Buy Rough, Sell Smooth

Paul Glasserman[*] Pu He[†]

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Abstract

Recent work has documented *roughness* in the time series of stock market volatility and investigated its implications for option pricing. We study a strategy for trading stocks based on measures of their implied and realized roughness. A strategy that goes long the roughest-volatility stocks and short the smoothest-volatility stocks earns statistically significant excess annual returns of 6% or more, depending on the time period and strategy details. The profitability of the strategy is not explained by standard factors. We compare alternative measures of roughness in volatility and find that the profitability of the strategy is greater when we sort stocks based on implied rather than realized roughness. We interpret the profitability of the strategy as compensation for near-term idiosyncratic event risk.

Key words: Volatility, fractional Brownian motion, trading strategies, volatility skew.

1 Introduction

A recent line of research has found evidence that stock price volatility is *rough*, in the sense that the evolution of volatility is rougher than the paths of ordinary Brownian motion. The evidence for rough volatility comes from two sources: the time series behavior of realized volatility, and an empirical regularity of option-implied volatility at short maturities that turns out to be well explained by roughness. See Gatheral, Jaisson, and Rosenbaum (2018), Bayer, Friz, and Gatheral (2016), Fukasawa (2017), Bennedsen, Lunde, and Pakkanen (2016), and El Euch, Fukasawa, Gatheral, and Rosenbaum (2018a) for background and further references.

Rough models of stochastic volatility replace an ordinary Brownian motion driving the dynamics of volatility with a *fractional* Brownian motion (fBM). The fBM family, indexed by a single parameter, includes ordinary Brownian motion and also processes with smoother and rougher paths. Empirical estimates here and in Gatheral et al. (2018) and Bennedsen et al. (2016) find parameter values smaller than 1/2 (the case of ordinary Brownian motion), corresponding to rougher paths. We refer to these estimates as measures of *realized* roughness.

[*]Columbia Business School, pg20@columbia.edu
[†]Columbia Business School, phe19@gsb.columbia.edu

By *implied* roughness, we mean estimates extracted from option prices. Implied volatilities from equity put options are ordinarily skewed, meaning that they are larger at lower strikes, particularly at short maturities. But the steepness of this skew typically falls quickly as the maturity extends — more quickly than predicted by most stochastic volatility models. Rough volatility models capture this feature. Stocks with greater realized roughness exhibit fast mean-reversion in volatility; stocks with greater implied roughness exhibit a fast decay in their implied-volatility skew.

The implications of these empirical regularities have received little attention beyond option markets. In this article, we seek to shed light on the possible sources and consequences of rough volatility by studying a trading strategy that trades stocks — not options — based on roughness in volatility. We sort stocks based on measures of realized or implied roughness and analyze a strategy that goes long the roughest quintile and short the smoothest quantile. When sorted on implied roughness, the strategy earns excess returns of 6% or more, after controlling for standard factors. The strategy is profitable in 13 out of the 17 years in our sample, including 2007, 2008, and 2009. The strategy based on realized roughness earns somewhat lower returns and is less robust to standard controls.

These results have several implications. First, they show that roughness matters for stock returns and is not just a feature of option markets. Second, they point to potential differences between implied and realized roughness, though in theory the two should coincide. Third, we will argue that the profitability of our implied rough-minus-smooth strategy reflects compensation for near-term idiosyncratic event risk. The fast decay in the implied volatility skew associated with implied roughness indicates near-term downside uncertainty that will be resolved quickly. We support this interpretation by examining the performance of our strategy near two types of events: our strategy earns higher returns near earnings announcements (which mainly resolve company-specific uncertainty) and lower returns near interest rate announcements by the Federal Reserve (which resolve market-wide uncertainty).

Efforts to date to model an underlying source of roughness have focused on market microstructure and the splitting of large orders, particularly El Euch et al. (2018b), Jusselin and Rosenbaum (2018). However, these models do not offer clear predictions on what types of stocks should exhibit greater roughness, which limits their application to our setting. Nevertheless, we investigate possible connections between roughness and market liquidity. We confirm a positive association between roughness and illiquidity (which may be seen as consistent with Jusselin and Rosenbaum (2018)); but we also find that controlling for illiquidity reduces but does not eliminate the profitability of our implied strategy. Moreover, this strategy is limited to stocks with significant options trading, and these are generally larger and more liquid stocks. The profitability of our strategy therefore cannot be explained by an illiquidity premium.

Our results present an interesting contrast to the work of Xing, Zhang, and Zhao (2010). They find that a steep skew (corresponding to expensive puts at low strikes) forecasts negative earnings surprises, a finding we confirm in more recent data. This pattern supports a strategy of buying stocks with lower skews and selling stocks with steeper skews. One might expect stocks with a faster skew decay (greater implied roughness) to start with a steeper skew, in which case the strategy of Xing et al. (2010) would lead to selling rough and buying smooth, just the opposite of the strategy we find profitable. Moreover, we find that roughness does not forecast earnings surprises, reinforcing the notion that the profitability of rough-minus-smooth reflects compensation for risk rather than cash flow predictability. Together these patterns indicate that the information in roughness is distinct from the skewness measure in Xing et al. (2010).

Section 2 provides background on realized and implied roughness, and it explains the procedures we use to estimate both quantities. In Section 3, we evaluate the performance of strategies that buy the roughest quintile of stocks and short the smoothest quintile of stocks each month. We evaluate strategies using realized and implied measures of roughness, after controlling for standard factors. In Section 4, we control for additional factors through double sorts that hedge out other effects, including several measures of illiquidity and the levels of implied volatility and skewness. We find that returns on the implied strategy are robust to these controls. We also test robustness to these controls using Fama and MacBeth (1973) time-series averages of cross-sectional regressions. In Section 5, we find that the performance of our strategy is enhanced when restricted to stocks with earnings announcements in the subsequent month and diminished near Federal Reserve announcements. We interpret these findings as evidence that rougher stocks (particularly as measured by implied roughness) are those facing near-term downside uncertainty.

2 Realized and Implied Roughness

2.1 Realized Roughness

To discuss roughness, we first recall the definition of fractional Brownian motion; for additional background, see Mandelbrot and Van Ness (1968) and Section 7.2 of Samorodnitsky and Taqqu (1994). A fractional Brownian motion with Hurst parameter $H \in (0,1)$ is a mean-zero Gaussian process $\{W_t^H, -\infty < t < \infty\}$ with stationary increments and covariance function given by

$$\mathbb{E}[W^H(t)W^H(s)] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right). \quad (1)$$

The case $H = 1/2$ corresponds to ordinary Brownian motion. With $H \in (1/2, 1)$, fractional Brownian motion exhibits long-range dependence; processes with $H \in (0, 1/2)$ have paths that are rougher than those of ordinary Brownian motion, with small H indicating greater roughness.

As one indication of greater roughness, we have the following property of the moments of the increments of fractional Brownian motion. For any $t \in \mathbb{R}$, and $\Delta \geq 0$, and any $q > 0$,

$$\mathbb{E}[|W^H_{t+\Delta} - W^H_t|^q] = \mathbb{E}[|Z|^q]\Delta^{qH}, \quad Z \sim N(0,1). \tag{2}$$

With smaller H, increments over a short interval Δ have larger moments.

As an example of a rough volatility model for an asset price $\{S_t, t \geq 0\}$, we could set

$$d \log S_t = \mu \, dt + \sigma_t \, dW_t \tag{3}$$

$$d \log \sigma_t = \nu \, dW^H_t; \tag{4}$$

this is a special case of a single-factor version of what Gatheral et al. (2018) call the rough Bergomi model, after Bergomi (2009). More generally, the model specifies a mean-reverting log volatility process

$$d \log \sigma_t = -\kappa(\log \sigma_t - m) + \nu \, dW^H_t. \tag{5}$$

Here, μ, κ, m, and ν are constants, W is an ordinary Brownian motion, W^H is a fractional Brownian motion with $H \in (0, 1/2)$, and W and W^H may be correlated. The parameter H determines the roughness of the volatility process.

Empirical evidence for roughness in the time series of volatility can be found in Gatheral et al. (2018), Bennedsen et al. (2016), Livieri, Mouti, Pallavicini, and Rosenbaum (2018), and later in this paper. Abi Jaber and El Euch (2018) present an approximation method for rough volatility models that suggests a simple interpretation: rough volatility arises from mixing mean-reverting volatility processes with different speeds of mean reversion, driven by an ordinary Brownian motion, including components with arbitrarily fast mean reversion. The connection between roughness and fast mean reversion is also supported by the analysis of option prices in Garnier and Sølna (2018).

If we could observe $\log \sigma_t$ at times $t = 0, \Delta, 2\Delta, \ldots$ for some small $\Delta > 0$, we could estimate H by estimating

$$\mathbb{E}[|\log \sigma_{t+\Delta} - \log \sigma_t|^q] \tag{6}$$

for various values of $q > 0$, and then applying (2) to extract H. This is the method of Gatheral et al. (2018), which they apply more generally to estimate roughness, without necessarily assuming the specific model in (3)–(4) or (5).

In practice, σ_t cannot be observed and must be estimated, so we proceed as follows. Using trades from the Trade and Quote (TAQ) data, we apply the realized kernel method of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2009) to estimate the daily integrated variance of returns; taking the square root yields our estimated daily volatility.[1] We obtained similar results using the realized variance of 5-minute returns, but the realized kernel method is designed to be less sensitive to microstructure noise.

The rest of the estimation procedure works with these daily volatilities, which we write as $\hat{\sigma}_d$, with d indexing days. We apply (6) with $q = 2$, estimating second moments over intervals of ℓ days, $\ell = 1, 2, \ldots, 10$. In each month, for each stock and each lag ℓ, we calculate

$$z_2(\ell) = \frac{1}{T-\ell} \sum_{d=1}^{T-\ell} (\log \hat{\sigma}_{d+\ell} - \log \hat{\sigma}_d)^2, \tag{7}$$

where T is the number of days in the month. Based on (2), we expect

$$z_2(\ell) \approx \nu^2 \ell^{2H}.$$

We therefore run a regression

$$\log z_2(\ell) = \beta_1 + \beta_2 \log \ell + \epsilon, \tag{8}$$

to estimate H as $\beta_2/2$. We also estimate the volatility of volatility ν by setting $\log \nu = \beta_1/2$. This procedure yields an estimate of H (and ν) for each stock in each month.

Gatheral et al. (2018) estimate (7) and (8) for moments of several orders q and then run a regression of the slope in (8) against q. We find that using several moments rather than just $q = 2$ leads to very similar estimates of H. [2]

2.2 Implied Roughness

By implied roughness we mean the value of H obtained by fitting option prices to a rough volatility model.

A conventional approach to evaluating an implied parameter would proceed as follows. Choose a specific model with some free parameters — in this case, a rough volatility model; find the parameters that bring the model's option prices closest to a set of market prices.

[1] We use the non-flat Parzen kernel as implemented in Kevin Sheppard's toolbox at https://www.kevinsheppard.com/MFE_Toolbox.

[2] In tests of alternative estimation methods on simulated data, for which we know H, we have found that the main source of error is the estimation of the daily integrated variances $\hat{\sigma}_d^2$ from intraday returns, rather than the estimation of H from the daily volatilities.

Applying this approach to extract H from option prices raises two issues. The first is a practical consideration: pricing options in rough volatility models requires Monte Carlo simulation, so inverting prices to evaluate H for hundreds of stocks and months is computationally daunting. The second issue is more fundamental: a misspecified model may lead to an incorrect value of H, even if the "true" volatility process in rough.

To circumvent these issues, we follow a simpler and more robust approach, based on the term structure of the at-the-money (ATM) skew. Write $\sigma_{BS}(k,\tau)$ for the Black-Scholes implied volatility of an option with time-to-maturity τ and log-moneyness $k = \log(K/S)$, where K is the option's strike price and S is the current level of the underlying. The ATM skew at maturity τ is given by

$$\phi(\tau) = \left| \frac{\partial \sigma_{BS}(k,\tau)}{\partial k} \right|_{k=0} \tag{9}$$

An empirical regularity of the ATM skew is that it flattens at longer maturities. This pattern is illustrated in Figure 1, which shows fitted implied volatilities for JPMorgan Chase on June 5, 2012, using data from OptionMetrics. (We discuss the details of the fitting procedure below.) The horizontal axis shows the ratio of the strike price to the current stock price, so the ATM skew is the slope at a ratio of 1. The different curves correspond to different maturities. The slope is steepest (most negative) at the shortest maturity of three days and quickly flattens as we move to longer maturities.

The expansions of Fukasawa (2011), Bayer et al. (2016), El Euch et al. (2018a), and Forde and Zhang (2017) characterize the rate of decay of the ATM skew for a very broad range of rough volatility models. These results (in particular as in Fukasawa (2011)) show that the ATM skew admits an approximation of the form

$$\phi(\tau) \approx \text{constant} \times \tau^{H-1/2}, \quad \text{as } \tau \downarrow 0. \tag{10}$$

In other words, the ATM skew exhibits a power law decay at short maturities, with an exponent determined by H.

This idea is illustrated in Figure 2, which replicates similar figures in Gatheral et al. (2018). The horizontal axis records time-to-maturity τ, and the vertical axis records ATM skew $\phi(\tau)$. Each dot in the figure shows an estimate of $\phi(\tau)$, all calculated on September 15, 2005 (left panel) or June 20, 2013 (right panel), based on OptionMetrics data. The smooth curve in the figure shows a power law fit to the data, from which we estimate the exponent. In this example, the exponents are -0.48 (left) and -0.452 (right) corresponding to $H = .02$ and $H = 0.048$, respectively.

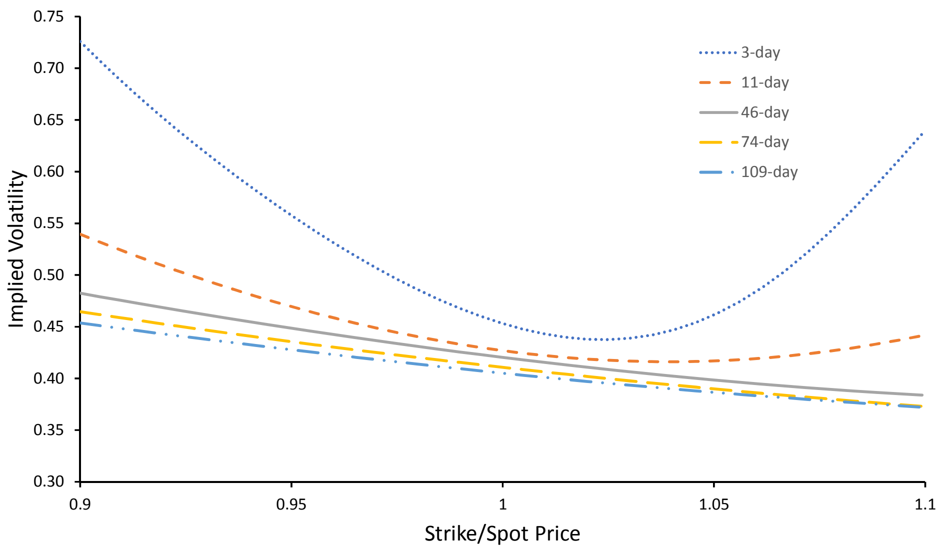

Figure 1: JPM implied volatilities on June 5, 2012. The curves show cubic spline fits at various maturities using raw data from OptionMetrics, plotted against the ratio of the put strike K to the spot price S. The ATM skew is the slope at $K/S = 1$. Its absolute value falls quickly as the maturity increases.

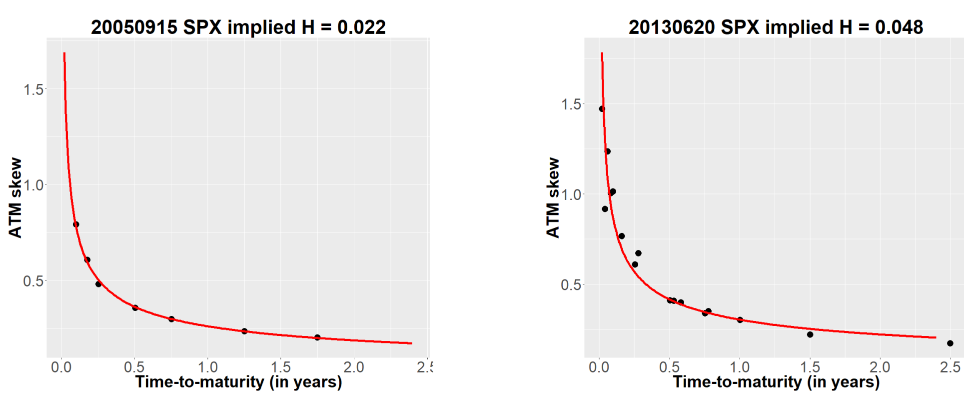

Figure 2: Term structure of the ATM skew for the S&P 500 index, as in similar figures in Gatheral et al. (2018). The charts plot the slope of the ATM skew against option maturity on Sep 15, 2005 (left) and Jun 20, 2013 (right), using OptionMetrics data.

This is the approach we will use to calculate an option-implied value of H, after providing details of the calculation. The method is easy to use and readily lends itself to evaluating an implied H for hundreds of stocks, each day for nearly 20 years. The method is robust because it exploits the general property of rough volatility models in (9) rather than the detailed structure of a specific model.

Some may object to using the rate of decay of the ATM skew to extract an implied measure of roughness on the grounds that certain stochastic volatility models driven by ordinary Brownian motion may also be able to fit the term structure of ϕ. For example, Bergomi and Guyon (2012) fit what appears to be a power law decay using a linear combination of two exponentials. Some might prefer to follow the more conventional approach with which we began this section, fitting a specific model to market prices and finding the value of H that fits best. But if the model fits option prices well, *that approach will lead to the same value of H* because if the model fits the data, then the market prices satisfy (10). Using (10) directly is simply a more efficient and more robust way of arriving at the implied H. Calling it implied roughness is also much simpler than calling it the rate of decay of the ATM skew (plus 1/2).

To carry out this approach, we proceed as follows. First, we merge CRSP and OptionMetrics data to link stock prices and option prices. Next, we filter out options following standard rules in the literature; these are detailed in the appendix. On each day for each stock, using only the filtered data, we use a cubic spline to fit implied volatility as a function of $\log(K/S)$. We take the derivative of the spline at $\log(K/S) = 0$ as the ATM skew $\phi(\tau)$. Then we run a regression

$$\log \phi(\tau) = c + (H - 1/2) \log \tau + \epsilon;$$

that is, we add 1/2 to the estimated slope in this regression to evaluate the implied H.

In addition to the realized measure discussed in Section 2.1 and the implied measure discussed here, we have tested a third measure — realized roughness of implied volatility, as in Livieri et al. (2018). In this approach, for each stock we take the ATM implied volatility, and we evaluate the realized roughness (following (7)–(8)) from the stock's time series of implied volatility. We have found that investment results based on this measure are very similar to those using realized roughness, so we do not discuss them further.

2.3 Descriptive Statistics of Realized and Implied Roughness

Our focus is on the cross-sectional relationship between roughness and stock returns, so in Table 1 we present summary statistics on the cross-sectional variation of implied and realized roughness. In each month we calculate the mean, standard deviation and several quantiles

(25%, 50%, 75%) of implied and realized roughness measures for all stocks; we then take the time-series average of these summary statistics and report them in the table.

As discussed in Section 2.2, we have values of implied roughness for only a subset of stock-month pairs. We refer to this subset as the "implied universe." In contrast, by the "full universe" we mean the larger set of stock-month pairs for which we have sufficient data to calculate a realized H and link TAQ, CRSP, and Compustat data. See the appendix for details on the filters applied.

In the last column of Table 1 we report summary statistics for realized roughness on the implied universe. The results in the table indicate that implied estimates of H are a bit larger than realized estimates and that this may be partly due to differences in the implied and realized universes, but the differences are small. Livieri et al. (2018) find that values of realized H estimated from the time series of implied volatility are generally larger than values estimated from realized volatility, and they attribute the difference to a smoothing effect over an option's time to maturity. This effect may play some role in our estimates of implied H.

	Implied H	Realized H	Realized H on Implied Universe
avg Mean	0.18	0.07	0.09
avg S.D.	0.21	0.10	0.10
avg 25th pctile	0.06	0.00	0.02
avg median	0.18	0.06	0.08
avg 75th pctile	0.30	0.14	0.15

Table 1: Monthly averages of cross-sectional summary statistics. The last column shows statistics for realized H estimated from the subset of stocks for which implied estimates are available.

Table 2 reports time-series averages of cross-sectional means and standard deviations by industry, using industry classifications from Ken French's website.[3] The estimates are very consistent across different sectors.

3 Sorted Portfolios

In this section, we test the performance of trading strategies that pick stocks based on realized or implied roughness. Each month, we sort stocks based on roughness (realized or implied) and group them into quintile portfolios. We evaluate the performance of a strategy that buys the roughest (smallest H) quintile and shorts the smoothest (largest H) quintile, holding these

[3]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Industry	Implied H Avg Mean	Implied H Avg S.D.	Realized H Avg Mean	Realized H Avg S.D.	Realized H on Imp. Univ. Avg Mean	Realized H on Imp. Univ. Avg S.D.
Consumer NonDurables	0.18	0.23	0.07	0.10	0.08	0.10
Consumer Durables	0.17	0.19	0.07	0.10	0.09	0.09
Manufacturing	0.17	0.19	0.08	0.10	0.09	0.10
Energy	0.20	0.20	0.08	0.10	0.09	0.09
Chemicals	0.19	0.19	0.08	0.10	0.09	0.09
Business Equipment	0.18	0.21	0.08	0.10	0.09	0.10
Telecom	0.19	0.22	0.08	0.10	0.09	0.10
Utilities	0.17	0.22	0.08	0.10	0.10	0.09
Shops	0.18	0.20	0.07	0.10	0.08	0.10
Health	0.17	0.23	0.07	0.10	0.09	0.10
Finance	0.18	0.19	0.07	0.10	0.10	0.10
Other	0.17	0.21	0.07	0.10	0.09	0.10

Table 2: Monthly averages of cross-sectional summary statistics by industry. The last two columns show statistics for realized H estimated from the subset of stocks for which implied estimates are available.

positions for one month. We calculate value-weighted returns in the month following the month in which portfolios are formed, and then repeat the procedure for the next month.

In addition to calculating average returns, we calculate excess returns (alphas) relative to various factor models: a single-factor (CAPM) model using the overall return of the market, net of the risk-free rate; the three-factor Fama and French (1993) model (with factors for the market, size, and book-to-market) augmented with a momentum factor, as in Carhart (1997); the five-factor model of Fama and French (2015) (with factors for the market, size, book-to-market, earnings robustness, and investment conservativeness), again augmented with momentum.

We use stock prices from CRSP, factor returns from Ken French's website, and option implied volatilities from OptionMetrics. The OptionMetrics data starts in 1996, but we start from 2000 because much more data is available after 2000 than in the earler years.

Table 3 shows results for stocks sorted on implied roughness. The columns show results for the quintile portfolios, sorted from smoothest (highest H) to roughest (lowest H). The last column shows results for the long-short strategy. The strategy earns an average monthly return of 0.49% (5.9% annually). Its alphas with respect to the various factor models range from 0.47% to 0.52% monthly, or 5.6% to 6.2% annually. The numbers in brackets are Newey and West (1987) t-statistics, and show that these excess returns are all statistically significant. Statistical significance at the 10%, 5% and 1% levels is indicated by *, **, and ***, respectively.

The lower half of Table 3 shows features of the quintile portfolios. By construction, the

	1 Smooth	2	3	4	5 Rough	5-1
Mean	0.22	0.39	0.37	0.33	0.71	0.49
Std. Dev.	4.82	4.76	4.69	5.08	5.26	2.63
CAPM Alpha	-0.28**	-0.11	-0.13	-0.19	0.19	0.47**
	[-2.51]	[-1.48]	[-1.39]	[-1.35]	[1.37]	[2.43]
FF-3-MOM Alpha	-0.33***	-0.07	-0.07	-0.07	0.16	0.49***
	[-2.88]	[-0.94]	[-0.94]	[-0.64]	[1.22]	[2.63]
FF-5-MOM Alpha	-0.29***	-0.04	-0.04	0.03	0.24*	0.52***
	[-2.74]	[-0.58]	[-0.47]	[0.30]	[1.70]	[2.76]
Implied H	0.46	0.27	0.18	0.09	-0.11	
Size in billion $	14.64	18.87	19.07	15.79	7.84	
Book-to-Market	0.48	0.44	0.42	0.41	0.43	
Number of stocks	153	152	153	152	152	
Portfolio persistence	61%	75%	76%	74%	63%	

Table 3: Performance of portfolios sorted on implied roughness. Alphas are monthly values in percent. Numbers in brackets are t-statistics.

average implied H values decrease from left to right. The smoothest quintile has H close to the Brownian value of 1/2, and the roughest quintile has a negative average H. A negative H is not meaningful as a Hurst parameter, but can certainly arise as an implied parameter through (10).

We see from Table 3 that the average book-to-market ratio is quite consistent across the quintiles, but size (measured by market cap) seems to be positively correlated with H, a point we will investigate further. The last row shows the percentage of stocks in each quintile that remain in the quintile from one month to the next.

Table 4 reports corresponding results using realized roughness. Panel A uses the full universe of CRSP stocks; Panel B limits the set of stocks used each month to the "implied universe," meaning those that pass the filters we use for the implied roughness portfolios in Table 4.

Both panels of Table 4 show that stocks with rougher volatility (smaller realized H) tend to outperform stocks with smoother volatility (larger realized H). Comparing the last column of Table 4 (showing performance of the rough-minus-smooth long-short strategy), with the last column of Table 3, indicates that the effect is not quite as strong and not quite as statistically significant sorting on realized as sorting on implied roughness. Portfolio persistence is a bit greater using realized roughness, indicating that this strategy has somewhat lower turnover.

Comparing Panels A and B of Table 4, we find that sorting on realized roughness yields higher alphas when we limit the universe of stocks to those for which we can also calculate

		1 Smooth	2	3	4	5 Rough	5-1
PANEL A	Mean	0.22	0.52	0.59	0.71	0.60	0.38
	Std. Dev.	4.99	4.69	4.65	4.33	4.94	2.66
	CAPM Alpha	-0.30***	0.02	0.09	0.24**	0.10	0.40**
		[-3.08]	[0.18]	[1.20]	[2.45]	[0.71]	[2.03]
	FF-3-MOM Alpha	-0.27***	0.00	0.05	0.24***	0.03	0.31
		[-2.97]	[0.00]	[0.61]	[2.71]	[0.22]	[1.56]
	FF-5-MOM Alpha	-0.19**	0.01	0.02	0.13	-0.01	0.17
		[-2.13]	[0.09]	[0.23]	[1.52]	[-0.10]	[0.97]
	Realized H	0.23	0.12	0.06	0.01	-0.05	
	Size in billion $	6.67	5.18	4.62	4.03	3.36	
	Book-to-Market	0.69	0.67	0.64	0.65	0.70	
	Number of stocks	611	613	614	614	614	
	Portfolio persistence	79%	80%	80%	80%	80%	
PANEL B	Mean	0.11	0.27	0.51	0.58	0.59	0.47
	Std. Dev.	5.28	4.89	4.85	4.58	4.89	2.89
	CAPM Alpha	-0.42***	-0.24**	0.00	0.10	0.09	0.51**
		[-3.55]	[-2.16]	[0.01]	[0.94]	[0.71]	[2.51]
	FF-3-MOM Alpha	-0.38***	-0.19*	0.01	0.15	0.13	0.51***
		[-3.49]	[-1.75]	[0.06]	[1.50]	[1.00]	[2.73]
	FF-5-MOM Alpha	-0.25**	-0.13	0.00	0.17	0.08	0.33*
		[-2.31]	[-1.19]	[-0.02]	[1.59]	[0.60]	[1.73]
	Realized H	0.24	0.14	0.08	0.03	-0.04	
	Size in billion $	18.38	16.37	15.31	13.69	12.18	
	Book-to-Market	0.44	0.43	0.43	0.43	0.44	
	Number of stocks	151	151	151	151	152	
	Portfolio persistence	80%	82%	82%	82%	81%	

Table 4: Performance of portfolios sorted on realized roughness. Alphas are monthly values in percent. Panel A shows results for all stocks and Panel B is limited to the stocks used in Table 3 for comparison.

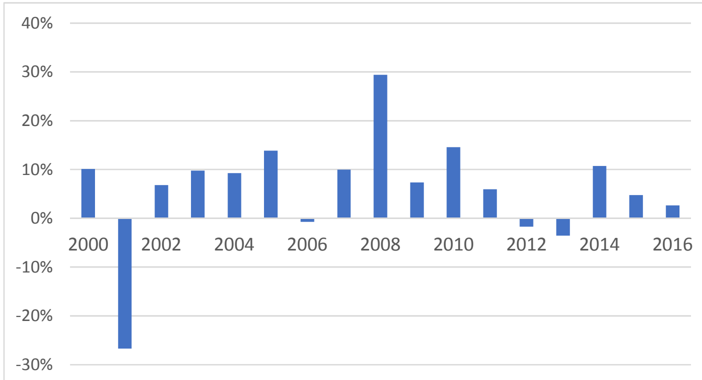

Figure 3: Annual performance of rough-minus-smooth strategy based on implied roughness.

implied roughness. This is surprising because the stocks in the more limited universe are larger on average and have lower book-to-market ratios; smaller stocks and high book-to-market stocks generally have higher expected returns. We see a similar effect in Table 3, where controlling for the Fama-French factors improves performance.

It is worth noting that in both panels of Table 4 the highest returns are generally associated with the fourth quintile of realized H rather than the fifth quintile. The performance of the realized strategy could be substantially improved by buying the fourth quintile, rather than the fifth, and shorting the first. For consistency and to avoid data snooping, we work exclusively with the original long-fifth, short-first strategy; however, this may underestimate the efficacy of trading on realized roughness.

Tables 3 and 4 show average performance over the full period 2000–2016. To illustrate how performance varies over time, Figure 3 shows annual performance by year for the implied strategy. Remarkably, sorting on implied roughness, the rough-minus-smooth strategy is profitable in 13 out of the 17 years, including 2007–2009; indeed, 2008 was the strategy's best year. The strategy's only large significant loss is in 2001, and the loss that year is almost entirely attributable to September, the month of the 9/11 attacks. We return to this point in Section 5.

Figure 4 shows annual performance of the strategy based on realized roughness. The figure shows performance of the realized strategy on the full universe and on the implied universe. Except in the early part of the sample, where the option data is more limited, the realized strategy generally performs similarly on the full and restricted sets of stocks. This confirms that the performance in Figure 3 is not attributable to the set of stocks included in the implied universe. Indeed, comparing Figures 3 and 4 shows that the realized and implied strategies have done well at different times, suggesting that combining the two signals could lead to even

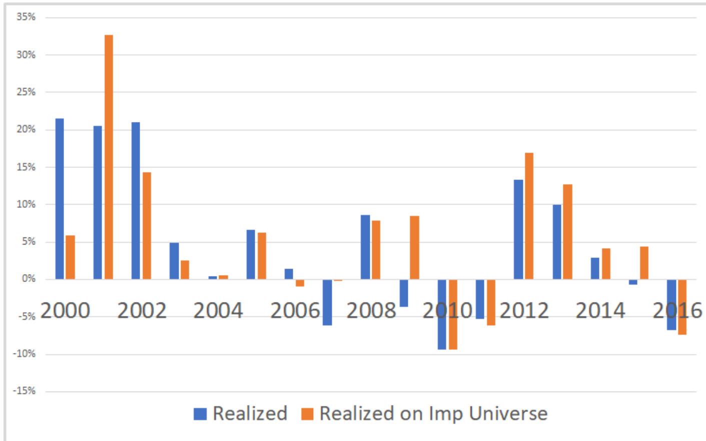

Figure 4: Annual performance of rough-minus-smooth strategy based on realized roughness, using all stocks or just the implied universe.

better performance. However from Table 4 we see that the realized strategy provides a smaller FF-5-MOM alpha than the implied strategy. We will see in Section 4 that the realized strategy is also less robust to controls for other factors.

The performance of the implied strategy in 2008 raises the question of whether sorting on roughness implicitly tilts the long-short portfolio to favor some industries over others. For example, a strategy that shorts bank stocks would have performed well in 2008. However, we saw in Table 2 that roughness estimates are similar across industries. Moreover, the average implied and realized H estimates for finance companies in particular are in the middle of the ranges across industries, indicating that a rough-minus-smooth strategy does not tend to favor or disfavor financial stocks.

4 Controlling for Other Factors

To better understand the performance of the rough-minus-smooth strategies, in this section we add controls for additional factors. We first discuss factors that might influence performance and then evaluate their impact using two methods — double sorts and Fama and MacBeth (1973) regressions.

4.1 Liquidity

We observed previously that in Table 3 the average market cap across the five quintiles increases with H: rougher stocks tends to be smaller on average. This pattern suggests the possibility that roughness may reflect lower liquidity and therefore that a rough-minus-smooth strategy

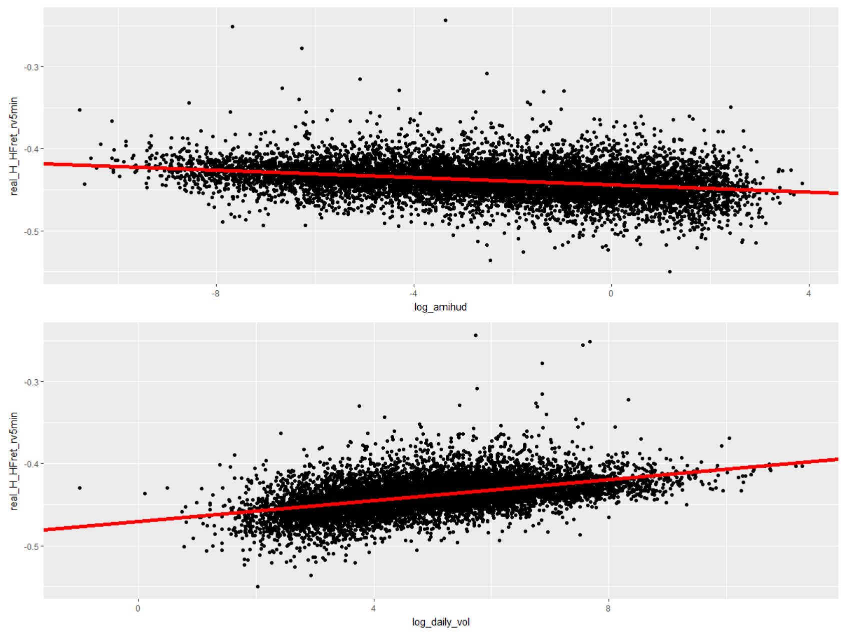

Figure 5: Realized roughness and liquidity. The figures plot realized roughness against the log of the Amihud illiquidity measure (top) and log daily volume (bottom). Each point shows a single stock in a single month.

earns an illiquidity premium. This possibility is tempered by the fact that the stocks that pass the filters for calculating implied roughness are larger, on average, than those that do not. The question therefore requires a more systemic investigation.

A connection between realized roughness and liquidity was noted in an early version of Bennedsen et al. (2016), but it was removed from subsequent versions of that paper. Bennedsen et al. (2016) compare estimates of realized roughness with daily volume of trading in a stock.

In addition to trading volume, we consider the widely-used Amihud (2002) illiquidity measure. The Amihud measure for a single stock in a single month sums the absolute values of the daily returns and divides the sum by the dollar volume for the month. Larger values of the Amihud measure are interpreted as indicating lower liquidity, whereas larger values of trading volume are associated with greater liquidity.

Figures 5 and 6 compare, respectively, realized and implied estimates of H with the log of the Amihud measure and log daily volume. Each dot in the figure corresponds to a single stock in a single month. Consistent with the earlier version of Bennedsen et al. (2016), we find a positive correlation (0.55) between realized H and log daily volume. Consistent with this pattern, we find a negative correlation (-0.46) between realized H and the log Amihud measure.

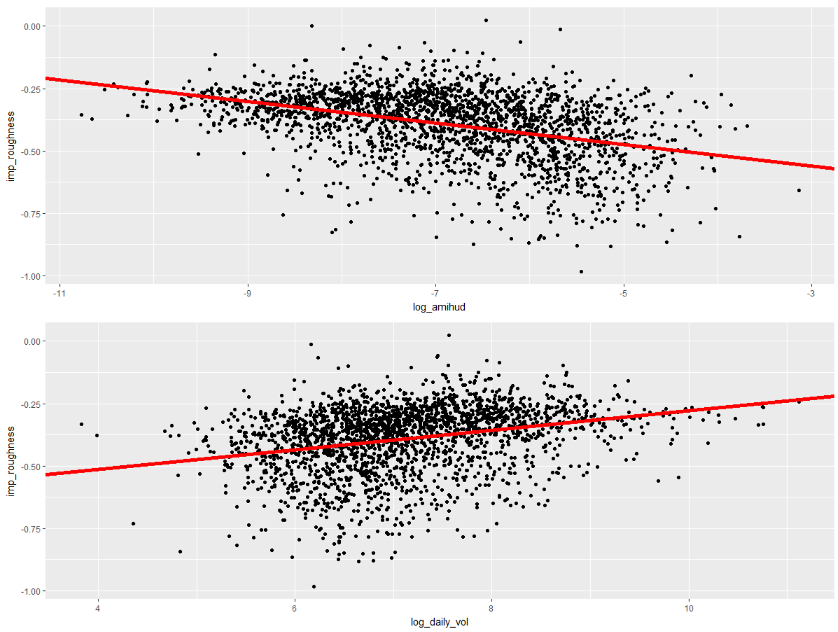

Figure 6: Implied roughness and liquidity. The figures plot implied roughness against the log of the Amihud illiquidity measure (top) and log daily volume (bottom). Each point shows a single stock in a single month.

The results using implied roughness in Figure 6 are qualitatively similar but not as strong. The correlation between implied H and log daily volume is 0.28, and the correlation with the log Amihud measure is -0.40.

Beyond these empirical patterns, a potential link between roughness and liquidity is interesting because of efforts to explain realized roughness through market microstructure; see El Euch et al. (2018b) and Jusselin and Rosenbaum (2018). However, the explanations developed to date are highly stylized, and they do not make clear predictions about whether greater roughness should be associated the more or less liquidity.[4]

4.2 Implied Volatility and Skewness

Implied roughness is a relatively complex feature of a stock's implied volatility surface, involving differences in implied volatilities across both strikes and maturities. To try to isolate the source of alpha in the implied rough-minus-smooth strategy, we will therefore control for more basic features — the level of the ATM implied volatility and the shape of implied volatility skew.

Several authors (particularly Conrad et al. (2013) and Xing et al. (2010)) have documented

[4]According to Mathieu Rosenbaum (personal communication), Jusselin and Rosenbaum (2018) implies a longer transient price impact when H is smaller, which would be consistent with the correlations we find.

predictability in stock returns using measures of implied volatility and skewness. A fast decay in the ATM skew (low implied H) is potentially associated with high degree of near-term skewness or implied volatility. We therefore control for these factors.

As our measure of ATM implied volatility, we use the implied volatility for a one-month call with strike closest to the spot price as reported in implied volatility surface data set from OptionMetrics. We denote this by $\sigma_{1m}^{Call}(\frac{K}{S}=1)$. Similar to Xing et al. (2010), we use as our measure of implied volatility skew

$$\text{XZZ-skew} = \sigma_{1m}^{Put}(\frac{K}{S}=0.9) - \sigma_{1m}^{Call}(\frac{K}{S}=1), \tag{11}$$

the difference between the one-month implied volatility for a put with moneyness closest to 0.9 and the one-month implied volatility for a call with strike closest to the spot price.

Xing et al. (2010) find that larger values of their skew measure predict lower stock returns in the cross section, a pattern that we find holds up as well using more recent data and a slightly different skew measure. Interestingly, this effect appears to run in the opposite direction of what we find using implied roughness. A smaller implied H indicates a faster decay of the ATM skew. If this indicates a higher initial value of the ATM skew, then the finding of Xing et al. (2010) would suggest that stocks with smaller implied H have lower stock returns, yet we find exactly the opposite. This suggests that the performance of the rough-minus-smooth strategy is not explained by the XZZ-skew, a hypothesis we will check in the next sections.

4.3 Double Sorts

To control for factors like liquidity or skewness that might influence the returns on our roughness quintile portfolios, in this section, we apply a standard double-sorting procedure.

Suppose, for example, that we want to control for illiquidity, using the Amihud measure. For each month, we proceed as follows. We sort stocks into deciles according to the Amihud measure. Within each of these illiquidity deciles, we sort stocks by roughness (realized or implied). We then take the roughest quintile from each of the illiquidity deciles — this is our rough portfolio. Similarly, we form our smooth portfolio by grouping all stocks that are in the smoothest quintile of any of the illiquidity deciles.

Under this construction, all levels of illiquidity are represented in the rough and smooth portfolios, so the performance of the rough-minus-smooth strategy should be unaffected by illiquidity: we have hedged out illiquidity. We sort into ten portfolios based on illiquidity in the first step in order to achieve a better balance of the conditioning factor between our controlled rough portfolio and smooth portfolio. The same procedure allows us to hedge out the effect of any other factor by first sorting on that factor.

We apply double sorts that condition on the following variables, one at a time:

- Average daily volume for each stock;
- The Amihud illiquidity measure;
- Turnover, measured as a stock's monthly trading volume divided by the average shares outstanding of that stock during the month;
- ATM implied volatility, as measured by the implied volatility for a 30-day option with strike closest to spot price;
- XZZ-skew, as defined in (11);
- Size (as measured by log market cap), book-to-market, and trailing 12-month return.

Table 5 shows the performance of the rough-minus-smooth strategy based on implied roughness after controlling for each of these factors through double sorts. The table shows average returns and alphas using either FF3-Mom or FF5-Mom factor models.

The first three rows of the table consider liquidity measures. Sorting first on average daily volume or the Amihud illiquidity measure reduces but does not eliminate the profitability of the strategy. Some reduction in performance is to be expected, given the correlation we documented in Section 4.1 between implied roughness and these measures. But the profitability of the strategy remains significant, particularly as measured by alpha relative to the Fama-French 5-factor with momentum, ranging from 3.1% to 5.4% per year, depending on the measure used, with t-statistics ranging from 2.0 to 3.0. Controlling for turnover actually increases the mean return of the strategy, with average monthly returns of 0.54%, and increases the t-statistics to around 4.0. In short, liquidity by itself cannot account for the performance of the rough-minus-smooth strategy.

The next two rows of the table control for implied volatility and the ATM skew. Controlling for ATM implied volatility improves the average return and alphas to 7%, except for the FF5Mom alpha, which decreases a bit to 4.9% annually. Controlling for the XZZ-skew measure of Xing et al. (2010) has only a small effect on the average return, alphas and t-statistics, and all alphas remain statistically significant. Thus, these well-known features of the implied volatility surface — the level of ATM volatility and skewness in implied volatility — cannot account for the performance of the rough-minus-smooth strategy.

Conditioning Variable	Mean Return	CAPM Alpha	FF3Mom Alpha	FF5Mom Alpha
Average Daily Volume	0.23*	0.21	0.23*	0.26**
	[1.84]	[1.60]	[1.81]	[2.01]
Average Daily Amihud	0.45***	0.46***	0.46***	0.40***
	[3.23]	[3.31]	[3.21]	[2.65]
Turnover	0.54***	0.53***	0.52***	0.45***
	[3.96]	[3.90]	[3.53]	[2.98]
XZZ Skew	0.46***	0.44**	0.49***	0.45***
	[2.79]	[2.54]	[2.96]	[2.61]
ATM Implied Volatility	0.59***	0.59***	0.59***	0.41**
	[3.54]	[3.51]	[3.32]	[2.28]
Size	0.41***	0.42***	0.46***	0.42***
	[3.24]	[3.31]	[3.53]	[3.16]
Book-to-Market	0.34**	0.32**	0.36**	0.35**
	[2.44]	[2.17]	[2.57]	[2.34]
12-Month Return	0.45***	0.43***	0.43***	0.48***
	[3.29]	[3.06]	[3.17]	[3.38]

Table 5: Performance of rough-minus-smooth portfolios using implied roughness, constructed through double sorts on various factors, for the period Jan 2000 through Jun 2016. Mean return and alphas are monthly values in percent. Numbers in brackets are t-statistics based on Newey-West standard errors.

The last three factors in the table serve as robustness checks. Sorting on size, book-to-market, and trailing returns may slightly reduce the performance of the strategy but does not eliminate — and may even strengthen — statistical significance.

Table 6 shows corresponding results based on realized roughness, using the full universe of stocks (Panel A) or the implied universe (Panel B). Here we find that controlling for liquidity (through average daily volume or the Amihud measure) removes the significance of returns and alphas of the rough-minus-smooth strategy. Controlling for size does as well in Panel A. These results suggest a strong association between realized roughness and illiquidity. In contrast, controlling for implied volatility and the ATM skew actually enhances the performance of the strategy. This further indicates that the effect of roughness, whether realized or implied, is not already reflected in the ATM volatility or the ATM skew.

Conditioning Variable	Mean Return	CAPM Alpha	FF3Mom Alpha	FF5Mom Alpha
PANEL A: Full Universe				
Average Daily Volume	0.14	0.15	0.14	0.05
	[1.47]	[1.63]	[1.51]	[0.60]
Average Daily Amihud	0.12	0.17	0.12	-0.05
	[0.94]	[1.37]	[1.03]	[-0.45]
Turnover	0.38***	0.38**	0.33**	0.20
	[2.60]	[2.49]	[2.34]	[1.46]
XZZ Skew	0.54***	0.57***	0.53***	0.30*
	[3.10]	[3.22]	[3.36]	[1.95]
ATM Implied Volatility	0.54***	0.56***	0.59***	0.41**
	[2.81]	[2.92]	[3.17]	[2.15]
Size	0.12	0.18	0.13	-0.04
	[0.90]	[1.47]	[1.27]	[-0.37]
Book-to-Market	0.35***	0.38***	0.35***	0.20
	[2.63]	[2.90]	[2.73]	[1.55]
12-Month Return	0.51***	0.54***	0.50***	0.37**
	[3.06]	[3.12]	[3.19]	[2.34]
PANEL B: Implied Universe				
Average Daily Volume	0.22	0.25	0.21	0.08
	[1.43]	[1.64]	[1.48]	[0.56]
Average Daily Amihud	0.40*	0.47**	0.41**	0.22
	[1.94]	[2.31]	[2.40]	[1.35]
Turnover	0.60***	0.61***	0.63***	0.53***
	[3.13]	[3.09]	[3.38]	[2.79]
XZZ Skew	0.49**	0.53**	0.51***	0.27
	[2.41]	[2.51]	[2.88]	[1.54]
ATM Implied Volatility	0.62***	0.63***	0.62***	0.43*
	[2.81]	[2.77]	[2.88]	[1.95]
Size	0.49**	0.55***	0.53***	0.35**
	[2.41]	[2.70]	[3.13]	[2.10]
Book-to-Market	0.31*	0.34**	0.33**	0.14
	[1.83]	[2.05]	[2.05]	[0.88]
12-Month Return	0.55***	0.60***	0.57***	0.41**
	[2.96]	[3.08]	[3.47]	[2.47]

Table 6: Performance of rough-minus-smooth portfolios using realized roughness, constructed through double sorts on various factors, for the period Jan 2000 through Jun 2016. Mean return and alphas are monthly values in percent. Numbers in brackets are t-statistics based on Newey-West standard errors.

4.4 Fama-MacBeth Regressions

To further investigate whether the performance of the rough-minus-smooth strategy is explained by other factors, we run regressions based on the specification

$$Ret_{i,t} = b_{0t} + b_{1t} H_{i,t} + b'_{2t} CONTROLS_{i,t-1} + e_{i,t}, \qquad (12)$$

where $Ret_{i,t}$ is the return of stock i in month t; $H_{i,t}$ is either realized or implied roughness of stock i in month t; $CONTROLS_{i,t-1}$ is a vector of controls; and the $e_{i,t}$ are error terms. We estimate coefficients and their standard errors through Fama and MacBeth (1973) regressions: in each month t, we run cross-sectional regressions to estimate b_{0t}, b_{1t}, and b_{2t}; we then take the time-series averages of these regression coefficients and use their time-series variation to estimate standard errors. Compared to the double sorts tested previously, these regressions have the advantage of allowing the simultaneous inclusion of multiple controls, but they have the disadvantage of imposing linearity on the relationship between returns and controls.

An alternative approach would be to run a panel regression to estimate (12) with no dependence on t in the coefficients. Since we are mainly interested in the cross-sectional relationship between roughness and returns, we would include month fixed-effects; and since monthly returns have very low autocorrelation, we would estimate standard errors clustered by month, following Petersen (2009). However, as also discussed in Petersen (2009), Section 3, Fama-MacBeth standard errors are more accurate than panel regressions with clustered standard errors under two conditions that are appropriate to our setting: (1) the main source of dependence in error terms comes from time effects (correlations in returns of different stocks in the same month); and (2) the number of time periods (201 months) is not very large compared with the number of stocks per month (up to 1108 stocks per month in the implied universe and 3577 per month for the full universe). The dependence in (1) is dealt with effectively by Fama-MacBeth regressions. The values in (2) would require the estimation of a very large covariance matrix between different stocks based on limited data in order to cluster by time. In light of these considerations, we use Fama-MacBeth regressions.

Table 7 shows the results. Panel A tests implied H; Panel B test realized H on the implied universe; and Panel C tests the realized H on the full universe of stocks. Each panel shows two regressions, one including only the corresponding roughness measure, and one including multiple controls. All explanatory variables have been standardized (cross-sectionally in each month) to make the coefficients comparable. Returns are in decimals, so a return of 5% is recorded as 0.05.

Panel A confirms the negative relationship between returns and implied H; including controls increases the magnitude and significance of the coefficient. Panel B shows that realized H has a significant relationship with returns when restricted to the implied universe, but this relationship is eliminated by the controls. In Panel C we find no significant relationship between realized H and returns on the full universe of stocks, with or without controls. Interestingly, our results confirm a strong negative relationship between returns and the skewness measure of Xing et al. (2010), while also showing in Panel A that this control does not explain the effectiveness of implied roughness.

Our controls include return volatility and implied volatility, so the regressions in Table 7 also control for the volatility risk premium (Carr and Wu (2008)) measured as the difference between implied and realized volatility. In particular, Panel A shows that the profitability of the implied strategy cannot be attributed to the volatility risk premium.

	PANEL A		PANEL B		PANEL C	
Variable	Reg 1	Reg 2	Reg 3	Reg 4	Reg 5	Reg 6
Intercept	0.0043	0.0046	0.0043	0.0046	0.0088*	0.0078
	[0.83]	[0.91]	[0.83]	[0.90]	[1.66]	[1.49]
Implied H	-0.0010**	-0.0014***				
	[-2.04]	[-3.43]				
Realized H			-0.0015**	-0.0003	-0.0003	-0.0002
			[-2.10]	[-0.68]	[-0.51]	[-0.56]
XZZ Skew		-0.0034***		-0.0034***		-0.0036***
		[-5.30]		[-5.20]		[-6.56]
ATM volatilities		-0.0063***		-0.0062**		-0.0047**
		[-2.62]		[-2.56]		[-2.25]
Log Option Volume		-0.0033*		-0.0034*		-0.0015
		[-1.85]		[-1.91]		[-1.44]
Log Option Open Interest		0.0025		0.0024		-0.0012
		[1.58]		[1.53]		[-1.22]
Log Stock $ Volume		0.0044		0.0046		0.0006
		[1.38]		[1.45]		[0.25]
Log Stock Volume		0.0019		0.0018		0.0061***
		[1.06]		[1.03]		[3.58]
Turnover		-0.0019		-0.0020		-0.0027**
		[-1.47]		[-1.54]		[-2.51]
Book-to-Market		-0.0003		-0.0002		-0.0010
		[-0.29]		[-0.21]		[-0.36]
Log Size		-0.0095***		-0.0094***		-0.0079***
		[-2.89]		[-2.84]		[-3.01]
Past 6M Return		-0.0006		-0.0007		-0.0007
		[-0.49]		[-0.56]		[-0.59]
Past 12M Return		0.0010		0.0011		0.0010
		[0.93]		[0.98]		[1.01]
Past Return Volatility		-0.0024*		-0.0024		-0.0043***
		[-1.65]		[-1.63]		[-2.76]
Past Return Skew		-0.0005		-0.0004		-0.0002
		[-0.95]		[-0.88]		[-0.60]
Adj. R^2	0.29%	13.15%	0.46%	13.18%	0.14%	9.21%

Table 7: Fama-MacBeth return regressions. Panel A, B, C each have two regression results, one with only one regressor (either implied or realized H) and the other including a complete set of controls. Panel A shows results for implied H. Panel B presents results for realized H on the implied universe. Panel C uses realized H and the unrestricted universe. Numbers in brackets are t-statistics based on Newey-West standard errors.

5 Event Risk: Earnings Announcements and FOMC Meetings

In this section, we argue that cross-sectional differences in implied roughness of individual stocks reflect differences in near-term downside risk; we interpret the profitability of the rough-minus-smooth strategy as compensation for bearing this risk. We support this interpretation by considering the performance of the strategy around two types of events: company-specific earnings announcements, and interest rate announcements by the Federal Reserve's Open Markets Committee (FOMC). We present three pieces of evidence to support our argument. The strategy's profitability is greatest when restricted to stocks with earnings announcements in the subsequent month, when the potential for near-term idiosyncratic risk is high; roughness does not forecast earnings, suggesting that the strategy's profitability reflects compensation for risk rather superior selection of profitable companies; the strategy is not profitable in the lead-up to FOMC announcements — a period of elevated aggregate near-term risk rather than idiosyncratic near-term risk.

5.1 Earnings Announcements

5.1.1 Testing for Earnings Surprise Predictability

We begin by testing whether roughness predicts earnings surprises, as a possible explanation for the profitability of our strategy. Positive earnings surprises tend to be followed by stock price appreciation, so a signal that forecasts earnings surprises can serve as the basis for a profitable trading strategy. We will see, however, that this does not explain the profitability of the roughness signal.

We focus on the subset of data defined by

$$I^{ea} = \{(i,t)\text{: stock } i \text{ has an earnings announcement in month } t\},$$

using earnings announcement data from IBES. Letting I denote the full universe of stock-month pairs for which we have an implied roughness measure, $I \setminus I^{ea}$ denotes the subset that do not have an earnings announcement.

To measure earnings surprises, we use the standardized unexpected earnings (SUE) score from IBES. SUE measures the difference between a company's actual earnings and the mean forecast by analysts, normalized by the standard deviation of analyst forecasts in the previous quarter. To test for a relation between SUE and roughness, we use the Fama-MacBeth regression approach, meaning that we first run the following regression for every month t,

$$SUE_{i,t} = b_{0t} + b_{1t}H_{i,t-1} + e_{i,t}, \quad (i,t) \in I^{ea},$$

FM Regression		Portfolio Sorting
Variable	Coef	Difference in SUE
Implied H	0.035	-0.087
	[0.155]	[-0.324]
ATM skew	-2.744***	-0.300***
	[-2.827]	[-3.277]

Table 8: Left panel shows coefficients and t-statistics in Fama-MacBeth regressions of standardized unexpected earnings (SUE) on implied roughness and ATM skew. Right panel shows the difference in average SUE in the top and bottom quintiles of stocks sorted by implied roughness or ATM skew. Numbers in brackets are t-statistics based on Newey-West standard errors.

where $H_{i,t-1}$ denotes the implied roughness calculated for stock i in month $t-1$. We then average the b_{1t} over all months t and calculate standard errors adjusted for autocorrelation.

For comparison, we run the same analysis replacing implied roughness with the ATM skew in (11). Using data through 2005, Xing et al. (2010) show that a greater ATM skew forecasts negative earnings surprises. In other words, before companies report disappointing earnings, low-strike puts become more expensive. Xing et al. (2010) interpret this as evidence that investors with inside information trade on that information through options and that the stock market is slow to incorporate the information in option prices.

The left panel of Table 8 reports estimated coefficients and t-statistics for the two regressions. The bottom row confirms the finding of Xing et al. (2010), with the benefit of more than ten years of additional data. The coefficient on the ATM skew is large, negative, and statistically significant. In contrast, the coefficient on implied roughness is indistinguishable from zero. Implied roughness does not forecast earnings surprises, and the implied roughness signal is distinct from the information in the ATM skew.

The right panel of Table 8 further supports these conclusions. In this analysis, in each month t we limit ourselves to stocks with earnings announcements in month $t+1$. We sort these stocks into quintile portfolios based on roughness in month t. The table shows the difference in average SUE (in month $t+1$) between the highest and lowest roughness quintiles. The table shows the same comparison for stocks sorted on ATM skew in month t. We again see that a higher ATM skew forecasts negative earnings surprises whereas there is no relation between roughness and SUE. The profitability of the rough-minus-smooth strategy is not grounded in forecasting earnings.

5.1.2 Strategy Performance Near Earnings Announcements

Next we compare the performance of the rough-minus-smooth strategy when restricted to subsets of stocks based on the timing of earnings announcements. Specifically, we evaluate performance in three cases:

I^{ea}:	sort stocks with announcements in month t based on roughness in month $t-1$;
$I \setminus I^{ea}$:	sort stocks without announcements in month t based on roughness in month $t-1$;
$I^{ea,100}$:	same as I^{ea} but only if at least 100 stocks in I have announcements in month t.

In all cases, portfolios are formed in month $t-1$ and returns are evaluated in month t.

Performance results under these restrictions are shown in the top panel of Table 9. Compared with the right-most column of Table 3, restricting attention to earnings-announcement stocks I^{ea} improves monthly alphas by roughly 40%, from around 0.50 to around 0.70. The estimated alphas are now only marginally significant, but this may be because the sample size (the number of stocks available each month) is now smaller. The results for $I^{ea,100}$ support this hypothesis: in months with at least 100 stocks available, the estimated monthly alpha goes above 1.0 (an annual alpha of more than 12%) and is highly significant. (These results are not sensitive to the choice of 100 as threshold.) In contrast, when we exclude stocks with earnings announcements, the $I \setminus I^{ea}$ alphas are smaller than the alphas in Table 3 and not statistically significant.

Taken together, the results in the top panel of the table show that sorting on roughness is most effective when applied to stocks facing a near-term idiosyncratic risk in the form of an earnings surprise. We interpret this to mean that greater roughness signals greater near-term downside risk, and that this risk is compensated with a price discount and a subsequent higher average return.

The analysis in the top panel is necessarily restricted to the universe I of stock-month pairs for which implied roughness is available. As a benchmark, the second panel shows market returns and alphas for the restricted sets of stocks used in the top panel. The second panel treats each restricted set as a long-only portfolio. The bottom row shows that stocks without earnings announcements earn lower returns; but the main implication of the second panel is that the results in the top panel cannot be attributed to the restrictions in the definitions of I^{ea}, $I^{ea,100}$, and $I \setminus I^{ea}$. Moreover, the average implied H values in these three sets are nearly identical and all in 0.17–0.18.

This point is reinforced by the bottom panel. Here we drop the restriction to I and compare performance on the full universe of stocks with earnings announcements F^{ea} and without

	Mean Return	CAPM Alpha	FF3Mom Alpha	FF5Mom Alpha
Rough Minus Smooth (implied roughness universe)				
Earnings Announcement Stocks	0.71*	0.71*	0.70*	0.74*
(I^{ea})	[1.71]	[1.68]	[1.73]	[1.72]
EA Stocks – Threshold 100	1.00***	1.03***	1.07***	1.11***
($I^{ea,100}$)	[2.60]	[2.67]	[2.91]	[2.95]
No Earnings Announcement	0.30	0.28	0.29	0.29
($I \setminus I^{ea}$)	[1.41]	[1.23]	[1.25]	[1.22]
Long Only (implied roughness universe)				
Earnings Announcement Stocks	0.63*	0.12	0.14	0.20
(I^{ea})	[1.70]	[0.88]	[1.08]	[1.49]
EA Stocks – Threshold 100	0.47	-0.03	-0.01	0.09
($I^{ea,100}$)	[1.13]	[-0.28]	[-0.06]	[0.76]
No Earnings Announcement	0.29	-0.22***	-0.17***	-0.15**
($I \setminus I^{ea}$)	[0.81]	[-3.23]	[-2.69]	[-2.53]
Long Only (full universe)				
Earnings Announcement Stocks	0.77**	0.24**	0.21**	0.20**
(F^{ea})	[2.23]	[2.33]	[2.20]	[2.12]
No Earnings Announcement	0.38	-0.16***	-0.17***	-0.18***
($F \setminus F^{ea}$)	[1.12]	[-2.91]	[-2.79]	[-3.07]

Table 9: Top panel: Implied roughness strategy performance on stocks with earnings announcements in the next month (I^{ea}), in months with at least 100 candidate stocks ($I^{ea,100}$), and on stocks without earnings announcements $I \setminus I^{ea}$. Middle panel: Long-only performance on the same sets of stocks. Bottom panel: Long-only comparison of stocks with and without earnings announcements in the full universe of stock-month pairs. Numbers in brackets are t-statistics based on Newey-West standard errors.

$F \setminus F^{ea}$. Stocks with earnings announcements earn higher returns than stocks without. Put differently, investors are compensated for bearing earnings announcement risk. Sorting on roughness identifies the stocks where this risk compensation is greatest.

These observations invite speculation on the implied strategy's losses in September 2001, which we mentioned in our discussion of Figure 3. Based on quintiles formed in August, the strategy would be long stocks facing near-term downside uncertainty. These stocks may have proved to be the most vulnerable to the disruptions and shock of the 9/11 attacks, leading the strategy to incur large losses.

5.2 Strategy Performance Near FOMC Announcements

We now turn from considering individual corporate events to FOMC announcements, which are among the most important scheduled events for the aggregate market. Indeed, Lucca and Moench (2015) find that the excess return of the stock market is mainly earned during the 24-hour window before the earnings announcement; in other periods the average excess return is not statistically different from zero. If, as we have suggested, implied roughness ranks stocks on near-term idiosyncratic risk, then our strategy should not be expected to enhance returns in the lead-up to FOMC announcements.

Following Lucca and Moench (2015), we consider announcements for the eight scheduled FOMC meetings each year. (Public announcements began in 1994, and our sample starts in 2000.) We define the pre-announcement period as the interval from the close of trading on day $d-2$ to the close on day d, where d denotes the FOMC announcement date. We compare the performance of our strategy when it is restricted to invest in (or outside of) the pre-announcement period.

Our strategy is based on monthly data, so these timing restrictions require some explanation. When we limit ourselves to investing in pre-announcement periods, we evaluate performance only in the eight months of the year with scheduled announcements. In each such month, we take the return for the month to be the return over the two days that make up the pre-announcement period. We can apply this restriction to stock-month pairs in the implied roughness universe, in which case we label it $I^{preFOMC}$, and we can apply the restriction to the full universe of stock-month pairs and label it $F^{preFOMC}$.

We label the opposite restrictions $I^{nonFOMC}$ and $F^{nonFOMC}$. For the four months of each year without an FOMC announcement, the "nonFOMC" return is the just the ordinary monthly return. For the other eight months, the "nonFOMC" return is the return for the month excluding the two-day pre-announcement window.

	Mean Return	CAPM Alpha	FF3Mom Alpha	FF5Mom Alpha
Rough Minus Smooth (implied roughness universe)				
pre FOMC ann	0.09	0.11*	0.08	0.11*
($I^{preFOMC}$)	[1.53]	[1.72]	[1.40]	[1.78]
non pre-FOMC ann	0.43**	0.40**	0.42**	0.43**
($I^{nonFOMC}$)	[2.42]	[2.16]	[2.36]	[2.38]
Long Only (implied roughness universe)				
pre FOMC ann	0.41***	0.27*	0.33**	0.32**
($I^{preFOMC}$)	[3.20]	[1.92]	[2.24]	[2.12]
non pre-FOMC ann	0.12	-0.36***	-0.35***	-0.30***
($I^{nonFOMC}$)	[0.34]	[-3.58]	[-3.25]	[-2.70]
Long Only (full universe)				
pre FOMC ann	0.41***	0.26	0.37*	0.36*
($F^{preFOMC}$)	[2.59]	[1.38]	[1.81]	[1.76]
non pre-FOMC ann	0.26	-0.25**	-0.29***	-0.29***
($F^{preFOMC}$)	[0.78]	[-2.49]	[-2.80]	[-2.66]

Table 10: Top panel: Implied roughness strategy performance in the pre-announcement period ($I^{preFOMC}$) and outside the pre-announcement period ($I^{nonFOMC}$). Middle panel: Long-only performance of the implied universe I during the same time periods. Bottom panel: Long-only performance of the full universe during the same time periods.

The results are shown in Table 10, which has the same format as Table 9. The top panel compares the rough-minus-smooth strategy with the "preFOMC" and "nonFOMC" restrictions; the second panel shows long-only results with the same restrictions and limited to the universe of stock-month pairs for which we have implied roughness; the bottom panel shows long-only results when the restrictions are applied to the full universe of stock-month pairs.

The bottom panel is closest to the work of Lucca and Moench (2015) and consistent with their conclusions: stocks earn higher returns during the pre-announcement period than at other times. The pattern is nearly identical in the middle panel, indicating that the I universe is representative of the full universe in its response to FOMC announcements.

In the top panel, the results flip. Sorting on implied roughness is not profitable during the pre-announcement period, when all stocks are facing a high degree of near-term systematic risk. The rough-minus-smooth strategy earns its returns the rest of the year, away from the pre-announcement period.

Recall that implied roughness measures the rate of decay of the ATM skew. A larger ATM skew indicates greater concern for downside risk, so a projected rapid decay in the ATM skew suggests concerns for downside risk that will be resolved quickly. Taking the results of this section together with those of Section 5.1.2, we see that proximity to a company-specific event enhances the performance of our strategy whereas proximity to an aggregate event has the opposite effect. This pattern suggests that the near-term downside risk captured by implied roughness is idiosyncratic. Moreover, the profitability of the rough-minus-smooth strategy suggests that investors are compensated for bearing this particular type of risk.

Our investigation does not explain why this near-term idiosyncratic risk should earn a risk premium. But the puzzle is not specific to our setting. Leaving aside roughness, the bottom panel of Table 9 records a well-known phenomenon of stocks earning higher returns around earnings announcements. Sorting on implied roughness pushes this effect further.

6 Conclusions

We have investigated strategies for trading stocks based on measures of roughness in their volatility. We have compared long-short strategies based on realized roughness (calculated from high-frequency stock returns) and implied roughness (calculated from option prices). Both measures support a strategy of buying stocks with rougher volatilities and selling stocks with smoother volatilities; but sorting on implied roughness yields higher returns and is more robust to controlling for other factors. In particular, it is robust to controlling for illiquidity and the level of the ATM skew.

We have argued that implied roughness provides a measure of near-term idiosyncratic risk: a stock with greater implied roughness is one that the market perceives to have downside uncertainty that will be resolved quickly. On this interpretation, the profitability of our rough-minus-smooth strategy reflects compensation for bearing this risk. The performance of our strategy is enhanced near earnings announcements, when stocks face elevated idiosyncratic risk, and it is suppressed near FOMC announcements, when the dominant near-term risk is systematic.

Our work raises interesting questions for the rough volatility framework. Part of the appeal of this framework is that it simultaneously explains key features of realized volatility and the implied volatility surface extracted from option prices. Yet we find important differences in working with realized and implied measures of roughness. Estimating either measure of roughness from limited data presents significant difficulties, so it is unclear if the differences we observe present a challenge to the theoretical framework or simply call for better estimation methods.

Appendix: Filtering of Option Data

We apply some filtering rules when computing implied roughness to avoid using questionable data from illiquid options. We largely follow the rules in Xing et al. (2010), which are quite standard in the empirical literature on options. We require the following features:

- Underlying stock volume for that day > 0;
- Underlying stock price for that day $> \$5$;
- Implied volatility of the option $\geq 3\%$ and $\leq 200\%$;
- The option's open interest > 0;
- The option's volume can be 0 but has to be non-missing;
- The option has time to maturity $\tau \geq 5$ and $\tau \leq 365$ calendar days.

In addition, when estimating non-parametrically the ATM skew for each time-to-maturity τ, we set the minimal number of implied volatilities needed to measure the ATM skew for a particular time-to-maturity (for a particular stock on a particular day) at four.

When running a regression of the ATM skew term structure to estimate an implied H, we use the following filtering rules: The minimal number of ATM skews along the dimension of time-to-maturity (for a particular stock on a particular day) is three, meaning that there must be at least three points in the regression

$$\log \phi(\tau) = c + (H - 1/2) \log \tau + \epsilon.$$

For each day, we apply these filters to call and put options separately. If for a stock, both calls and puts pass the filtering rules on a given day, we use the average implied roughness $(H^{call} + H^{put})/2$ as the implied measure on that day; otherwise we use whichever type of option passes, and if neither passes the filters, we mark the value as NA for that stock on that day.

When forming monthly portfolios, we need to aggregate daily implied roughness measures into a monthly measure. We include a stock only if it has more than 15 non-NA daily implied roughness estimates for that month. (This is similar to what is used by Ang, Hodrick, Xing, and Zhang (2006).) Otherwise, we mark the implied measure for that stock and month as NA. These restrictions define our implied universe of stock-month pairs.

In estimating daily realized variance $\hat{\sigma}_d^2$ in Section 2.1, we use trade data only and we apply the data cleaning steps in Barndorff-Nielsen et al. (2009).

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