{t,T})$ in panel B. The variance risk premia RP show large kurtosis and sometimes large skewness. The skewness and kurtosis are much smaller for the log variance risk premia LRP.

[Insert Table 3 about here.]

The sample averages of the variance risk premia and log variance risk premia are negative for all the five stock indexes and most of the individual stocks. Table 3 also reports the t-statistics on the significance of the mean risk premia, adjusted for serial dependence according to Newey and West (1987) with 30 lags. The largest t-statistics come from the S&P 500 and S&P 100 indexes and the Dow Jones Industrial Average, which are strongly significant for both variance risk premia and log variance risk premia. The t-statistics for the Nasdaq-100 index and its tracking stock are lower, but remain strongly significant.[1].

The two definitions of variance risk premia in Table 3 represent two ways of computing returns for variance swap investments. The mean estimates in panel A, $(RV - SW) \times 100$, represent the average dollar profit and loss for each \$100 notational investment in the variance swap contract. Thus, if we long a 30-day variance swap contract with a notional of \$100 on S&P 500 index and hold the contract to maturity, during our sample period the average return per \$100 notional investment is $-$\$2.74.

Alternatively, if we regard the variance swap rate as the forward cost of the variance swap contract, the log variance risk premium $\ln(RV/SW)$ in panel B can be thought of as the continuously compounded excess return to going long the variance swap contract and holding it to maturity. Based on this calculation, the average excess return is -66% for long 30-day variance swap contracts on the S&P 500 index. The different magnitudes in the two panels mainly come from different scaling. Panel A regards the \$100 notional as the initial investment whereas panel B uses the forward cost (i.e., the variance swap rate) as the initial investment. For the S&P 500 index, a \$100 notional corresponds to an average forward cost of \$6.81 (Table 2). For the same dollar profit and loss, the smaller base number generates larger return estimates in panel B.

Despite the different representations, it is clear that investors are willing to accept a significantly negative average return to long variance swaps on the S&P and Dow indexes. Accordingly, shorting variance swap contracts on the indexes generates significantly positive average excess returns during our sample period. To gauge the profitability of such a trading strategy, we estimate the annualized Sharpe ratio on shorting the 30-day variance swap contracts, and report them in the last column of Table 3 under “IR.” The Sharpe ratio is computed as the sample mean of the log excess return $-\ln(RV/SW)$ divided by its standard deviation and multiplied by $\sqrt{365/30}$ for annualization. The standard deviation is adjusted for serial dependence according to Newey and West (1987) with 30 lags. The Sharpe ratio estimates are 0.98, 0.85, and 0.87 for shorting variance swaps on the S&P 500, the S&P 100, and the Dow Indexes, significantly higher than an average stock portfolio investment.

Nevertheless, it is important to point out that the Sharpe ratios are computed using synthetic variance swap rates. The actual profitability depends on several practical factors, such as the actual availability of variance swap quotes, their bid-ask spreads, and their similarity to our synthetic values. Furthermore, given the nonlinear payoff structure, caution should be applied when interpreting Sharpe ratios on derivative trading strategies, e.g., Goetzmann, Ingersoll Jr., Spiegel, and Welch (2002).

The average variance risk premia and log variance risk premia on individual stocks show large cross-sectional variation. The standard deviation estimates on the variance risk premia (*RP*) of the individual stocks are all larger than those on the S&P and Dow indexes. As a result, out of the 35 individual stocks, only seven generate variance risk premia that are significantly negative at the 95% confidence level. By contrast, the standard deviation estimates on the log variance risk premia (*LRP*) are much more uniform across all stock indexes and individual stocks. For 23 out of 35 individual stocks, the mean log variance risk premia are significantly negative at the 95% confidence level.

The cross-sectional variation of the variance risk premia possibly suggests that the market does not price all return variance risk in each single stock, but only prices a systematic variance risk component in the stock market portfolio. Based on this hypothesis, the average variance risk premium on each stock is not proportional to the total variation of the return variance, but to the covariation of the return variance with the market portfolio return variance. To test this hypothesis, we use the realized variance on S&P 500 index return as the market portfolio variance, and estimate the "variance beta" for each stock as,

$$\beta_j^V = Cov(\ln RV_j, \ln RV_{SPX})/Var(\ln RV_{SPX}), \tag{12}$$

where the variance and covariance are measured using the common sample of the two realized variance series. We estimate the variance beta using log variance for better distributional behaviors.

Given the variance beta estimates, our hypothesis suggests that the average variance risk premia are more negative on stocks with higher variance beta. Regressing the average log variance risk premia on the variance beta across the 40 stocks and stock indexes generates the following estimates,

$$\begin{array}{cccccccc} \overline{LRP}_j & = & 0.0061 & - & 0.3283 & \beta_j^V & +e, & R^2 = 18.4\%, \\ & & (0.09) & & (-2.96) & & & \end{array} \tag{13}$$

with t-statistics reported in the parentheses below the estimates. Consistent with our hypothesis, the slope estimate is negative and statistically significant.

Therefore, we identify a systematic variance risk factor that the market prices heavily. The negative sign on the market variance risk premia suggests that investors are willing to pay a premium to hedge away upward movements in the return variance of the stock market. In other words, investors regard increases in market volatility as unfavorable shocks to the investment opportunity and demand a high premium for bearing such shocks.

Table 3 also reports the average non-overlapping 30-day autocorrelation for the variance risk premia. The autocorrelation estimates are low, averaging at -0.023 for RP and -0.006 for LRP. Therefore, although return variance is strongly predictable, investors have priced this predictability into options, so that excess returns on synthetic variance swap investments are no longer strongly predictable.

3.2. Can classic risk factors explain the variance risk premia?

Return variance can vary either by itself as in stochastic volatility models of Heston (1993) and Hull and White (1987), or it can vary as a function of the stock price as in the constant elasticity of variance model of Cox (1996) and the local volatility model of Dupire (1994) and Derman and Kani (1994). In the first case, the independent variance variation represents an additional source of risk (in addition to the return risk), which can ask for a risk premium in addition to the premium on the return risk. In the latter case, the variance risk premium is induced purely by the underlying return risk and return risk premium.

The classic capital asset pricing model (CAPM) argues that the expected excess return on an asset is only proportional to the beta of the asset on the market portfolio. Under this model, variance risk premium cannot come from an independent source of risk, but can only come from the variance swap's correlation with the market portfolio. Qualitatively, the negative excess return on the variance swap contract on the stock indexes is consistent with the well-documented negative correlation between index returns and index return variance. The question is whether this negative correlation can fully account for the negative variance risk premia.

To answer this question, we estimate the following CAPM regressions,

$$\ln RV_{t,T}/SW_{t,T} \quad = \quad \alpha + \beta_j ER^m_{t,T} + e, \tag{14}$$

for the five stock indexes and 35 individual stocks, where ER^m denotes the excess return on the market portfolio, for which we consider two proxies. First, we use the S&P 500 index to proxy for the market portfolio and compute the excess return as $ER^m_{t,T} = \ln S^m_T/F^m_{t,T}$. Our second proxy is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates). This excess return is publicly available at Kenneth French's online data library.[2] The data are monthly. The sample period that matches our options data is from January 1996 to December 2002. We estimate the regressions using the generalized methods of moments (GMM), with the weighting matrix computed according to Newey and West (1987) with 30 lags for the overlapping daily series and six lags for the non-overlapping monthly series.

Table 4 reports the estimates (and t-statistics in parentheses). The results from the two market portfolio proxies are similar. The β estimates are strongly negative for all the stock indexes and most of the individual stocks. The estimates are the most negative for S&P and Dow indexes. Nevertheless, the intercept α estimates remain strongly negative, especially for the S&P and Dow indexes, indicating that the negative beta cannot fully account for the negative variance risk premia. Indeed, the estimates for α are not much smaller than the mean variance risk premia reported in Table 3. The results call for additional risk factors.

[Insert Table 4 about here.]

Kraus and Litzenberger (1976) propose a three-moment capital asset pricing model, in which the excess return on a security is proportional not only to the excess return on the market portfolio, but also to the squared deviation of the market portfolio return from its expected value,

$$\ln RV_{t,T}/SW_{t,T} \quad = \quad \alpha + \beta ER^m_{t,T} + \gamma \left(R_m - \overline{R}_m\right)^2 + e, \tag{15}$$

where R_m denotes the market portfolio return and $\overline{R}_m$ denotes its expected value. We use ER_m to proxy R_m in constructing the squared deviation factor. The loading coefficient estimates for γ are mostly insignificant and the α estimates are close to what we have obtained from the regression in (14). Hence, $(R_m - \bar{R}_m)^2$ is not the factor that we are looking for in explaining the variance risk premia. To save space, we do not report the estimation results but they are available upon request.

Fama and French (1993) identify two additional risk factors in the stock market that are related to the firm size (SMB) and book-to-market value (HML), respectively. We investigate whether these additional common risk factors explain the variance risk premia. We estimate the following relations:

$$\ln RV_{t,T}/SW_{t,T} \quad = \quad \alpha + \beta ER^m_{t,T} + sSMB_{t,T} + hHML_{t,T} + e. \tag{16}$$

Data on all three risk factors are available on Kenneth French's online data library. We refer the interested readers to Fama and French (1993) for details on the definition and construction of these common risk factors. The sample period that overlaps with our options data is monthly from January 1996 to December 2002. Again, ER^m denotes the excess return to the market portfolio. Furthermore, both SMB and HML are in terms of excess returns on zero-cost portfolios. Therefore, the intercept α represents the expected excess return on an investment that is neutral to all three risk factors. Table 5 reports the GMM parameter estimates and t-statistics. The intercept estimates for the indexes remain strongly negative, the magnitudes only slightly smaller than the average variance risk premia in Table 3. Thus, the Fama-French risk factors can only explain a small portion of the variance risk premia.

[Insert Table 5 about here.]

In the regression, both the market ER^m and the size SMB factors generate significantly negative loadings, indicating that the return variance is not only negatively correlated with the market portfolio return, but also with the SMB factor. Hence, going long the variance swap contract also serves as an insurance against the SMB factor going up. The loading estimates on the HML factor are mostly insignificant.

Fama and French (1993) also consider two bond-market factors, related to the bond maturity ($TERM$) and default (DEF) risks. Furthermore, Jegadeesh and Titman (1993) identify a momentum phenomenon that past winners often continue to outperform past losers. We construct the $TERM$ and DEF factors using Treasury and corporate yield data from the Federal Reserve Statistical Release. Kenneth French's data library also provides a momentum factor (UMD) similar to that from Carhart (1997). However, single-factor regressions on these three factors show that none of these factors have a significant loading on the variance risk premia. Therefore, they cannot explain the variance risk premia, either.

The bottom line story here is that classic risk factors cannot fully account for the negative variance risk premia on the stock indexes. Either there exists a large inefficiency in the market for index variance or else the majority of the variance risk is generated by an independent risk factor that the market prices heavily. Investors are willing to receive a negative excess return to hedge against market volatility going up, not only because market volatility movement is negatively correlated with stock market portfolio return, but also because investors regard market volatility hikes by themselves as unfavorable shocks and demand high compensation for bearing such shocks.

There are several potential reasons for the negative variance risk premia. Take the market portfolio of stocks as an example, which the market holds in aggregate. With the same expected return, the increase in return variance implies a decline in performance in terms of the Sharpe ratio. Hence, one way to guarantee a minimum performance is to buy options to hedge against return variance increases. Furthermore, going long the variance swap contrast is an effective strategy to hedge against risks associated with the random arrival of discontinuous price movements. Finally, considerations on meeting value-at-risk requirements and preventing shortfalls and draw-downs also make long variance swap an attractive strategy that investors are willing to take even with a negative expected excess return.

3.3. Are variance risk premia constant or time-varying?

To understand the dynamic behavior of variance risk premia, we run the expectation hypothesis regression,

$$RV_{t,T} \quad = \quad a + b\,SW_{t,T} + e. \tag{17}$$

Under the null hypothesis of zero variance risk premia in dollar terms: $\mathrm{Cov}_t^{\mathbb{P}}(m_{t,T}, RV_{t,T}) = 0$ as defined in equation (8), we have $a = 0$ and $b = 1$. In particular, the slope estimate deviating from zero would suggest that the variance risk premia are time-varying and correlated with the variance swap rate.

Table 6 reports the GMM estimates of equation (17) and the t-statistics under the null hypotheses of $a = 0$ and $b = 1$ in panel A. All the slope estimates are positive, but many are lower than one. The t-statistics show that over half of the stock indexes and individual stocks generate regression slopes that are significantly lower than the null value of one.

[Insert Table 6 about here.]

Since the variance and variance swap rates show positive skewness (Table 2), we also run the expectation hypothesis regression in log terms and report the results in panel B of Table 6:

$$\ln RV_{t,T} = a + b \ln SW_{t,T} + e. \tag{18}$$

Under the null hypothesis of zero variance risk premia in return terms: $\mathrm{Cov}_t^{\mathbb{P}}(m_{t,T}, \frac{RV_{t,T}}{SW_{t,T}}) = 0$ as defined in equation (9), the slope estimate b should be zero and the intercept estimate should be lower than zero due to the convexity term induced by the variance of the log variance risk premia. The estimation results in panel B of Table 6 show that for all the stock indexes and many of the individual stocks, the slope estimates are no longer significantly different from one at the 95% confidence level. The difference between the slope estimates of the two regressions indicates that the risk premia defined in log returns is closer to a constant or independent series than the risk premia defined in dollar terms.

4. Robustness Analysis

Our results on variance risk premia rely on the accuracy of the variance swap rates that we synthesize from the options market. For robustness check, we first gauge the approximation error of the synthetic variance swap rate due to price jumps and discretization. Second, we analyze the impact of options bid-ask spreads on our results. Third, we evaluate the impacts of error-in-variable problems on our expectation hypothesis regressions where the synthetic variance swap rate is used as a regressor. Finally, we analyze whether the variance risk premia behavior varies significantly over different subsample periods.

4.1. Replication errors due to price jumps and discretization

The replication of the payoff to a variance swap in equation (5) has an instantaneous error of order $O((\frac{dF_t}{F_{t-}})^3)$. We refer to this error as *jump error* as it vanishes under continuous path monitoring if there are no jumps. Furthermore, equation (5) asks for a continuum of option prices at all strikes. We use a simple interpolation/extrapolation scheme to generate 2,000 option prices over ± 8 standard deviations from the available option quotes. We then sum over the 2,000 option prices to replace the integration in equation (5). The scheme introduces a second source of error due to the interpolation/extrapolation and the discrezation of the integral. We refer to this error as the *discretization error*.

To gauge the magnitude of these two sources of errors, we numerically illustrate three standard option pricing models: (1) the Black-Scholes model (BS), (2) the Merton (1976) jump-diffusion model (MJD), and (3) a combination of the MJD model with Heston (1993) stochastic volatility (MJDSV), as in Bates (1996) and Bakshi, Cao, and Chen (1997). The risk-neutral dynamics of the underlying futures price process under these three models are:

$$\begin{aligned} \text{BS:} \quad & dF_t/F_t &=& \ \sigma dW_t, \\ \text{MJD:} \quad & dF_t/F_{t-} &=& \ \sigma dW_t + dJ(\lambda) - \lambda g dt, \\ \text{MJDSV:} \quad & dF_t/F_{t-} &=& \ \sqrt{v_t} dW_t + dJ(\lambda) - \lambda g dt, \end{aligned} \tag{19}$$

where W_t denotes a standard Brownian motion and $J(\lambda)$ denotes a compound Poisson jump process with constant intensity λ. Conditional on a jump occurring, the MJD model assumes that the size of the jump in log price is normally distributed with mean μ_J and variance σ_J^2, with the mean percentage price change induced by a jump given by $g = e^{\mu_J + \frac{1}{2}\sigma_J^2} - 1$. In the MJDSV model, the diffusion variance rate v_t is stochastic and follows a mean-reverting square-root process:

$$dv_t = \kappa(\theta - v_t)\,dt + \sigma_v \sqrt{v_t}\,dZ_t, \tag{20}$$

where Z_t is another standard Brownian motion, correlated with W_t by $\mathbb{E}[dZ_t dW_t] = \rho dt$.

The MJDSV model nests the MJD model, which in turn nests the BS model. We regard the progression from BS to MJD and then from MJD to MJDSV as one of increasing complexity. All three models are analytically tractable, allowing us to numerically calculate risk-neutral expected values of variance. The difference in the BS model between the synthetic variance swap rate and the constant variance rate are purely due to the discretization. The increase in the error due to the use of the MJD model instead of BS allows us to numerically gauge the magnitude of the jump error in the presence of discrete strikes. The change in approximation error from MJD to MJDSV allows us to numerically gauge the impact of stochastic volatility in the presence of discrete strikes and jumps.

For the numerical analysis, we normalize the current futures price to \$100 and assume a constant riskfree rate at $r = 5.6\%$. We set $\sigma = 0.37$ in the BS model and 0.35 in the MJD model. The other parameters are set to $\lambda = 0.4$, $\mu_J = -0.09$, $\sigma_J = 0.18$,$\kappa = 1.04$, $\theta = 0.35$, $\sigma_v = 0.9$, and $\rho = -0.7$. These parameters reflect approximately those estimated from S&P 500 index option prices, e.g., in Bakshi, Cao, and Chen (1997).

In parallel to our empirical study, we fix the option maturity to 30 days. We assume that only five option quotes are available at this maturity at strikes of \$80, \$90, \$100, \$110, and \$120. Since all the stock indexes and individual stocks in our data sample average no less than five strikes at each chosen maturity, the choice of five options for the numerical analysis is reasonable and conservative. First, we compute the prices of the five options under each model and convert them into implied volatilities. Second, we employ the same interpolation/extrapolation method as in our empirical study to obtain a fine grid of 2,000 implied volatilities. Third, we convert the fine grid of implied volatilities into out-of-the-money option prices and approximate the integral in equation (5) with a summation. From this procedure, we compute the synthetic variance swap rate, $\widehat{SW}_{t,T}$, where the hat stresses the approximations involved. The difference between this approximate synthetic variance swap rate $\widehat{SW}$ and the analytically computed variance swap rate $\mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}]$ represents the total approximation error.

Under the BS model, the annualized return variance rate is constant at σ^2. Under MJD, this variance rate is constant at $\sigma^2 + \lambda\left(\mu_J^2 + \sigma_J^2\right)$. In both cases, the variance swap rate equals to the constant variance. Under MJDSV, the variance rate is stochastic, and the variance swap rate depends on the current level of the instantaneous variance rate v_t,

$$\mathbb{E}_t^{\mathbb{Q}}\left[RV_{t,T}\right] = \sigma_t^2 + \lambda\left(\mu_J^2 + \sigma_J^2\right), \tag{21}$$

where σ_t^2 is given by

$$\sigma_t^2 \equiv \frac{1}{T-t}\mathbb{E}_t^{\mathbb{Q}}\int_t^T v_s ds = \theta + \frac{1-e^{-\kappa(T-t)}}{\kappa(T-t)}\left(v_t - \theta\right). \tag{22}$$

Our replicating strategy for the variance swap contract is exact when the underlying dynamics are purely continuous, but has a higher order approximation error in the presence of jumps. Thus, under the BS model, the theoretical approximation error is zero: $\varepsilon = 0$. Under the two jump-diffusion models MJD and MJDSV, the compound Poisson jump component has the following compensator:

$$\nu(x) = \lambda\frac{1}{\sqrt{2\pi\sigma_J^2}}e^{-\frac{(x-\mu_J)^2}{2\sigma_J^2}}, \tag{23}$$

from which we can compute the jump-induced error ε according to equation (6):

$$\varepsilon = 2\lambda\left(g - \mu_J - \sigma_J^2/2\right).$$

Table 7 reports the analytical variance swap rate ($\mathbb{E}_t^{\mathbb{Q}}[RV]$), the synthetic variance swap rate ($\widehat{SW}$), the total approximation error ($\mathbb{E}_t^{\mathbb{Q}}[RV] - \widehat{SW}$), and the jump-induced error (ε) under each model. Under the BS model, the jump error (ε) is zero. Furthermore, since the implied volatility is constant and equal to σ at all strikes, there is no interpolation or extrapolation error. The only potential error comes from the discretization of the integration. Table 7 shows that this error is practically zero.

[Insert Table 7 about here.]

Under MJD, the jump error (ε) is 0.0021, which is merely 1.51 percent of the variance level at 0.1387. The total error is also 0.0021, indicating that the interpolation and extrapolation scheme does not introduce any noticeable additional errors in this case.

Under MJDSV, we consider different instantaneous variance levels, represented by its log difference from the mean, $\ln(v_t/\theta)$. As the variance level v_t varies, the jump error is fixed at 0.0021 because the jump arrival rate does not change. Table 7 indicates that the total approximation error increases with the volatility level. The largest absolute error is 0.0221 when the variance swap rate reaches as high as 2.3782. The error is less than one percent of the variance level. Therefore, even under stochastic volatility and when the volatility level is very high, the interpolation and extrapolation across the five implied volatility quotes do not add much additional approximation error. The numerical exercise shows that our simple interpolation and extrapolation method works well.

4.2. Bid-ask spreads

We synthesize variance swap rates by interpolating implied volatilities computed from the mid-quotes of the option prices. The mid-quote may not reflect the fair price if the bid and ask quotes are not symmetric around the fair price. To gauge how much our conclusions are affected by the mid-quote choice, we reconstruct the synthetic variance swap rates using bid and ask option prices, respectively. When direct quotes are not available, we can regard these as synthetic bid and ask swap rate quotes, respectively. For European options, we directly convert the bid and ask option prices into bid and ask implied volatilities, and perform the interpolation and extrapolation on each side. For American options, we first convert the OptionMetrics implied volatility into a mid-quote European option price. Then we superimpose the bid-ask spread of the American option quotes on this mid price to generate bid and ask European option prices, from which we compute the bid and ask implied volatilities, respectively.

Table 8 reports the sample averages of the synthetic bids and asks of the variance swap rates, as well as the variance risk premia defined in both dollar terms and log returns. For the risk premia, we also report in parentheses the serial-dependence adjusted *t*-statistics on the significance of the mean value. The bid-ask spreads for the synthetic variance swap rates range from 0.82 to 16.75. Converting the variance swap rates into volatility percentage points per industry quoting convention, we obtain the average bid-ask spreads ranging from 1.55 to 8.28 volatility percentage points. Overall, the spreads are larger for individual stocks than for stock indexes.

[Insert Table 8 about here.]

It is important to point out that currently there exists an active over-the-counter market for variance swap contracts on stock indexes. Although it is difficult to retrieve long histories, current quotes from several broker dealers are readily available from common financial data sources. The bid-ask spreads on these variance swap rate quotes are normally within one volatility percentage point. Our synthetic variance swap bid-ask spreads for the five stock indexes are from 1.55 to 4.62 volatility points, much wider than the actual spreads from the over-the-counter market.

Nevertheless, even with the exaggerated bid-ask spreads, our main conclusions on the variance risk premia remain valid whether we measure the premia using the synthetic bid swap rates or ask swap rates. Using the synthetic ask rates makes the variance risk premia even more negative. Using the bid swap rates lowers the absolute magnitude of the negative risk premia. However, even when we use the synthetic bid swap rates to compute the variance risk premia, the premia remain significantly negative for S&P and Dow indexes, whether the premia are measured in dollar terms or log returns.

4.3. Errors in variables

Since the synthetic variance swap rates are measured with error, the error-in-variable issue arises when they are used as regressors in the expectation hypothesis regressions in (17) - (18). Thus, the fact that the slope estimate for equation (17) is significantly below the null hypothesis of one for S&P and Dow indexes could be either due to time-varying risk premium, as we have conjectured, or simply due to the bias induced by the error-in-variable problem.

To gauge the size of the bias caused by the error-in-variable problem in equation (17), we propose the following expanded formulation:

$$\begin{aligned} RV_{t,T} &= a + bSW_{t,T} + e, \\ \widehat{SW}_{t,T} &= SW_{t,T} + \eta, \end{aligned} \tag{24}$$

where SW denotes the true swap rate and $\widehat{SW}$ denotes the synthetic swap rate, which is regarded as a noisy estimator of the true swap rate, with η capturing the measurement error. Furthermore, we specify an auxiliary AR(1) dynamics for the true 30-day variance swap rate:

$$SW_{t+1} = \theta(1-\phi) + \phi SW_{t+1} + \varepsilon_{t+1}. \tag{25}$$

If we assume independent normal distributions on the error terms (e, η, ε) with variance $(\sigma_e^2, \sigma_\eta^2, \sigma_\varepsilon^2)$, respectively, we can use the maximum likelihood method joint with Kalman filter to estimate the parameters of the system. In this estimation, we regard equation (25) as the state-propagation equation and equation (24) as the measurement equation. Given initial parameter guesses, we use Kalman filter to obtain the forecasted mean values and variances on the measurement series. Then, we construct the likelihood based on the forecasting errors, which are normally distributed under our assumption. The model parameters $(a, b, \theta, \phi, \sigma_e^2, \sigma_\eta^2, \sigma_\varepsilon^2)$ are estimated by maximizing the likelihood value. Using this method, we learn not only the bias-corrected expectation hypothesis coefficients (a, b), but also the variances of the measurement errors and the true swap rates.

We perform the likelihood estimation on the three S&P and Dow indexes that have generated regression slope coefficients significantly lower than one. Table 9 reports the maximum likelihood estimates and standard errors of the parameters. Take the S&P 500 index as an example. The slope estimate is 0.618, larger than the least square estimate of 0.455 reported in Table 6. The difference between the two estimates captures the bias induced by the measurement errors in the synthetic variance swap rates. Nevertheless, after correcting for this bias, the slope coefficient on the S&P 500 index remains significantly lower than the null value of one. The results are similar for the other two indexes. Therefore, our earlier conclusion remains valid after controlling for the error-in-variable issue. Especially for the S&P and Dow indexes, the expectation hypothesis regression slope estimate is significantly below the null value of one, suggesting that the variance risk premium in dollar terms is time-varying and correlated with the variance swap rate.

[Insert Table 9 about here.]

4.4. Subsample analysis

The stock market had been largely bullish since the beginning of our sample in 1996 until the burst of the Nasdaq bubble in March 2000, after which the stock market has been going down till the end of our sample in 2003. As a concrete example, the S&P 500 index started at around 600 in January 1996, and climbed over 1500 points before it started to fall after March 2000. By the end of our sample in February, 2003, the S&P 500 index retreated to around 800. Thus, we can largely divide our whole sample into two subsample periods, a bullish period from 1996 to March 2000, and a bearish period after March 2000.

To study whether the variance risk premia behavior varies in bullish versus bearish market conditions, we divide our sample into two subsamples, with March 24, 2000 as the dividing point. The first subsample includes dates before March 23, 2000. The second subsample includes March 24, 2000 and after. Table 10 reports the summary statistics of the realized variance, variance swap rates, and the variance risk premia under the two subsample periods. On average, both the realized variance and the variance swap rates are higher during the bearish period (the second subsample) than during the bullish period. Nevertheless, the variance risk premia are strongly negative under both market conditions for the S&P and Dow indexes.

[Insert Table 10 about here.]

5. Conclusion

In this paper, we propose a direct and robust method to quantify the variance risk premia on financial assets underlying options. Our method uses the notion of a variance swap, which is an over-the-counter contract that pays the difference between the realized variance and the fixed swap rate. Since the variance swap rate represents the risk-neutral expected value of the realized variance, we propose to use the difference between the realized variance and the variance swap rate to quantify the variance risk premium. We show that the variance swap rate can be well approximated by the value of a particular portfolio of options. Using a large options data set, we synthesize variance swap rates and analyze variance risk premia on five stock indexes and 35 individual stocks.

We find that the variance risk premia are strongly negative for the S&P and Dow indexes. Further analysis shows that there exists a systematic variance risk factor in the stock market that asks for a highly negative risk premium. When we investigate whether the classic asset pricing model can explain the negative variance risk premia, we find that the well-documented negative correlation between index returns and volatility generates a strongly negative beta, but this negative beta can only explain a small portion of the negative variance risk premia. The Fama-French factors cannot account for the strongly negative variance risk premia, either. Therefore, we conclude that either there is a large inefficiency in the market for index variance or else the majority of the variance risk is generated by an independent risk factor that the market prices heavily. The negative sign on the variance risk premia indicates that investors regard market volatility going up as unfavorable shocks, and are willing to pay a large premium to hedge against market volatility going up.

To analyze the dynamic properties of the variance risk premia, we formulate expectation-hypothesis regressions. When we regress the realized variance on the variance swap rate, we obtain slope estimates that are significantly lower than one, the null value under the hypothesis of constant or independent variance risk premia. The slope estimates become closer to one when the regression is on the logarithm of variance. These regression results indicate that although the log variance risk premia are strongly negative, they are not strongly correlated with the logarithm of the variance swap rate.

The simple, direct, and robust method that we propose to measure variance risk premium opens fertile ground for future research. Given the evidence on stochastic variance and strongly negative variance risk premia, it is important to understand the pricing kernel behavior as a function of both the market portfolio return and return variance. Recent studies, e.g., Jackwerth (2000) and Engle and Rosenberg (2002) have found some puzzling behaviors on the pricing kernel projected on the equity index return alone. Accurately estimating the pricing kernel as a joint function of the index return and return variance can prove fruitful not only for understanding the variance risk premia behavior, but also for resolving the puzzling behaviors observed on the pricing kernels projected on the index return alone.

The empirical analysis in this paper focuses on the variance swap rate and variance risk premium over a fixed 30-day horizon. As over-the-counter variance swap rate quotes are becoming increasingly available at many different maturities, an important line for future research is to design and estimate stochastic return variance models that can capture the term structure of variance swap rates and variance risk premia, e.g., Egloff, Leippold, and Wu (2006).

Appendix A. Synthesizing variance swap contracts

Let $f(F)$ be a twice differentiable function of F. By Itô's lemma for semi-martingales:

$$\begin{aligned} f(F_T) &= f(F_t) + \int_t^T f'(F_{s-})dF_s + \frac{1}{2}\int_t^T f''(F_{s-})\sigma_{s-}^2 ds \\ &\quad + \int_t^T \int_{\mathbb{R}^0} [f(F_{s-}e^x) - f(F_{s-}) - f'(F_{s-})F_{s-}(e^x - 1)]\mu(dx, ds), \end{aligned} \tag{A1}$$

Applying equation (A1) to the function $f(F) = \ln F$, we have:

$$\ln(F_T) = \ln(F_t) + \int_t^T \frac{1}{F_{s-}} dF_s - \frac{1}{2}\int_t^T \sigma_{s-}^2 ds + \int_t^T \int_{\mathbb{R}^0} [x - e^x + 1]\mu(dx, ds). \tag{A2}$$

Adding and subtracting $2[\frac{F_T}{F_t} - 1] + \int_t^T x^2\mu(dx, ds)$ and re-arranging, we obtain the following representation:

$$\begin{aligned} V_{t,T} &\equiv \int_t^T \sigma_{s-}^2 ds + \int_t^T x^2 \mu(dx, ds) = 2\left[\frac{F_T}{F_t} - 1 - \ln\left(\frac{F_T}{F_t}\right)\right] + 2\int_t^T \left[\frac{1}{F_{s-}} - \frac{1}{F_t}\right] dF_s \\ &\quad -2\int_t^T \int_{\mathbb{R}^0} \left[e^x - 1 - x - \frac{x^2}{2}\right] \mu(dx, ds). \end{aligned} \tag{A3}$$

A Taylor expansion with remainder of $\ln F_T$ about the point F_t implies:

$$\ln F_T = \ln F_t + \frac{1}{F_t}(F_T - F_t) - \int_0^{F_t} \frac{1}{K^2}(K - F_T)^+ dK - \int_{F_t}^\infty \frac{1}{K^2}(F_T - K)^+ dK. \tag{A4}$$

Combining equations (A3) and (A4) and noting that $F_T = S_T$, we have:

$$\begin{aligned} V_{t,T} &= 2\left[\int_0^{F_t} \frac{1}{K^2}(K - S_T)^+ dK + \int_{F_t}^\infty \frac{1}{K^2}(S_T - K)^+ dK\right] \\ &\quad +2\int_t^T \left[\frac{1}{F_{s-}} - \frac{1}{F_t}\right] dF_s \\ &\quad -2\int_t^T \int_{\mathbb{R}^0} \left[e^x - 1 - x - \frac{x^2}{2}\right] \mu(dx, ds). \end{aligned} \tag{A5}$$

Thus, we can replicate the return quadratic variation up to time T by the sum of (i) the payoff from a static position in $\frac{2dK}{K^2}$ European options on the underlying spot at strike K and maturity T (first line), (ii) the payoff from a dynamic trading strategy holding $2B_s(T)\left[\frac{1}{F_{s-}} - \frac{1}{F_t}\right]$ futures at time s (second line), and (iii) a higher-order error term induced by the discontinuity in the futures price dynamics (third line). The options are all out-of-the-money forward, i.e., call options when $F_t > K$ and put options when $K \leq F_t$.

Taking expectations under measure $\mathbb{Q}$ on both sides, we obtain the risk-neutral expected value of the quadratic variation on the left hand side. We also obtain the forward value of the startup cost of the replicating strategy and the replication error on the right hand side:

$$\mathbb{E}_t^{\mathbb{Q}}\left[V_{t,T}\right] = \int_0^\infty \frac{2\Theta_t(K,T)}{B_t(T)K^2}dK - 2\mathbb{E}_t^{\mathbb{Q}}\int_t^T \int_{\mathbb{R}^0}\left[e^x - 1 - x - \frac{x^2}{2}\right]\nu_s(x)dxds.$$

By the martingale property, the expected value of the gains from dynamic futures trading is zero under the risk-neutral measure. Dividing by $(T-t)$ on both sides, we obtain the result on the annualized return quadratic variation.

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Notes

[1]The variance risk premia on the Nasdq-100 index and its tracking stock QQQ also show some differences due to, among other things, their different sample periods

[2]The address is: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 1
List of stocks and stock indexes used in our study

No.	Ticker	Starting Date	Ending Date	N	NK	Name
1	SPX	04-Jan-1996	28-Feb-2003	1779	26	S&P 500 Index
2	OEX	04-Jan-1996	28-Feb-2003	1780	27	S&P 100 Index
3	DJX	06-Oct-1997	28-Feb-2003	1333	12	Dow Jones Industrial Average
4	NDX	04-Jan-1996	28-Feb-2003	1722	19	Nasdaq 100 Stock Index
5	QQQ	10-Mar-1999	28-Feb-2003	978	22	Nasdaq-100 Index Tracking Stock
6	MSFT	04-Jan-1996	28-Feb-2003	1766	9	Microsoft Corp
7	INTC	04-Jan-1996	28-Feb-2003	1653	8	Intel Corp
8	IBM	04-Jan-1996	28-Feb-2003	1768	9	International Business Machines Corp
9	AMER	04-Jan-1996	28-Feb-2003	1648	9	Nanobac Pharmaceuticals Inc
10	DELL	04-Jan-1996	28-Feb-2003	1650	7	Dell Inc
11	CSCO	04-Jan-1996	28-Feb-2003	1554	7	Cisco Systems Inc
12	GE	04-Jan-1996	28-Feb-2003	1458	6	General Electric Co
13	CPQ	04-Jan-1996	03-May-2002	1272	6	Compaq Computer Corp
14	YHOO	09-Sep-1997	28-Feb-2003	1176	14	Yahoo! Inc
15	SUNW	04-Jan-1996	28-Feb-2003	1395	8	Sun Microsystems Inc
16	MU	04-Jan-1996	28-Feb-2003	1720	8	Micron Technology Inc
17	MO	04-Jan-1996	28-Feb-2003	1474	5	Altria Group Inc
18	AMZN	19-Nov-1997	28-Feb-2003	1078	12	Amazon.Com Inc
19	ORCL	04-Jan-1996	28-Feb-2003	1104	6	Oracle Corp
20	LU	19-Apr-1996	28-Feb-2003	981	7	Lucent Technologies Inc
21	TRV	04-Jan-1996	28-Feb-2003	1279	5	Thousand Trails Inc
22	WCOM	04-Jan-1996	21-Jun-2002	1104	6	WorldCom Inc
23	TYC	05-Jan-1996	28-Feb-2003	979	6	Tyco International Ltd
24	AMAT	04-Jan-1996	28-Feb-2003	1671	8	Applied Materials Inc
25	QCOM	04-Jan-1996	28-Feb-2003	1613	8	Qualcomm Inc
26	TXN	04-Jan-1996	28-Feb-2003	1610	7	Texas Instruments Inc
27	PFE	04-Jan-1996	28-Feb-2003	1420	6	Pfizer Inc
28	MOT	04-Jan-1996	28-Feb-2003	1223	6	Motorola Inc
29	EMC	04-Jan-1996	28-Feb-2003	1188	7	EMC Corp
30	HWP	04-Jan-1996	28-Feb-2003	1395	6	Hewlett-Packward Co
31	AMGN	04-Jan-1996	28-Feb-2003	1478	6	Amgen Inc
32	BRCM	28-Oct-1998	28-Feb-2003	1003	12	Broadcom Corp
33	MER	04-Jan-1996	28-Feb-2003	1542	6	Merill Lynch & Co Inc
34	NOK	04-Jan-1996	28-Feb-2003	1176	6	Nokia OYJ
35	CHL	04-Jan-1996	28-Feb-2003	1422	5	China Mobile Hong Kong Ltd
36	UNPH	16-Sep-1996	28-Feb-2003	745	12	JDS Uniphase Corp
37	EBAY	01-Feb-1999	28-Feb-2003	1000	12	eBay Inc
38	JNPR	07-Oct-1999	28-Feb-2003	627	15	Juniper Networks Inc
39	CIEN	14-May-1997	28-Feb-2003	998	9	Ciena Corp
40	BRCD	30-Nov-1999	28-Feb-2003	693	10	Brocade Communications Systems Inc

Entries list the ticker, the starting date, the ending date, the sample length (N), the average number of available strikes per maturity (NK), and the full name for each of the five stock indexes and 35 individual stocks used in our study.

Table 2
Summary statistics for the realized variance and the synthetic variance swap rate

Ticker	Panel A: Realized variance, $RV \times 100$					Panel B: Variance swap rate, $SW \times 100$				
	Mean	Std	Auto	Skew	Kurt	Mean	Std	Auto	Skew	Kurt
SPX	4.07	3.43	0.98	2.23	8.76	6.81	3.87	0.88	2.45	13.95
OEX	4.53	3.82	0.98	2.13	8.26	6.90	3.65	0.96	1.77	6.94
DJX	4.39	3.66	0.98	2.15	7.55	6.98	3.60	0.93	2.18	9.59
NDX	16.69	15.62	0.98	2.31	9.61	19.12	11.86	0.98	0.96	3.32
QQQ	22.61	16.53	0.98	1.60	5.32	26.54	11.29	0.94	0.82	3.58
MSFT	16.59	13.44	0.98	2.33	9.10	19.79	11.80	0.76	4.80	63.40
INTC	27.67	23.25	0.98	2.25	8.49	25.17	14.06	0.94	1.80	7.97
IBM	15.15	11.39	0.97	1.76	6.52	16.83	8.56	0.89	2.19	11.64
AMER	41.13	25.49	0.97	1.12	4.07	44.64	21.04	0.90	1.25	5.37
DELL	32.90	21.95	0.97	1.60	5.48	37.33	17.29	0.93	2.24	12.97
CSCO	31.10	28.53	0.98	2.09	7.83	33.40	20.96	0.94	2.01	7.92
GE	11.91	8.83	0.98	1.84	6.54	14.15	7.93	0.90	1.38	5.53
CPQ	31.01	21.46	0.96	1.66	5.94	32.14	16.48	0.81	2.06	10.81
YHOO	72.25	43.52	0.97	0.91	3.29	73.92	43.08	0.78	2.78	18.63
SUNW	36.71	27.14	0.98	1.83	6.61	37.63	23.13	0.84	3.22	25.06
MU	56.68	31.16	0.97	1.43	5.37	59.43	25.53	0.87	1.75	12.10
MO	13.63	11.04	0.96	1.99	8.24	15.44	9.79	0.84	2.41	16.68
AMZN	88.62	48.69	0.96	0.63	2.99	103.81	55.77	0.95	1.80	7.60
ORCL	43.72	33.83	0.97	1.70	5.84	48.11	43.62	0.94	4.40	29.09
LU	31.43	30.86	0.97	3.22	17.31	31.25	29.02	0.68	6.22	79.86
TRV	19.36	17.40	0.95	2.46	9.24	19.02	11.44	0.93	2.83	15.27
WCOM	26.84	24.12	0.97	2.76	13.82	27.81	20.81	0.92	2.22	8.94
TYC	32.61	38.22	0.98	2.32	8.71	40.74	53.24	0.90	4.46	30.54
AMAT	43.89	26.91	0.97	1.89	7.37	45.78	20.18	0.91	1.30	4.98
QCOM	46.98	32.35	0.98	1.31	4.26	48.70	23.65	0.93	1.41	5.45
TXN	37.24	25.65	0.98	1.90	7.24	35.40	18.21	0.94	1.27	4.65
PFE	12.65	7.96	0.97	1.69	7.06	14.19	5.88	0.88	0.97	4.69
MOT	29.28	25.44	0.96	2.04	7.23	27.09	18.70	0.84	1.97	9.56
EMC	41.93	37.67	0.98	2.62	10.39	38.05	22.29	0.91	1.71	6.10
HWP	25.19	17.24	0.96	1.34	4.34	24.94	13.52	0.91	1.58	6.18
AMGN	23.78	17.93	0.98	1.82	6.75	25.66	15.64	0.95	1.42	4.71
BRCM	91.22	57.64	0.98	1.68	6.06	90.56	45.18	0.95	1.68	6.56
MER	23.26	15.12	0.97	1.77	7.12	24.05	11.51	0.93	1.61	8.81
NOK	33.67	20.99	0.96	0.97	3.32	32.15	15.99	0.83	1.34	7.90
CHL	19.23	17.65	0.98	3.12	16.78	20.10	14.30	0.96	2.62	13.02
UNPH	83.74	59.74	0.97	1.47	5.40	77.16	42.70	0.95	1.02	3.44
EBAY	69.16	59.28	0.98	1.29	4.18	73.10	42.97	0.96	1.00	3.81
JNPR	104.72	56.79	0.97	1.02	3.30	114.50	49.41	0.93	0.99	3.54
CIEN	96.26	65.37	0.97	1.26	4.39	91.04	47.70	0.92	1.05	5.51
BRCD	110.11	65.55	0.97	1.01	3.45	100.75	40.78	0.92	0.38	2.51

Entries report summary statistics for the annualized realized variance RV and the synthetic variance swap rate SW. Columns under Mean, Std, Auto, Skew, Kurt report the sample average, standard deviation, daily autocorrelation, skewness, and excess kurtosis, respectively.

Table 3
Summary statistics of variance risk premia

Ticker	Panel A: $(RV - SW) \times 100$						Panel B: $\ln(RV/SW)$						IR
	Mean	Std	Auto	Skew	Kurt	t	Mean	Std	Auto	Skew	Kurt	t	
SPX	-2.74	3.63	-0.04	-1.44	17.86	-8.39	-0.66	0.57	0.05	0.18	3.23	-11.83	0.98
OEX	-2.36	3.57	-0.07	0.21	6.69	-7.02	-0.58	0.56	0.06	0.36	2.90	-10.34	0.85
DJX	-2.58	3.86	-0.05	-0.15	8.28	-6.37	-0.61	0.58	0.07	0.63	3.31	-9.07	0.87
NDX	-2.43	10.24	0.05	1.49	9.42	-2.54	-0.28	0.47	0.11	0.40	3.41	-6.49	0.55
QQQ	-3.93	12.55	0.12	0.83	4.68	-2.62	-0.29	0.48	0.21	0.16	2.88	-4.91	0.55
MSFT	-3.20	12.31	-0.10	-1.91	57.68	-3.32	-0.30	0.52	-0.08	0.08	3.48	-6.62	0.55
INTC	2.49	19.07	-0.13	1.91	8.96	1.34	-0.02	0.51	-0.19	0.42	3.31	-0.44	0.04
IBM	-1.68	10.24	-0.04	0.67	9.38	-1.80	-0.24	0.60	0.02	0.01	2.94	-4.35	0.36
AMER	-3.51	23.76	-0.08	0.63	4.62	-2.05	-0.17	0.57	-0.06	0.03	3.06	-3.79	0.33
DELL	-4.43	21.35	0.14	0.49	7.05	-2.15	-0.23	0.55	0.18	0.22	3.07	-4.17	0.36
CSCO	-2.30	20.31	-0.11	1.51	10.80	-1.42	-0.27	0.83	0.05	-6.14	70.41	-4.06	0.36
GE	-2.24	7.63	-0.14	0.59	7.68	-3.52	-0.25	0.49	-0.04	0.28	3.32	-5.60	0.51
CPQ	-1.14	20.66	-0.02	0.17	7.66	-0.62	-0.13	0.59	-0.07	0.14	3.27	-2.54	0.25
YHOO	-1.67	43.58	-0.21	-1.17	16.82	-0.42	-0.09	0.56	-0.16	0.07	2.97	-1.63	0.17
SUNW	-0.92	20.32	0.03	-1.97	40.32	-0.53	-0.11	0.48	-0.01	-0.08	3.32	-2.54	0.24
MU	-2.75	29.38	-0.12	0.68	6.71	-1.12	-0.10	0.47	-0.12	0.22	3.25	-2.70	0.23
MO	-1.81	11.76	0.10	0.52	9.99	-1.66	-0.24	0.69	0.05	0.21	3.29	-3.79	0.34
AMZN	-15.19	59.66	-0.06	-0.30	5.14	-2.14	-0.22	0.59	-0.14	0.10	2.75	-3.33	0.35
ORCL	-4.39	46.26	0.04	-4.55	33.85	-0.82	-0.14	0.66	0.04	-1.59	8.93	-1.92	0.20
LU	0.18	29.35	0.06	-3.43	79.81	0.08	-0.08	0.54	0.18	0.05	3.53	-1.55	0.17
TRV	0.35	16.04	0.20	2.38	10.88	0.22	-0.12	0.64	0.14	0.93	5.03	-1.85	0.18
WCOM	-0.97	21.22	-0.11	1.39	11.29	-0.38	-0.13	0.63	-0.07	-0.10	2.84	-1.78	0.19
TYC	-8.13	48.26	0.06	-2.16	26.08	-1.55	-0.34	0.74	-0.13	0.90	4.13	-4.00	0.45
AMAT	-1.89	24.20	-0.08	1.08	7.08	-0.93	-0.11	0.48	-0.14	0.20	3.21	-2.72	0.23
QCOM	-1.73	28.20	-0.11	0.82	5.18	-0.69	-0.15	0.59	-0.06	-0.14	3.77	-2.77	0.24
TXN	1.84	19.93	-0.07	1.22	7.33	1.00	-0.02	0.47	-0.11	0.16	2.97	-0.59	0.05
PFE	-1.54	8.08	-0.06	1.44	7.92	-1.92	-0.21	0.60	0.03	-0.14	4.17	-3.35	0.31
MOT	2.19	20.13	-0.12	0.09	10.91	1.23	-0.02	0.57	-0.08	-0.29	3.43	-0.31	0.03
EMC	3.87	28.91	0.35	2.25	11.62	1.21	-0.02	0.53	0.05	0.55	3.64	-0.47	0.05
HWP	0.25	14.56	-0.01	0.71	7.28	0.19	-0.08	0.54	-0.01	0.27	3.72	-1.67	0.16
AMGN	-1.88	15.11	-0.07	0.71	5.94	-1.26	-0.16	0.55	-0.02	0.10	2.76	-3.01	0.27
BRCM	0.66	48.32	0.07	0.64	4.89	0.12	-0.05	0.48	0.17	0.20	2.41	-0.99	0.11
MER	-0.79	13.11	0.05	1.04	5.90	-0.68	-0.11	0.50	-0.13	0.26	2.99	-2.50	0.22
NOK	1.51	18.73	0.07	-0.11	7.86	0.79	-0.03	0.55	0.08	0.07	3.31	-0.54	0.05
CHL	-0.87	15.10	-0.11	1.91	14.02	-0.60	-0.15	0.54	-0.14	0.36	3.61	-2.84	0.26
UNPH	6.59	49.42	-0.16	0.79	4.88	1.12	-0.01	0.58	-0.04	-0.32	3.00	-0.25	0.03
EBAY	-3.95	45.88	-0.05	1.44	6.54	-0.69	-0.27	0.58	0.17	0.35	2.97	-3.48	0.38
JNPR	-9.79	54.78	-0.15	-0.21	3.74	-1.27	-0.14	0.52	-0.12	-0.54	3.20	-2.07	0.29
CIEN	5.22	62.53	-0.07	0.80	6.64	0.68	-0.03	0.61	-0.07	0.47	4.18	-0.33	0.04
BRCD	9.36	59.99	0.10	0.59	3.59	1.08	0.00	0.56	0.08	-0.25	2.63	0.02	-0.00

Entries report summary statistics of variance risk premia, defined as the difference between the realized variance and the variance swap rate in panel A and as the log difference in panel B. Columns under Mean, Std, Auto, Skew, Kurt report the sample average, standard deviation, average non-overlapping 30-day autocorrelation, skewness, and excess kurtosis, respectively. Columns under t report the t-statistics of the mean risk premia, which are adjusted for serial dependence according to the Newey-West method with a lag of 30 days. The last column of the table under "IR" reports the annualized Sharpe ratio of shorting the 30-day variance swap contracts, computed as the mean of $-\ln(RV/SW)$ divided by its Newey-West standard deviation (with 30 lags), and then annualized by $\sqrt{365/30}$.

Table 4
Explaining variance risk premia with CAPM beta

Proxy	Panel A: S&P 500 Index					Panel B: Valued-Weighted Market Portfolio				
	α		β		R^2	α		β		R^2
SPX	-0.646	(-13.554)	-4.510	(-5.644)	0.173	-0.641	(-10.236)	-5.508	(-4.751)	0.245
OEX	-0.562	(-11.630)	-4.473	(-5.653)	0.175	-0.571	(-9.112)	-5.536	(-4.936)	0.247
DJX	-0.613	(-10.820)	-4.681	(-5.165)	0.206	-0.617	(-9.362)	-4.668	(-3.828)	0.205
NDX	-0.273	(-6.689)	-2.450	(-3.742)	0.078	-0.237	(-5.998)	-3.617	(-3.225)	0.182
QQQ	-0.301	(-5.257)	-1.157	(-1.707)	0.018	-0.320	(-5.330)	-2.964	(-1.702)	0.117
MSFT	-0.287	(-6.722)	-2.168	(-4.134)	0.048	-0.339	(-5.981)	-2.495	(-2.870)	0.068
INTC	-0.011	(-0.252)	-2.211	(-2.706)	0.051	-0.045	(-0.912)	-3.793	(-3.021)	0.150
IBM	-0.235	(-4.302)	-2.181	(-2.737)	0.037	-0.269	(-4.448)	-2.185	(-1.759)	0.041
AMER	-0.164	(-3.595)	-2.133	(-3.161)	0.038	-0.242	(-5.077)	-1.695	(-1.604)	0.026
DELL	-0.214	(-4.145)	-2.723	(-3.582)	0.068	-0.271	(-3.645)	-3.527	(-3.673)	0.130
CSCO	-0.269	(-3.583)	-0.966	(-0.603)	0.004	-0.272	(-3.948)	-1.998	(-0.888)	0.043
GE	-0.240	(-5.855)	-2.648	(-3.868)	0.087	-0.287	(-5.461)	-1.648	(-1.488)	0.046
CPQ	-0.107	(-1.959)	-2.426	(-2.336)	0.039	-0.028	(-0.558)	-3.292	(-2.646)	0.099
YHOO	-0.089	(-1.630)	-0.582	(-0.800)	0.003	-0.171	(-2.540)	0.752	(0.791)	0.006
SUNW	-0.095	(-2.250)	-2.295	(-3.378)	0.054	-0.111	(-1.977)	-3.978	(-3.162)	0.178
MU	-0.096	(-2.576)	-1.215	(-2.055)	0.018	-0.105	(-2.364)	-2.452	(-3.664)	0.093
MO	-0.246	(-3.818)	0.375	(0.400)	0.001	-0.253	(-3.980)	0.539	(0.473)	0.002
AMZN	-0.217	(-3.323)	0.075	(0.072)	0.000	-0.164	(-1.722)	0.486	(0.464)	0.002
ORCL	-0.125	(-1.805)	-2.311	(-2.706)	0.032	-0.179	(-2.488)	-3.606	(-2.408)	0.099
LU	-0.065	(-1.269)	-1.463	(-1.740)	0.018	-0.047	(-0.877)	-2.713	(-2.757)	0.099
TRV	-0.122	(-1.885)	-2.075	(-2.584)	0.034	-0.167	(-1.647)	-1.025	(-0.626)	0.010
WCOM	-0.099	(-1.432)	-3.588	(-3.550)	0.078	-0.066	(-0.715)	-4.256	(-2.821)	0.140
TYC	-0.349	(-4.025)	-1.726	(-1.591)	0.017	-0.405	(-3.885)	0.778	(0.260)	0.002
AMAT	-0.104	(-2.641)	-0.972	(-1.700)	0.011	-0.119	(-2.482)	-2.839	(-3.515)	0.110
QCOM	-0.149	(-2.747)	-1.062	(-1.358)	0.009	-0.164	(-2.629)	-2.573	(-2.129)	0.059
TXN	-0.022	(-0.546)	-0.603	(-1.150)	0.005	-0.086	(-1.596)	-0.783	(-1.046)	0.009
PFE	-0.206	(-3.307)	-2.016	(-1.862)	0.035	-0.230	(-3.541)	-1.907	(-1.213)	0.035
MOT	-0.003	(-0.055)	-1.955	(-1.956)	0.030	-0.024	(-0.329)	-3.479	(-1.926)	0.099
EMC	-0.007	(-0.150)	-2.865	(-2.967)	0.079	-0.069	(-0.845)	-4.090	(-3.950)	0.167
HWP	-0.070	(-1.444)	-1.669	(-2.037)	0.024	-0.048	(-0.802)	-1.996	(-1.650)	0.052
AMGN	-0.159	(-2.979)	-0.947	(-1.028)	0.009	-0.128	(-1.478)	-0.164	(-0.116)	0.000
BRCM	-0.046	(-0.867)	0.861	(1.140)	0.010	-0.082	(-0.950)	-0.824	(-0.423)	0.006
MER	-0.106	(-2.419)	-1.184	(-1.601)	0.016	-0.145	(-2.354)	-1.294	(-1.147)	0.021
NOK	-0.031	(-0.537)	-1.708	(-1.959)	0.028	-0.037	(-0.563)	-2.132	(-1.765)	0.074
CHL	-0.144	(-2.752)	-1.662	(-1.840)	0.029	-0.105	(-2.405)	-1.968	(-2.173)	0.050
UNPH	-0.014	(-0.236)	-1.676	(-1.440)	0.023	-0.057	(-1.276)	-3.015	(-1.209)	0.074
EBAY	-0.266	(-3.401)	0.214	(0.219)	0.000	-0.293	(-2.267)	-0.145	(-0.122)	0.000
JNPR	-0.147	(-2.151)	-0.484	(-0.572)	0.003	-0.184	(-2.494)	-2.091	(-1.082)	0.042
CIEN	-0.020	(-0.258)	-2.320	(-1.738)	0.039	-0.062	(-0.653)	-4.433	(-1.929)	0.148
BRCD	0.004	(0.054)	0.182	(0.190)	0.000	0.006	(0.060)	-2.790	(-2.301)	0.104

Entries report the GMM estimates (and t-statistics in parentheses) of the following relation,

$$\ln RV_{t,T}/SW_{t,T} = \alpha + \beta_j ER^m_{t,T} + e,$$

where ER^m denotes the excess return on the market portfolio, which is proxyed by the return on the S&P 500 index forward in panel A and the excess return on the CRSP valued-weighted stock portfolio in panel B. The t-statistics are computed according to Newey and West (1987) with 30 lags for the overlapping daily series in panel A and six lags for the non-overlapping monthly series in panel B. Columns under "R^2" report the unadjusted R-squared.

Table 5
Explaining variance risk premia with Fama-French risk factors

Ticker	α		ER^m		SMB		HML		R^2
SPX	-0.633	(-9.070)	-5.205	(-3.853)	-2.858	(-2.098)	-0.195	(-0.227)	0.287
OEX	-0.561	(-8.294)	-5.268	(-4.030)	-3.292	(-2.472)	-0.443	(-0.477)	0.300
DJX	-0.604	(-8.251)	-4.601	(-3.246)	-3.603	(-3.074)	-1.346	(-1.743)	0.275
NDX	-0.235	(-5.943)	-2.851	(-2.614)	-1.958	(-2.216)	1.391	(1.780)	0.269
QQQ	-0.304	(-5.213)	-2.204	(-1.461)	-1.694	(-1.548)	1.543	(2.470)	0.229
MSFT	-0.321	(-6.617)	-2.530	(-2.758)	-4.960	(-4.839)	-1.855	(-2.164)	0.225
INTC	-0.037	(-0.886)	-3.875	(-3.119)	-2.961	(-3.026)	-1.177	(-1.530)	0.205
IBM	-0.259	(-4.963)	-2.027	(-1.567)	-3.063	(-1.944)	-0.660	(-0.428)	0.089
AMER	-0.229	(-5.607)	-1.692	(-1.225)	-3.084	(-2.154)	-0.964	(-0.930)	0.080
DELL	-0.268	(-3.664)	-2.987	(-3.459)	-2.968	(-2.293)	0.420	(0.344)	0.203
CSCO	-0.282	(-4.458)	-1.084	(-0.479)	1.461	(1.092)	2.130	(2.473)	0.081
GE	-0.267	(-6.495)	-1.537	(-1.281)	-2.834	(-2.871)	-0.818	(-0.973)	0.132
CPQ	-0.030	(-0.653)	-2.907	(-2.005)	0.790	(0.661)	0.910	(0.906)	0.106
YHOO	-0.170	(-2.554)	0.104	(0.087)	0.669	(0.432)	-0.876	(-0.783)	0.024
SUNW	-0.117	(-2.220)	-3.066	(-2.066)	-1.507	(-1.466)	1.119	(1.188)	0.227
MU	-0.101	(-2.400)	-2.650	(-3.792)	-0.646	(-0.811)	-0.745	(-1.166)	0.101
MO	-0.255	(-4.129)	0.709	(0.529)	-0.527	(-0.286)	0.556	(0.394)	0.005
AMZN	-0.173	(-2.025)	-0.110	(-0.069)	-1.775	(-0.746)	-1.828	(-1.182)	0.035
ORCL	-0.174	(-2.355)	-3.814	(-3.008)	-0.110	(-0.068)	-0.334	(-0.334)	0.100
LU	-0.046	(-0.859)	-3.377	(-2.586)	-0.918	(-0.546)	-1.333	(-1.287)	0.125
TRV	-0.129	(-1.581)	-0.496	(-0.283)	-5.906	(-4.769)	-1.194	(-1.066)	0.209
WCOM	-0.073	(-0.835)	-4.910	(-2.768)	-2.263	(-1.419)	-1.756	(-1.660)	0.167
TYC	-0.367	(-3.334)	0.253	(0.107)	-4.234	(-1.611)	-2.668	(-1.875)	0.078
AMAT	-0.100	(-2.769)	-2.617	(-2.867)	-4.062	(-4.045)	-1.103	(-1.667)	0.247
QCOM	-0.164	(-2.657)	-1.657	(-1.330)	-3.863	(-3.097)	0.749	(0.716)	0.136
TXN	-0.073	(-1.382)	-1.007	(-1.600)	-4.353	(-4.575)	-1.997	(-2.073)	0.163
PFE	-0.208	(-3.746)	-1.504	(-0.834)	-3.795	(-2.047)	-1.163	(-0.582)	0.107
MOT	-0.029	(-0.354)	-2.860	(-1.650)	0.706	(0.452)	1.002	(0.869)	0.109
EMC	-0.071	(-0.994)	-2.482	(-2.356)	-1.771	(-2.035)	2.003	(2.848)	0.270
HWP	-0.044	(-0.751)	-2.456	(-1.744)	-0.918	(-0.816)	-1.138	(-1.026)	0.067
AMGN	-0.122	(-1.555)	-0.421	(-0.276)	-1.367	(-1.030)	-1.184	(-1.352)	0.019
BRCM	-0.049	(-0.633)	-0.313	(-0.148)	-3.039	(-2.407)	-0.432	(-0.389)	0.079
MER	-0.138	(-2.573)	-0.899	(-0.737)	-2.030	(-1.382)	0.313	(0.346)	0.063
NOK	-0.023	(-0.339)	-2.001	(-1.745)	-2.276	(-2.111)	-0.626	(-0.800)	0.127
CHL	-0.102	(-2.498)	-1.830	(-1.927)	-2.866	(-2.234)	-0.565	(-0.568)	0.112
UNPH	-0.044	(-0.604)	-1.688	(-0.757)	-1.547	(-0.868)	1.382	(0.999)	0.152
EBAY	-0.263	(-2.003)	0.446	(0.412)	-2.835	(-2.125)	0.336	(0.296)	0.073
JNPR	-0.105	(-1.506)	-3.192	(-2.421)	-3.368	(-2.146)	-3.227	(-3.041)	0.158
CIEN	-0.040	(-0.452)	-5.340	(-2.403)	-2.960	(-2.350)	-2.574	(-1.565)	0.202
BRCD	0.060	(0.510)	-2.025	(-1.699)	-3.496	(-2.457)	-0.723	(-0.637)	0.223

Entries report the GMM estimates (and t-statistics in parentheses) of the following relation,

$$\ln RV_{t,T}/SW_{t,T} = \alpha + \beta ER^m_{t,T} + sSMB_{t,T} + hHML_{t,T} + e,$$

where the regressors are the three stock-market risk factors defined by Fama and French (1993): the excess return on the market portfolio (ER^m), the size factor (SMB), and the book-to-market factor (HML). Data are monthly from January 1996 to December 2002. The t-statistics are computed according to Newey and West (1987) with six lags. Columns under "R^2" report the unadjusted R-squared of the regression.

Table 6
Expectation hypothesis regressions on constant variance risk premia

Ticker	Panel A: $RV_{t,T} = a + bSW_{t,T} + e$					Panel B: $\ln RV_{t,T} = a + b\ln SW_{t,T} + e$				
	a		b		R^2	a		b		R^2
SPX	0.010	(1.416)	0.455	(-4.596)	0.262	-0.891	(-2.593)	0.919	(-0.684)	0.378
OEX	0.006	(0.981)	0.568	(-3.933)	0.294	-0.600	(-1.797)	0.992	(-0.065)	0.408
DJX	0.013	(1.524)	0.443	(-4.046)	0.190	-1.210	(-2.859)	0.781	(-1.467)	0.253
NDX	-0.023	(-1.329)	0.995	(-0.042)	0.571	-0.170	(-1.233)	1.060	(0.876)	0.672
QQQ	-0.027	(-0.887)	0.953	(-0.326)	0.424	-0.281	(-1.466)	1.007	(0.060)	0.445
MSFT	0.046	(1.804)	0.605	(-2.726)	0.282	-0.465	(-2.677)	0.903	(-1.040)	0.395
INTC	0.038	(1.067)	0.948	(-0.302)	0.328	-0.263	(-1.724)	0.839	(-1.922)	0.404
IBM	0.039	(2.164)	0.670	(-2.707)	0.253	-0.594	(-2.975)	0.814	(-1.881)	0.264
AMER	0.145	(4.138)	0.596	(-5.697)	0.242	-0.408	(-5.318)	0.743	(-3.014)	0.271
DELL	0.126	(3.260)	0.543	(-3.954)	0.183	-0.583	(-4.082)	0.668	(-2.934)	0.202
CSCO	-0.009	(-0.295)	0.957	(-0.419)	0.494	-0.117	(-0.863)	1.127	(0.983)	0.343
GE	0.026	(2.557)	0.657	(-4.075)	0.348	-0.660	(-4.239)	0.803	(-3.013)	0.455
CPQ	0.129	(2.922)	0.562	(-2.858)	0.187	-0.540	(-3.257)	0.673	(-2.455)	0.229
YHOO	0.354	(4.335)	0.499	(-4.679)	0.244	-0.204	(-3.569)	0.730	(-2.823)	0.299
SUNW	0.065	(1.295)	0.802	(-1.293)	0.468	-0.233	(-2.021)	0.892	(-1.134)	0.445
MU	0.221	(4.816)	0.582	(-5.174)	0.228	-0.309	(-4.773)	0.656	(-4.078)	0.255
MO	0.072	(5.406)	0.415	(-7.421)	0.135	-0.971	(-4.791)	0.641	(-3.795)	0.222
AMZN	0.565	(5.120)	0.309	(-8.470)	0.125	-0.246	(-4.035)	0.652	(-2.559)	0.238
ORCL	0.323	(3.758)	0.238	(-3.502)	0.095	-0.513	(-2.013)	0.600	(-1.847)	0.255
LU	0.141	(2.435)	0.554	(-2.232)	0.271	-0.330	(-2.348)	0.817	(-2.128)	0.454
TRV	0.066	(2.557)	0.673	(-2.921)	0.196	-0.722	(-3.327)	0.665	(-2.911)	0.220
WCOM	0.087	(3.201)	0.652	(-2.520)	0.316	-0.500	(-2.721)	0.751	(-2.305)	0.358
TYC	0.185	(3.862)	0.347	(-17.988)	0.233	-0.535	(-4.928)	0.851	(-2.009)	0.462
AMAT	0.132	(3.524)	0.670	(-3.464)	0.252	-0.377	(-4.362)	0.690	(-3.847)	0.275
QCOM	0.117	(2.474)	0.724	(-2.833)	0.281	-0.361	(-3.516)	0.747	(-2.439)	0.249
TXN	0.056	(1.489)	0.893	(-0.813)	0.402	-0.211	(-1.983)	0.839	(-1.971)	0.435
PFE	0.059	(4.270)	0.473	(-6.469)	0.122	-1.052	(-6.084)	0.587	(-4.779)	0.160
MOT	0.063	(2.393)	0.846	(-1.133)	0.387	-0.325	(-2.022)	0.794	(-2.348)	0.427
EMC	0.006	(0.099)	1.087	(0.461)	0.414	-0.203	(-1.450)	0.838	(-1.532)	0.399
HWP	0.069	(2.836)	0.733	(-2.849)	0.330	-0.378	(-2.659)	0.804	(-2.159)	0.349
AMGN	0.061	(2.617)	0.691	(-2.832)	0.363	-0.451	(-3.461)	0.809	(-2.395)	0.405
BRCM	0.240	(2.010)	0.742	(-1.692)	0.338	-0.112	(-1.802)	0.705	(-2.868)	0.322
MER	0.061	(2.844)	0.714	(-2.766)	0.295	-0.479	(-3.292)	0.758	(-2.726)	0.346
NOK	0.119	(2.872)	0.676	(-2.395)	0.265	-0.366	(-2.913)	0.734	(-2.776)	0.323
CHL	0.051	(2.525)	0.704	(-2.373)	0.326	-0.347	(-2.277)	0.889	(-1.377)	0.491
UNPH	0.213	(2.056)	0.809	(-1.080)	0.334	-0.092	(-1.050)	0.809	(-1.702)	0.372
EBAY	0.047	(0.506)	0.882	(-0.882)	0.408	-0.166	(-2.100)	1.207	(2.481)	0.637
JNPR	0.422	(2.837)	0.546	(-2.997)	0.226	-0.121	(-1.728)	0.553	(-3.302)	0.185
CIEN	0.435	(3.038)	0.580	(-2.784)	0.179	-0.103	(-1.362)	0.675	(-2.582)	0.305
BRCD	0.386	(2.561)	0.710	(-1.702)	0.195	-0.026	(-0.318)	0.666	(-2.341)	0.231

Entries report the GMM estimates (and t-statistics in parentheses) of the following relations,

$$\begin{aligned} \texttt{Panel A:} \quad RV_{t,T} &= a + bSW_{t,T} + e, \\ \texttt{Panel B:} \quad \ln RV_{t,T} &= a + b\ln SW_{t,T} + e. \end{aligned}$$

The t-statistics are calculated according to Newey and West (1987) with 30 lags, under the null hypothesis of $a = 0, b = 1$. Columns under "R^2" report the unadjusted R-squared.

Table 7
Numerical illustration of the approximation error for variance swap rates

$\ln v_t/\theta$	$\mathbb{E}^{\mathbb{Q}}[RV]$	$\widehat{SW}$	Total Error ($\mathbb{E}^{\mathbb{Q}}[RV] - \widehat{SW}$)	Jump Error (ε)
The Black-Scholes Model:				
0.0	0.1369	0.1369	0.0000	0.0000
The Merton Jump-Diffusion Model:				
0.0	0.1387	0.1366	0.0021	0.0021
The MJD-Stochastic Volatility Model:				
-3.0	0.0272	0.0273	-0.0001	0.0021
-2.5	0.0310	0.0313	-0.0003	0.0021
-2.0	0.0372	0.0376	-0.0004	0.0021
-1.5	0.0475	0.0477	-0.0001	0.0021
-1.0	0.0645	0.0637	0.0008	0.0021
-0.5	0.0925	0.0905	0.0020	0.0021
0.0	0.1387	0.1356	0.0031	0.0021
0.5	0.2148	0.2107	0.0041	0.0021
1.0	0.3403	0.3353	0.0051	0.0021
1.5	0.5472	0.5410	0.0062	0.0021
2.0	0.8884	0.8799	0.0085	0.0021
2.5	1.4509	1.4377	0.0132	0.0021
3.0	2.3782	2.3561	0.0221	0.0021

Entries report the analytical 30-day variance swap rate ($\mathbb{E}^{\mathbb{Q}}[RV]$), the synthetic approximation of the variance swap rate ($\widehat{SW}$) based on interpolation and extrapolation over five implied volatility quotes, the total approximation error (Total Error = $\mathbb{E}^{\mathbb{Q}}[RV] - \widehat{SW}$), and the error induced by jumps in the underlying asset price (ε) under each model. For the MJD-stochastic volatility model, the first column denotes the log difference between the current instantaneous variance level v_t and its long-run mean θ. For ease of comparison, we represent all swap rates and errors in volatility percentage points.

Table 8
Mean synthetic variance swap rates and variance risk premia from bid and ask option prices

Ticker	Panel A: $SW \times 100$		Panel B: $(RV - SW) \times 100$				Panel C: $\ln(RV/SW)$			
	Ask	Bid	Ask		Bid		Ask		Bid	
SPX	7.52	6.41	-3.45	(-9.81)	-2.34	(-7.44)	-0.76	(-13.48)	-0.60	(-10.81)
OEX	7.44	6.62	-2.90	(-8.32)	-2.08	(-6.26)	-0.65	(-11.73)	-0.54	(-9.53)
DJX	7.90	6.33	-3.51	(-8.15)	-1.94	(-4.94)	-0.73	(-10.93)	-0.51	(-7.60)
NDX	20.76	18.09	-4.06	(-4.29)	-1.40	(-1.43)	-0.36	(-8.29)	-0.23	(-5.30)
QQQ	29.48	24.68	-6.88	(-4.51)	-2.07	(-1.39)	-0.40	(-6.51)	-0.22	(-3.74)
MSFT	22.34	18.12	-5.75	(-5.59)	-1.53	(-1.62)	-0.41	(-9.18)	-0.21	(-4.61)
INTC	27.69	23.60	-0.03	(-0.01)	4.07	(2.15)	-0.11	(-2.37)	0.04	(0.90)
IBM	18.23	16.06	-3.08	(-3.24)	-0.92	(-0.99)	-0.32	(-5.73)	-0.20	(-3.50)
AMER	48.68	42.26	-7.55	(-4.37)	-1.13	(-0.65)	-0.26	(-5.73)	-0.12	(-2.49)
DELL	41.51	34.57	-8.61	(-4.17)	-1.67	(-0.81)	-0.33	(-6.14)	-0.15	(-2.69)
CSCO	37.03	31.15	-5.93	(-3.96)	-0.05	(-0.03)	-0.37	(-5.54)	-0.21	(-3.00)
GE	16.12	12.80	-4.21	(-5.97)	-0.89	(-1.46)	-0.37	(-8.16)	-0.14	(-3.35)
CPQ	37.18	28.60	-6.17	(-3.26)	2.40	(1.34)	-0.27	(-5.05)	-0.02	(-0.31)
YHOO	79.74	70.77	-7.49	(-1.82)	1.48	(0.38)	-0.16	(-2.94)	-0.05	(-0.83)
SUNW	43.28	33.69	-6.58	(-3.72)	3.02	(1.71)	-0.25	(-5.58)	0.01	(0.12)
MU	64.64	56.35	-7.96	(-3.21)	0.33	(0.13)	-0.18	(-4.93)	-0.05	(-1.24)
MO	17.84	13.72	-4.21	(-3.55)	-0.09	(-0.09)	-0.38	(-5.94)	-0.12	(-1.94)
AMZN	112.47	98.81	-23.84	(-3.08)	-10.18	(-1.53)	-0.30	(-4.39)	-0.17	(-2.66)
ORCL	53.46	44.59	-9.73	(-1.82)	-0.86	(-0.16)	-0.25	(-3.58)	-0.05	(-0.65)
LU	34.63	29.07	-3.20	(-1.52)	2.36	(1.07)	-0.17	(-3.45)	-0.01	(-0.23)
TRV	21.38	17.41	-2.02	(-1.26)	1.95	(1.20)	-0.24	(-3.70)	-0.03	(-0.40)
WCOM	32.49	24.44	-5.65	(-2.04)	2.40	(0.98)	-0.29	(-3.85)	0.02	(0.31)
TYC	45.10	37.94	-12.49	(-2.18)	-5.33	(-1.07)	-0.45	(-5.22)	-0.26	(-3.10)
AMAT	51.27	42.13	-7.38	(-3.59)	1.76	(0.86)	-0.23	(-5.74)	-0.01	(-0.36)
QCOM	53.38	45.76	-6.40	(-2.54)	1.22	(0.49)	-0.25	(-4.49)	-0.08	(-1.47)
TXN	39.23	32.91	-1.99	(-1.07)	4.32	(2.35)	-0.13	(-3.08)	0.05	(1.31)
PFE	16.14	12.82	-3.48	(-4.21)	-0.17	(-0.21)	-0.34	(-5.34)	-0.11	(-1.68)
MOT	30.43	24.80	-1.16	(-0.64)	4.48	(2.49)	-0.13	(-2.46)	0.08	(1.50)
EMC	42.83	34.83	-0.90	(-0.29)	7.10	(2.19)	-0.15	(-2.81)	0.08	(1.48)
HWP	27.60	23.20	-2.41	(-1.80)	1.99	(1.57)	-0.19	(-3.74)	-0.00	(-0.03)
AMGN	28.33	23.95	-4.55	(-3.02)	-0.17	(-0.11)	-0.27	(-5.10)	-0.08	(-1.42)
BRCM	97.50	86.72	-6.28	(-1.11)	4.50	(0.81)	-0.12	(-2.29)	-0.01	(-0.20)
MER	26.18	22.75	-2.92	(-2.50)	0.52	(0.44)	-0.20	(-4.48)	-0.05	(-1.10)
NOK	35.57	29.92	-1.90	(-0.97)	3.74	(1.95)	-0.13	(-2.18)	0.04	(0.67)
CHL	22.59	18.38	-3.36	(-2.18)	0.85	(0.59)	-0.27	(-5.02)	-0.05	(-1.05)
UNPH	82.99	74.07	0.75	(0.13)	9.68	(1.61)	-0.09	(-1.44)	0.03	(0.44)
EBAY	78.23	70.37	-9.07	(-1.59)	-1.22	(-0.21)	-0.34	(-4.37)	-0.23	(-2.95)
JNPR	122.76	110.14	-18.05	(-2.24)	-5.43	(-0.73)	-0.21	(-2.98)	-0.10	(-1.53)
CIEN	99.05	86.25	-2.79	(-0.34)	10.01	(1.33)	-0.11	(-1.38)	0.03	(0.38)
BRCD	110.90	94.16	-0.80	(-0.09)	15.95	(1.80)	-0.09	(-1.20)	0.07	(0.88)

Entries compare sample mean of synthetic variance swap rates, synthetic volatility swap rate, and variance risk premia when the synthesis is based on ask and bid option prices, respectively. We also report in parenthesis the t-statistics on the significance of the mean variance risk premia, which are adjusted for serial dependence according to the Newey-West method with a lag of 30 days.

Table 9
Maximum likelihood estimates of the expectation hypothesis regression on S&P 500 index

Ticker	SPX		OEX		DJX	
a	-0.144	(0.162)	-0.151	(0.160)	-0.110	(0.184)
b	0.618	(0.012)	0.698	(0.013)	0.569	(0.013)
θ	3.719	(1.383)	3.720	(1.422)	3.748	(0.921)
ϕ	0.988	(0.002)	0.990	(0.003)	0.995	(0.003)
σ_e^2	6.782	(0.152)	6.803	(0.136)	7.083	(0.168)
σ_η^2	1.989	(0.031)	1.851	(0.105)	1.122	(0.049)
σ_ε^2	0.549	(0.028)	0.422	(0.044)	0.536	(0.036)

Entries report the maximum likelihood estimates of the parameters (and standard errors in parentheses) of the following system of equations:

$$\begin{aligned} RV_{t,T} &= a + bSW_{t,T} + e, \\ \widehat{SW}_{t,T} &= SW_{t,T} + \eta, \\ SW_{t+1} &= \theta(1-\phi) + \phi SW_{t+1} + \varepsilon_{t+1}. \end{aligned}$$

where the error terms are independently normal with zero mean and variances $(\sigma_e^2, \sigma_\eta^2, \sigma_\varepsilon^2)$. The realized variance and swap rates are both scaled up by 100 for the estimation.

Table 10
Summary statistics of variance risk premia from different subsamples

Ticker	$RV \times 100$		$SW \times 100$		$(RV - SW) \times 100$		$\ln(RV/SW)$	
	S1	S2	S1	S2	S1	S2	S1	S2
SPX	3.23	5.31	6.12	7.83	-2.89 (-8.99)	-2.52 (-7.83)	-0.76 (-14.26)	-0.52 (-9.91)
OEX	3.51	6.06	5.84	8.48	-2.32 (-8.02)	-2.42 (-6.28)	-0.65 (-12.55)	-0.47 (-8.30)
DJX	3.84	4.88	6.91	7.03	-3.07 (-6.96)	-2.15 (-6.19)	-0.72 (-10.49)	-0.51 (-8.62)
NDX	9.84	26.51	12.19	29.05	-2.35 (-4.78)	-2.55 (-1.90)	-0.32 (-8.15)	-0.23 (-4.86)
QQQ	15.15	25.36	18.74	29.42	-3.59 (-5.45)	-4.06 (-2.42)	-0.31 (-8.93)	-0.28 (-4.37)
MSFT	12.95	22.12	16.37	24.96	-3.43 (-6.26)	-2.84 (-2.21)	-0.31 (-8.37)	-0.27 (-5.24)
INTC	18.03	42.50	18.06	36.13	-0.03 (-0.04)	6.37 (2.32)	-0.05 (-1.52)	0.03 (0.53)
IBM	13.06	18.26	14.55	20.21	-1.49 (-1.85)	-1.95 (-1.78)	-0.23 (-4.47)	-0.27 (-4.27)
AMER	44.84	34.67	48.24	38.35	-3.41 (-1.94)	-3.69 (-2.24)	-0.16 (-3.67)	-0.20 (-4.02)
DELL	30.24	36.85	37.71	36.75	-7.47 (-5.10)	0.10 (0.04)	-0.27 (-6.61)	-0.17 (-2.40)
CSCO	18.78	55.54	24.37	51.31	-5.59 (-5.97)	4.23 (1.94)	-0.40 (-5.67)	-0.02 (-0.54)
GE	8.63	15.99	10.33	18.91	-1.70 (-3.43)	-2.92 (-3.80)	-0.24 (-5.53)	-0.26 (-5.71)
CPQ	25.87	48.62	28.96	43.06	-3.09 (-2.32)	5.56 (2.08)	-0.17 (-3.56)	-0.01 (-0.13)
YHOO	67.28	77.79	65.33	83.51	1.95 (0.48)	-5.72 (-1.54)	-0.06 (-0.91)	-0.12 (-2.82)
SUNW	26.75	64.20	30.03	58.59	-3.29 (-2.57)	5.61 (2.38)	-0.17 (-3.74)	0.03 (0.92)
MU	49.94	67.42	50.57	73.56	-0.63 (-0.33)	-6.14 (-2.04)	-0.07 (-1.96)	-0.15 (-3.85)
MO	13.03	14.53	13.91	17.74	-0.88 (-0.77)	-3.21 (-3.26)	-0.16 (-2.55)	-0.37 (-5.93)
AMZN	99.63	77.32	93.34	114.57	6.30 (1.38)	-37.24 (-5.15)	0.00 (0.02)	-0.44 (-7.69)
ORCL	32.94	61.56	41.31	59.37	-8.37 (-1.34)	2.19 (0.70)	-0.17 (-2.15)	-0.07 (-1.50)
LU	21.70	61.12	23.49	54.93	-1.79 (-1.50)	6.19 (1.80)	-0.12 (-2.59)	0.06 (1.23)
TRV	19.25	19.47	18.76	19.26	0.49 (0.44)	0.21 (0.11)	-0.05 (-0.99)	-0.19 (-2.50)
WCOM	18.68	47.27	19.67	48.19	-0.99 (-0.73)	-0.91 (-0.21)	-0.15 (-2.22)	-0.08 (-0.97)
TYC	24.83	37.35	25.30	50.17	-0.47 (-0.22)	-12.81 (-2.03)	-0.21 (-2.68)	-0.42 (-4.88)
AMAT	37.21	54.36	37.99	57.98	-0.78 (-0.51)	-3.62 (-1.45)	-0.08 (-2.16)	-0.15 (-3.65)
QCOM	38.79	57.82	40.01	60.22	-1.22 (-0.54)	-2.40 (-0.86)	-0.15 (-2.55)	-0.16 (-3.09)
TXN	27.67	51.51	25.81	49.70	1.86 (1.43)	1.80 (0.75)	-0.00 (-0.06)	-0.06 (-1.44)
PFE	13.11	12.07	14.16	14.23	-1.05 (-1.66)	-2.16 (-2.24)	-0.12 (-2.96)	-0.33 (-4.08)
MOT	20.00	49.25	18.78	44.97	1.21 (1.34)	4.28 (1.55)	0.01 (0.29)	-0.08 (-1.19)
EMC	27.37	66.85	28.58	54.28	-1.20 (-0.83)	12.57 (2.74)	-0.08 (-1.82)	0.07 (1.15)
HWP	19.88	36.02	19.05	36.95	0.83 (0.62)	-0.94 (-0.78)	-0.06 (-1.17)	-0.12 (-3.01)
AMGN	21.81	26.00	19.52	32.60	2.29 (2.07)	-6.60 (-4.01)	-0.04 (-0.81)	-0.30 (-5.66)
BRCM	73.73	99.72	64.07	103.44	9.67 (3.55)	-3.72 (-0.60)	0.09 (2.43)	-0.12 (-2.22)
MER	23.24	23.30	21.65	27.01	1.59 (1.28)	-3.71 (-4.15)	-0.03 (-0.64)	-0.21 (-5.61)
NOK	25.34	43.30	23.27	42.43	2.08 (1.07)	0.86 (0.46)	-0.02 (-0.24)	-0.05 (-1.17)
CHL	15.39	25.06	14.85	28.09	0.54 (0.50)	-3.03 (-1.62)	-0.10 (-1.92)	-0.22 (-4.45)
UNPH	58.47	112.73	56.11	101.30	2.37 (0.53)	11.43 (1.61)	-0.04 (-0.65)	0.01 (0.23)
EBAY	94.39	58.85	94.19	64.49	0.21 (0.03)	-5.64 (-1.28)	-0.05 (-0.70)	-0.36 (-4.89)
JNPR	82.78	109.59	90.77	119.78	-7.99 (-1.25)	-10.19 (-1.32)	-0.14 (-1.83)	-0.14 (-2.20)
CIEN	80.18	115.23	71.03	114.64	9.15 (1.27)	0.59 (0.07)	0.03 (0.35)	-0.09 (-1.28)
BRCD	86.29	112.83	68.60	104.42	17.69 (3.46)	8.41 (0.97)	0.09 (1.61)	-0.01 (-0.11)

Entries report the sample averages of the annualized realized variance, the synthetic variance swap rates, and the (log) variance risk premia during two subsamples. The first subsample (S1) is from January 4, 1996 to March 23, 2000. The second subsample (S2) is from March 24, 2000 to February 28, 2003. We also report in parenthesis the t-statistics on the significance of the mean variance risk premia, which are adjusted for serial dependence according to the Newey-West method with a lag of 30 days.