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Valuing Options On Periodically-Settled Stocks

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SUMMARY

In some countries, for example France, stocks bought or sold during an account period have their settlement deferred to a designated settlement date. We explain how to value and hedge cash-settled European- or American-style options on stocks whose settlement is deferred. To do this we introduce the notion of "bare" (immediately settled) and "dressed" (deferred-settled) stock prices. It is the volatility of bare prices that is fundamental.

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THE PROBLEM

In France, options on the CAC-40 Index settle in cash and are hedged with stock or with futures contracts. At exercise or expiration, options settle in two business days. As far as stock settlement is concerned, the year is divided into *account periods* of several weeks; stock transactions occurring at any time within one account period all settle on a given *account period settlement date*. You can buy stock for hedging and hold it at no financing cost, sometimes for several weeks, until the account period's settlement date. Similarly, if you sell stock, you receive no cash or interest income until settlement.

These financing peculiarities affect hedging costs, which in turn affect options values, because the theory behind options valuation involves the cost of a riskless hedge. In this paper, we explain and evaluate the effects of periodic settlement on both European- and American-style cash-settled options.

A two-day delay between the exercise and settlement of options is negligible compared to the possibility of a delay of several weeks between the trade and settlement of stock. Therefore, to keep things simple, we'll assume below that options settlement always occurs immediately on exercise or expiration.

IMMEDIATE SETTLEMENT: BARE STOCK PRICES

We start by assuming we live in a world where stock trades settle immediately. Therefore, the stock price quoted by the market indicates the price for immediate settlement. We refer to this price as the *bare* stock price, because it hasn't yet been *dressed-up* by the complexities involved in quoting a price for deferred settlement.

The US market is a close approximation to this world. All stock transactions settle in five business days. Buyers of stocks gain five days worth of financing from the start of a transaction and lose five days worth when they unwind. To the extent that the difference between five days of interest at the beginning and at the end of the transaction is negligible, we can ignore the deferral of settlement, and regard trade and settlement as occurring at the same time.

The world of bare stock prices is the one we usually employ when applying the Black-Scholes equation or the Cox-Ross-Rubinstein binomial tree method to option pricing. The stock price at the nodes of the tree are bare prices; we usually ignore the effects of deferred settlement. It is the price of the bare stock which is assumed to evolve lognormally in time with a given volatility.

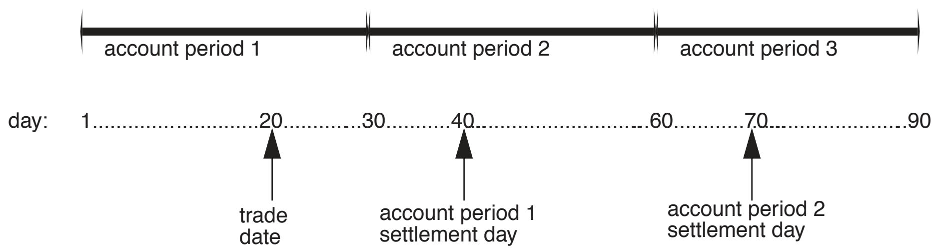
DEFERRED SETTLEMENT: DRESSED STOCK AND FORWARD PRICES

Now let's move to a world in which stock account periods exist. The stock price quoted by the market at any instant on any day in some account period is really the price demanded for settlement deferred to that account period's settlement day. We call these prices *dressed* stock prices because they are modified from the bare stock prices that refer to immediate settlement by taking into account the extra financing involved.

From Bare To Dressed Stock Prices

In Figure 1 we illustrate some hypothetical account periods and their settlement days. We use it to show the connection between bare and dressed stock prices. All trades in each account period settle on the corresponding settlement day.

FIGURE 1. Hypothetical Accounting Periods and Settlement Days



Let the bare price of the stock on the trade date in Figure 1 be S, quoted for settlement the same day. The dressed stock price S' on the trade date is quoted for settlement on the settlement day for account period 1, twenty days later. The relation between S and S' is $S' = S(1 + l)^t$, where l is the annually compounded stock loan rate and t is the time in years between trade and settlement. S' is the price that leaves a buyer indifferent to settling now for S, or investing S risklessly and settling later for S'. In Figure 1, t is 20 days. For a stock loan rate of 10% and a 30-360 day calendar convention, $S' = (1.0053) S$. The dressed price is 0.53% higher than the bare price. A stock with a dressed price of \$100 quoted for settlement 20 days later has an implied bare price of \$99.47.

Bare and dressed option prices are identical in this paper, because we ignore the small number of days between exercise and settlement of an option.

Forward Settlement and Forward Delivery

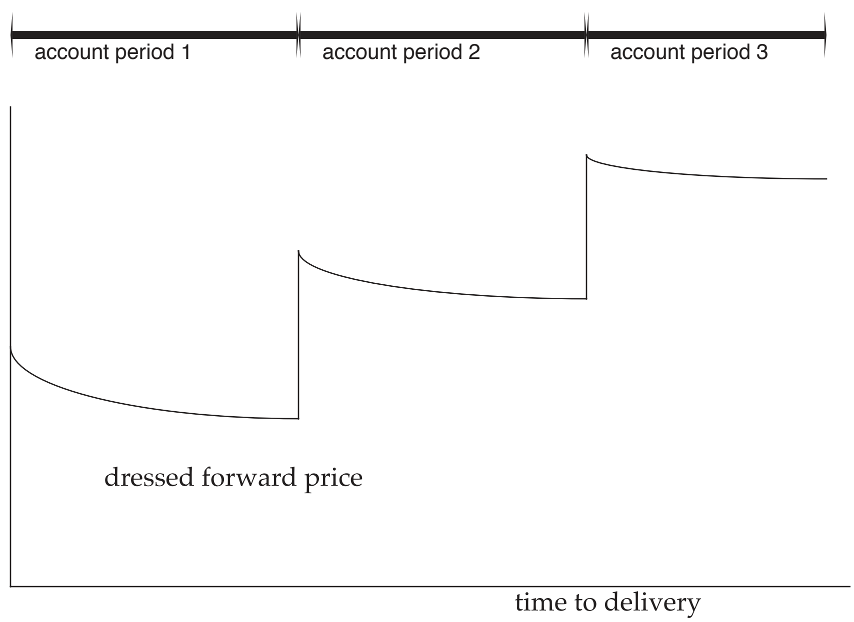
It is important to distinguish between forward settlement and forward delivery. If you buy a stock for forward delivery, you own the stock and dividends it pays only from the delivery date. In contrast, here we are concerned with *forward settlement*. If you buy the stock for forward settlement, you own the stock and receive dividends from the trade date forward; you only settle at a later date. The price for forward settlement exceeds the price for forward delivery on the same day in the future by the future value of all dividends for which the stock "went ex" between trade date and delivery date.

Dressed Forward Prices

A forward contract on a stock is a simpler derivative product than an option. It's value doesn't depend upon volatility at all, even though it can be synthesized from a put and a call. Understanding the behavior of forward prices for periodically-settled stocks will help in understanding options valuation.

The bare forward price of a stock for a given delivery date must be dressed to account for the financing to the next settlement date. Dressed forward prices, therefore, step up as the delivery date moves from one account period to the next. The step up reflects the increase in price necessary to make a buyer indifferent to settling now or investing risklessly and settling later. If the stock pays no dividends, the dressed forward price stays constant during account periods. Otherwise, it declines during account periods by the future value of the dividends paid in that period. This is illustrated schematically in Figure 2 below.

FIGURE 2. Dressed Forward Price as a Function of Time to Delivery



OPTIONS ON STOCKS WITH FORWARD SETTLEMENT

Assuming you already understand how to value options on stocks that settle immediately (bare stocks), we now explain how to value options on dressed stocks. In order to keep the explanation simple, we assume dividends are paid at a continuous rate in an amount proportional to the stock price.

European-Style Options

Take today as the trade date and let t_{ex} be the time to expiration, which we assume to lie beyond the first account period. Denote the quoted (dressed) stock price by S'_i, where the subscript i indicates the *initial* price. Then take the following steps:

1. Compute today's bare stock price $S_i = S'_i/(1+l_i)^t$, where t is the time from today to the next account period's settlement date, and l_i is the stock loan rate over this time.
2. Let this stock price evolve lognormally towards expiration with the stock loan rate and dividend rate applicable to the time to expiration.
3. At expiration, replace each bare stock price S_f in the lognormal distribution by the dressed stock price S'_f that would be quoted for settlement on the settlement day corresponding to the account period in which the expiration day lies. The dressed price is given by $S'_f = S_f(1+l_f)^T$, where l_f is the forward stock loan rate for the time T between expiration and the next settlement.
4. Now evaluate the option payoff over the resultant distribution of dressed stock prices S'_f.

Because of the initial and final transformations between bare and dressed prices, this is equivalent to replacing the initial dressed stock price S'_i by $S'_i(1+l_f)^T/(1+l_i)^t$, and then using this as the initial stock price in the usual Black-Scholes formula with immediate settlement. This procedure alters the forward price of the stock from the bare forward price. If the stock loan rate to expiration is l, this is equivalent to valuing the option with its quoted initial stock price S'_i and an effective loan rate l_{eff} given by

$$l_{\text{eff}} \approx l + (l_f T - l_i t)/t_{ex}$$

for small loan rates. This approximate equation is exact for continuously compounded rates.

What We Mean By Volatility

The volatility σ we use above is the volatility of bare stock prices. This describes the variation in stock prices after the vagaries of financing have been removed. To compute its historical value from a time series of dressed stock prices, which is what we observe, we must first extract bare from dressed prices. This procedure is similar to the one used to adjust for stock splits and dividends when calculating volatility.

American-Style Options

If you understand the binomial method for valuing American-style options, you can modify it to use dressed prices as follows:

1. Compute today's bare stock price as for European-style options above.
2. Diffuse this bare stock price forward on the nodes of a binomial tree towards the expiration date. Use the bare stock volatility σ and the stock loan rate l.
3. Replace the bare price at each node by the dressed price by adjusting for the account period's settlement date corresponding to that node.
4. Now evaluate the option in the usual binomial style by discounting its payoff down the tree, checking for early exercise.

We'll illustrate this in the next section with a numerical example. This method produces the correct dressed forward prices at each period in the tree. The key point is that we have assumed bare stock prices to be lognormally distributed throughout the tree. Since options are written on dressed stock prices, we have to convert the bare prices to quoted (dressed) prices at each node.

As one account period ends and the next begins, the dressed price of a stock jumps because of the sudden change in the number of financing days until settlement. Therefore, it may pay to delay exercising an American call option towards the end of an account period. In that case, the delta of the option will be lower than the delta computed in a model that ignores the effect of account periods.

Pricing From Futures

In some markets, futures prices are more efficient than stock prices. You can use the quoted futures price to extract the bare forward price for settlement at delivery, and then deduce the value of the bare stock price today. You can then continue from step 2 above.

AN EXAMPLE

In this section we explicitly evaluate an American-style option on a binomial tree. To keep things simple, we use a coarse grid with one-year steps. This makes the answer inaccurate but the method clear.

Let's look at a three-year American-style call option struck at \$100 on a stock whose price is quoted at \$100 today. Assume the bare stock has a volatility of 30% per year and pays a 10% dividend once a year. Let the term structure of riskless interest rates be 10% for one year, 10.5% for two years, 11% for three years and 11.49% for four years, and assume a zero fee for borrowing stock. These rates correspond to successive one-year forward rates of exactly 11%, 12% and 13%. Finally, assume that each account period runs for a full year, with a corresponding settlement day falling one year after the end of the account period.

Figure 3 displays the risk-neutral binomial trees for the bare and dressed stock prices, and the option valuation tree.The initial dressed stock price is \$100, quoted for settlement one year from today. It's bare value is \$100/1.1, or \$90.91. We assume that up and down moves on the tree take place with equal probability and 30% volatility. The expected return at each node is $(1+r)(1-d)$, where r denotes the forward riskless rate for each one-year period, and d the dividend rate. This allows us to generate the usual binomial tree of bare option prices from today's value.

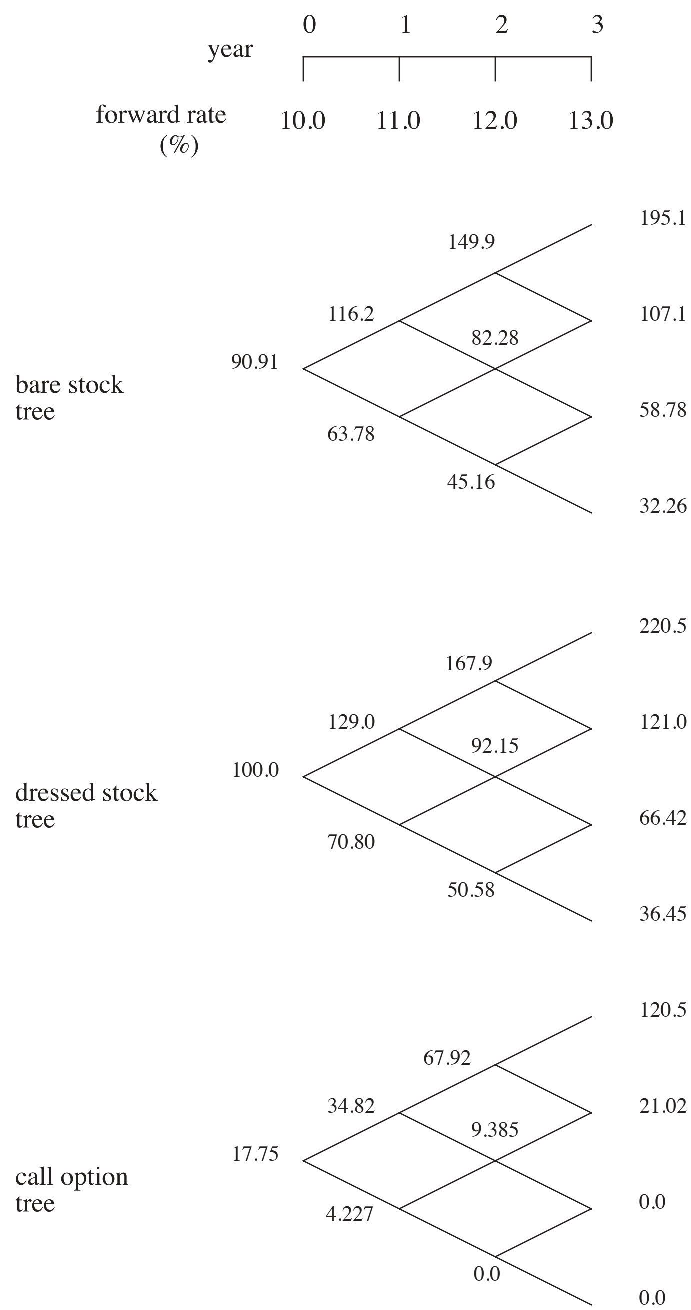
You can check that the expected return at the \$45.16 node at year 2 in the bare tree is (0.5)(32.26 + 58.78)/45.16 = 1.008, which is exactly $(1+r)(1-d)$ for $r=0.12$ (the 1-year forward rate at year 2) and $d = 0.1$.

The dressed stock tree in Figure 3 is generated from the bare stock tree by quoting each node's stock price for settlement one year forward. This means you have to multiply bare stock prices by $(1+r)$. The \$45.16 bare tree node at year 2 gets dressed by the factor 1.12 to yield the quoted stock price \$50.58 in the dressed tree.

The option is written on the dressed stock prices. You obtain the call option tree in Figure 3 by starting with the nodes at expiration in year 3 on the dressed stock tree, and subtracting the \$100 strike from each dressed stock price, keeping only the positive results. You compute the value at each earlier node by discounting the expected value of the two later nodes connected to it at the forward riskless rate. The option value at each node is then the greater of this holding value and the immediate exercise value. You can see that in this coarse tree, the call option is worth \$17.75.

It is easy to generalize this method to more finely-grained trees for greater accuracy, or indeed to other numerical methods of solution.

FIGURE 3. Binomial Trees for Bare Stock, Dressed Stock and Call Option



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