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High-Frequency copula-based pairs trading on U.S. Goldmine Stocks

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Abstract

Pairs trading is an often deployed trading strategy by hedge funds which exploits relative mispricing within two assets. In the present thesis, we empirically evaluate several copula-based pairs trading variants against the two most commonly used pairs trading frameworks, the distance and the cointegration approach. Additionally, we examine the use of the non-linear correlation measures Kendall's τ and Spearman's ρ as pairs selection criteria- next to the conventional methods, the Euclidean distance and the degree of spread mean-reversion. Overall, we compare the performance of either strategy and selection criterion by means of a high-frequency trading strategy on U.S. goldmine stocks, covering the time between January 1998 and April 2016. Before transaction costs, we find all pairs trading methodologies to be highly profitable with daily mean excess returns of 13 - 104 bps and annual Sharpe ratios of up to 6.25. Furthermore, neither strategy is greatly exposed to systematic risk factors, leading to economically and statistically significant alphas. The simple distance approach achieves highest excess returns, followed by the cointegration method. On the contrary, the copula based framework performs comparably poor due to falsely estimated parameter. Among the selection criteria, we find the degree of spread mean-reversion to be most lucrative, followed by Kendall's τ. After transaction costs, however, we observe a different picture. Strongly declining returns in recent years suggest that neither of the variants remain profitable. Furthermore, both non-linear correlation measures outperform the conventional criteria in terms of lower risk and higher average returns.

Keywords: pairs trading, distance, cointegration, copula, high-frequency, selection criteria
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1 Introduction

'Pairs trading' is an investment strategy that is often deployed by hedge funds and proprietary trading firms. It requires two closely related stocks that move together and trade at some spread. If both stocks diverge (the spread widens) one constructs a long-short position betting that the stock pair will converge eventually (the spread narrows).

Ever since Gerry Bamberger and Nunzio Tartaglia successfully pioneered pairs trading at Morgan Stanley in the 1980s (Gatev, Goetzmann and Rouwenhorst, 2006), several academics proposed techniques to compute the spread between stock pairs and consequently algorithms that exploit trading opportunities. Two of the most commonly applied algorithms in previous literature are the distance method introduced by Gatev at al. (2006) and the cointegration method best illustrated by Vidyamurthy (2004). Both techniques rely on the assumption that the selected stocks are linearly related, which only holds as long as the data are normally distributed. The fact that financial returns are rather leptokurtic than normally distributed in practice (Ling, 2006; Kat, 2003) raises concerns on whether trading signals created by the distance and cointegration methods are accurate (Liew and Wu, 2013). To overcome this limitation, Xie, Liew, Wu and Zou (2014) recently proposed an algorithm in which the stock pair dependency is modelled with copulas. Copulas are flexible tools that allow a separate estimation of marginal and joint distribution. Moreover, they are able to capture non-linear relations between random variables such as tail-dependence. Xie et al. (2014) select five stock pairs from the U.S. utility sector based on a minimum distance measure and fit parametric marginal distributions to their returns. Thereafter, they apply both their copula-based algorithm and the distance method on each of the five selected pairs. On a daily frequency, they find that the copula-based method generates higher excess returns and more trading opportunities than the distance method.

In this thesis, we aim to expand upon the research conducted by Xie et al. (2014). In particular, we (i) make use of three additional pairs selection criteria, (ii) estimate both parametric and non-parametric marginal distributions, (iii) utilize constant and time-varying copulas and most importantly (iv) apply their algorithm on a higher data frequency. Hence we can define two main research questions. First, we evaluate all copula variants based on an intraday strategy using minute resolution and check whether the copula-based algorithms remain superior to the benchmarks, the distance and the cointegration method. Second, we investigate how different pairs selection criteria affect the final trading performance of the algorithms. Specifically, we evaluate two commonly used selection criteria, the minimum Euclidean distance and the highest degree of spread mean-reversion criterion, against the two copula related non-linear correlation measures Kendall's tau and Spearman's rho.

An investigation of the above described research goals helps investors to accumulate wealth. Trading at higher frequency increases trading opportunities and thus might imply higher excess returns and information ratios (Aldridge, 2009). Similarly, a smart choice of pair-selection enhances trading performance as well.

We empirically answer the research questions by testing the three methodologies on U.S. goldmine stock pairs. Goldmine stocks are closely related by nature since they similarly react on gold price movements, making them decent candidates for intraday pairs trading. The minute resolution dataset comprises the time between January 1998 and April 2016 and can be viewed as a first of its kind dataset concerning high-frequency pairs trading applications in the previous literature.

On each trading day within the sample period, we select the top stock pair according to the four different selection criteria and consequently apply the three trading algorithms and their variants. Thereafter, we compute performance metrics of the return series and present risk-return characteristics, drawdown measures and common risk-factor dependencies of the trading strategies. Finally, we analyze the impact of transaction costs on the trading performance under two different transaction cost schemes.

Before transaction costs, we find all three trading frameworks to be highly profitable with daily mean excess returns of 13 - 104 bps. Among the three pairs trading approaches, the distance method performs best, followed by the cointegration method. The copula-based framework performs comparably poor due to falsely estimated parameter, which in turn led to wrong transaction signals. It turns out, that parametric marginals are superior to empirical ones and that there is no significant difference between a constant and a time-varying copula approach. Among the pairs selection criteria, we find the degree of spread mean-reversion to be most profitable, followed by the non-linear correlation measure Kendall's τ. Decisive for the superiority of both criteria are more generated trading opportunities and a higher rate of winning trades. All pairs trading variants show highly appealing risk-return characteristics with annual Sharpe ratios of up to 6.25. Furthermore, due to their low exposure to systematic risk, all variants generate economically and statistically significant alphas. Over time, we examine a sharp decline in pairs trading profitability and smaller differences in Sharpe ratios between the pairs trading frameworks. Finally, transaction costs severely affect the before-mentioned findings. While the distance and cointegration approaches still achieve positive daily mean returns of up to 26 bps over the *whole* sample, the declining returns in recent years suggest that neither of the pairs trading methods will be lucrative anymore. Moreover, the non-linear correlation measures outperform the conventional methods, both in terms of lower risk and higher average returns.

We structure the remainder of the thesis as follows. First, in Section 2, we position the thesis within the existing literature. In Section 3, we describe the methodology behind the three pairs trading methods. Thereafter, in Section 4, we apply the algorithms on U.S. goldmine stock pairs and present the results of our investigation. Finally, in Section 5, we conclude and provide further ideas for research.

2 Literature Review and Contribution

'Statistical arbitrage pairs trading' is an anomaly that is firmly related to other long-short anomalies such as lead-lag, reversal and violations of the law of one price, among others illustrated in Jacobs (2015). The literature on pairs trading is steadily growing, yet relatively small compared to momentum strategies, as introduced by Jegadeesh and Titman (1993). There are several different approaches of pairs trading, most of which summarized in Krauss (2015). In this thesis, however, we only focus on the two most common methods, the distance and cointegration method, and the more recent copula approach:

- **Distance approach:** The distance method is the most thoroughly researched pairs trading methodology. Its name stems from the simple way the spread is constructed. A large 'distance' between co-moving assets serves as an indication for relative mispricing.

  Gatev et al. (2006) introduce the distance approach in a seminal paper, which ever since represents the most cited paper in the pairs trading literature[1]. Their sample comprises

[1] As of 9^{th} of April 2015 Gatev et al. were cited 438 times on Google Scholar.

all liquid U.S. stocks from the CRSP, daily from 1962 to 2002. They divide their sample in formation periods of 12 months followed by trading periods of 6 months. During the formation periods, they first construct cumulative return series for each stock under consideration. Using these cumulative returns, they compute the sum of Euclidean squared distance for all possible pair combinations. In the subsequent trading period, they select the top 20 pairs with minimum distance, normalize prices with respect to the first day of the trading period and construct the spread by subtracting one normalized stock price from the other. Trades occur when the spread diverges by more than 2 historical σ and are reversed upon delisting of a stock, mean-reversion of the spread or at the end of the trading period. This strategy yields an annual excess return of 11%. Do, Faff and Hamza (2006) indicate that this approach has several advantages: It is model-free, simple to implement and robust to data-snooping. On the other hand, Krauss (2015) points out that selecting pairs according to minimum Euclidean distance is suboptimal. An Euclidean distance of zero, for instance, would yield no profit at all. A superior objective for pairs selection should be rather based on a highly volatile and mean-reverting spread.

Do and Faff (2010) wrote the second most influential paper on the distance approach. They replicate the paper by Gatev et al. (2006), however, extend the sample horizon by seven years. They find a decline in pairs trading profitability, mostly due to the fact of non-converging pairs and not due to increased hedge fund competition. Consequently, they study two changes concerning the pairs selection process to enhance trading performance. First, they restrict pairs to be only matchable within the same industry, hoping to eliminate spurious pairs. Second, they select pairs subject to the amount of zero-crossings of their spread in the formation period, which serves as an indication of more trading opportunity. Two years later, Do and Faff (2012) incorporate transaction costs in their analysis. Their findings show that the distance method becomes unprofitable when taking commissions, market impact and short selling fees into account.

Further applications of the Gatev et al. (2006) algorithm conduct Bianchi, Drew and Zhu (2009), Mori and Ziobrowski (2011), Broussard and Vaihekoski (2012) and Bowen and Hutchinson (2014). Most of the papers find significant excess returns on their tested samples.

All of the above mentioned papers test the distance approach on daily data. To the best of our knowledge, up until the time of writing there are only two high-frequency applications regarding the distance method. Nath (2003) is the first researcher, who applies the distance method on high-frequency. Specifically, Nath tests a variant of the baseline method of Gatev et al. (2006) on the secondary market of U.S. government debt from 1994 to 2000, using tick data. The strategy outperforms the benchmark, i.e. a duration matched portfolio, concerning Gain-Loss and Sharpe ratio. Furthermore, Bowen, Hutchinson and O'Sullivan (2010) inspect the performance of the distance method by applying it on the FTSE 100 constitutes. Their sample ranges from January to December 2007, in 60 minute frequency. Without transaction costs, the strategy produces remarkable excess returns of 20%. However, taking transaction costs and speed of execution into account, completely diminishes the profitability.

- **Cointegration approach:** The cointegration method represents the second largest pillar in pairs trading literature. As its name suggests, the main idea of this approach lies in finding cointegrated pairs and using the cointegrated relation between the pairs to construct the spread.

  The most influential work about the cointegration approach is written by Vidyamurthy (2004). Vidyamurthy provides a theoretical methodology without empirical application. It comprises and describes three steps. The first step describes the pre-selection of possibly cointegrated pairs. In this step, Vidyamurthy makes use of the Common Trends Model (CTM) of Stock and Watson (1988), to decompose the log return of an asset into a common trend return and a specific return. A pair is cointegrated, if the common trend returns of both assets are identical up to the cointegration coefficient. In order to find assets with similar common trend returns, Vidyamurthy applies Arbitrage Pricing Theory (APT) of Ross (1976). With APT, stock returns can be expressed as a factor model. Suggesting that the common factor returns of APT correspond to the common trend returns of the CTM, Vidyamurthy proposes to select pairs according to a distance metric, which is based on the Pearson correlation coefficient of the common factor returns of all assets under consideration. Pairs with the highest degree of correlated common factor returns are pre-selected for trading. The second step deals with the testing of tradability. Usually, this step would entail to apply standard cointegration tests on the spread of a pair. However, Vidyamurthy suggests to determine tradability by the time between two zero-crossings of the spread during the formation period. A small time interval between zero-crossings increases trading opportunities and decreases the holding period of both assets. Finally, in step three, Vidyamurthy determines a trading rule design. Similarly to Gatev et al. (2006), trades are executed when the spread deviates k historical σ from its mean and closed upon mean-reversion. However, contrary to Gatev et al. (2006), who fix k at 2σ, Vidyamurthy suggests to find an optimal threshold k for each pair. The optimal threshold is gained by taking that k, which maximizes profit during the formation period.

Girma and Paulson (1999) are one of the first who empirically apply the cointegration framework. They define the spread as the difference between petroleum futures and futures with its refined end products as underlying, ranging from 1983 to 1994. The resulting spread variants are determined stationary according to the Phillips-Perron and the Augmented Dickey-Fuller tests. Positions are opened when the spread deviates k cross-sectional standard deviations from its cross-sectional moving average. On the other hand, trades are reversed when the spread reverts to its own moving average. Girma and Paulson (1999) successfully try five and ten-day moving averages as well as five possible thresholds for k. Even with an inclusion of transaction costs, this strategy yields an average of around 15% annually. Using slightly different trading rules, Dunis, Laws and Evans (2006c) and Cummins and Bucca (2012) find similar profitability by trading the same spreads. Different spreads are successfully traded by Simon (1999) and Emery and Liu (2002), both using the same trading methodology as Girma and Paulson (1999). While Simon (1999) applies the framework on the difference between soybean futures and its end products, Emery and Liu (2002) examine the spread defined as the difference between natural gas and electricity futures. The only unsuccessful application concerning commodity spreads is documented by Wahab and Cohn (1994). They find the gold-silver spread to be unprofitable.

More recently, a couple of studies tested the cointegration framework on equities. For instance, Caldeira and Moura (2013) examine the 50 most liquid stocks of the Brazilian stock index IBovespa over the time-frame 2005 to 2010, yielding 15% excess returns. Moreover, Huck and Afawubo (2015) compare the performance of the distance and the cointegration methods by applying them on all S&P 500 constituents. They conclude that the cointegration method significantly outperforms the distance approach.

Several other papers investigate the cointegration approach by applying it on various equities. Among others, Gutierrez and Tse (2011), Bogomolov (2011) and Li, Chui and Li (2014), all confirming the profitability of the cointegration method.

Concerning high-frequency applications, we find three existing papers that address the cointegration approach. The first one is Dunis, Giorgioni, Laws and Rudy (2010), testing all constituents of the EuroStoxx 50 index. They test the algorithm on several high frequencies, ranging from 5 to 60 minutes. Unfortunately, their high-frequency sample comprises only five months within 2009, making their findings highly dependent on the short time-frame. Their contribution lies in the finding that pairs with high in-sample ADF t-statistics outperform out-of-sample in terms of information-ratio. The second high-frequency application is conducted by Kishore (2012), who apply the cointegration framework on the stock pair Exxon Mobil and Chevron during the year 2005. In their study, they focus on finding optimal trading signals for that pair. Finally, Miao (2014) applies the cointegration method on 177 U.S. gas and oil stocks, using 15 minute frequency from 2012 to 2013. Their strategy produces a remarkable annual Sharpe ratio of 9.25.

- **Copula approach:** Compared to the above described pairs trading methodologies, little research has been conducted concerning the copula method. This is due to the fact that the application of copulas in economics and finance is still a relatively new concept. Patton (2008a) provides an overview about application possibilities of copulas in both areas and Trivedi and Zimmer (2007) outline advantages of copulas in joint modelling of random variables.

  Ferreira (2008) introduces the idea of utilizing the dependence structure of copulas in pairs trading. Ferreira suggests to use conditional copulas to determine whether conditional prices are over or undervalued and illustrates the idea considering the stock pair Fannie Mae (FNM) and Freddie Mac (FRE), on daily prices from 2007 to 2008. The author makes use of only one copula and fits parametric distributions to the marginals. Ferreira (2008) concludes that further research needs to be conducted in understanding the relation between the copula method and fundamental analysis.

Stander, Marais and Botha (2013) propose a copula pairs trading strategy that triggers trades when stock returns fall outside a certain confidence level, which is derived using conditional copulas. Positions are closed after the stocks revert to their historical relation. They apply this strategy on several pairs listed in the Johannesburg Stock Exchange, daily from 2003 to 2008. In their application they make use of all 22 copulas listed in Nelsen (2006) and estimate marginal distributions in both, parametric and non-parametric ways. Their findings show that the strategy becomes unprofitable after transaction costs and that the number of trading opportunities depend on the set confidence interval.

Liew and Wu (2013) use a similar trading strategy as Stander et al. (2013). They compare the copula, distance and cointegration methods in means of one example stock pair (BKD-ESC) on daily data from 2009 to 2012. In their application, they utilize the five most commonly used copulas in the financial sector (Gaussian, Student-t, Gumbel, Clayton, Frank) and fit parametric marginal distributions. They conclude that the copula method implies more trading opportunity and is superior to the other approaches in terms of excess returns. This conclusion, however, is based on only that specific stock pair, which raises concerns of the validity of their results.

One year later, Xie et al. (2014) published the paper, most relevant in the content of this thesis. As outlined in the Introduction, they propose a new copula pairs trading strategy and apply it on five selected pairs from the U.S. utility sector, daily from 2003 to 2012. In terms of copula selection and fitting of marginals, they follow Liew and Wu (2013), as described above. Eventually, they compare the results to the conventional distance method and conclude a superiority of the copula method. While the distance framework leads to insignificant excess returns, the copula method yields 3.6% annual excess returns on the selected stock pairs. Interestingly, the more recent study by Rad, Low and Faff (2015) finds contradictory results by comparing a similar copula algorithm to the distance and cointegration approaches. Their daily data-set, comprising daily U.S. equity data from 1962 to

2014, suggests a relatively poor performance of the copula method compared to the other two frameworks.

While all of the above papers use conventional distance metrics for pairs selection, Krauss and Stuebinger (2015) propose an interesting integrated copula-based selection methodology and test their algorithm on the S&P 100 constituents, leading to a Sharpe ratio of 1.52.

All copula-based pairs trading papers use constant copula parameter in their application. In recent literature, a growing number of studies are devoted to time-varying copulas due to their ability of grasping changing dependency structures between random variables. Manner and Reznikova (2012) have summarized several estimation methods of time-varying copula parameter. They include, among others, autoregressive models, dynamic conditional correlations and structural break tests. In contrast to these sophisticated estimation methods, a rather simple way of modeling time variation is nowadays used in many financial institutions, that is updating the copula parameter on a frequent basis (Aussenegg and Cech, 2008). Aussenegg and Cech (2008) show that this approach of updating the copula parameter is promising regarding the ability of forecasting the probability of joint extreme co-movements of stocks. Due to this fact, its wide application, and its comparably low computation time, we utilize this method of estimating time-varying copula parameter in this thesis.

It becomes apparent that extensive research has been conducted in terms of lower frequency pairs trading applications. However, the literature clearly lacks tests conducted at higher frequency. Up until today, we count only two high-frequency applications of the distance approach, three applications of the cointegration approach and none of the copula-based framework. Among these applications, the longest time-span amounted to six years. Hence, this thesis fills the gap of a long-horizon high-frequency application by testing all three frameworks on minute data from January 1998 to April 2016. Concerning the copula method, we find no application that uses time-varying copula parameter. Furthermore, only Krauss and Stuebinger (2015) have tried a different pairs selection approach than using distance metrics. Therefore, we further contribute to the literature by testing a time-varying copula algorithm and by investigating whether non-linear correlation measures help in determining whether a pair is suitable for trading.

3 Methodology

In this Section we describe all three trading frameworks in detail. We commence by outlining the distance approach in Section 3.1. In Section 3.2, we sketch the cointegration approach. Finally, in Section 3.3, we describe the copula approach. The latter includes a short introduction to copulas, the estimation procedure of marginal distributions, a description of all utilized copulas, the copula selection criterion and the algorithm of Xie et al. (2014). Furthermore, in Section 3.4, we illustrate the four pairs selection criteria.

3.1 Distance Approach

Concerning the distance approach, we closely follow the algorithm provided by Gatev et al. (2006). In this framework, the spread is constructed by subtracting one normalised price (NP) from the other. Mathematically the spread at time t can be computed as

$$Spread_t = NP_{1,t} - NP_{2,t}, \tag{1}$$

where $NP_{i,t} = \frac{P_{i,t}-P_{i,0}}{P_{i,0}}$ denotes the cumulative return of stock $i = 1,2$ at time t with respect to the starting point 0. The spread tells us the degree of divergence with respect to the starting point. A positive spread, for instance, implies a relative over-pricing of asset 1 compared to asset 2. As in most trading algorithms, the trading sample is divided into formation and trading periods.

- *Formation period:* During the formation period, we aim to learn about the behaviour of the spread, in order to construct adequate trading signals. Setting the starting point $t = 0$ equal to the first day of the formation period, we first gather a series of the spread. Thereafter, we simply compute the standard deviation σ of the spread to gain a sense of the degree of pair-divergence.

- *Trading period:* During the trading period, we aim to profit from what we learnt before. Optimally, the spread shows strong mean-reverting patterns. In this case, we can short the 'winner' and buy the 'loser' stock when the spread deviates by more than $k\sigma$ and close positions upon mean-reversion of the spread, i.e. if the spread crosses 0, or at the end of the trading period. The spread in the trading period is computed by setting the starting point $t = 0$ equal to the first day of the trading period.

3.2 Cointegration Approach

Contrary to the distance method, which defines the spread as the distance between standardized prices, the cointegration approach derives the spread with the notion of error correction. The concept of error correction is built upon the long-run equilibrium of two time series in a cointegrated system (Vidyamurthy, 2004). A cointegrated relation between two series guarantees the long-run equilibrium to be restored, once there occurs a deviation from the long-run mean. In other words, once a cointegrated pair diverged, they are expected to return to their long-run equilibrium at some point, meaning they converge again.

Using log-price data $p_{1,t}$ and $p_{2,t}$, the spread is computed in two steps. First, we simply regress one series on another

$$p_{1,t} = \mu_1 + \gamma_1 p_{2,t} + \epsilon_t, \tag{2}$$

$$p_{2,t} = \mu_2 + \gamma_2 p_{1,t} + \eta_t. \tag{3}$$

In both equations, γ represents the cointegration factor of the time series. Due to a lower precision error, Vidyamurthy (2004) suggests to opt for estimating the equation containing the larger γ. Therefore, depending on the size of γ_1 and γ_2 the spread is either

$$Spread_t = \epsilon_t = p_{1,t} - \mu_1 - \gamma_1 p_{2,t}, \tag{4}$$

or

$$Spread_t = \eta_t = p_{2,t} - \mu_2 - \gamma_2 p_{1,t}. \tag{5}$$

If the spread is determined to be mean-reverting by an ADF or a Phillips-Perron test statistic, both stocks are said to be cointegrated (assuming that both stocks are integrated of order 1), a method known as the Engle and Granger (1987) test. The procedure during the formation and trading periods is rather similar to the distance approach. In the formation period, we create the spread in the above described manner and estimate its standard deviation. In the trading period, we again construct long-short positions when the spread deviates by $k\sigma$ and close positions upon mean-reversion of the spread or at the end of the trading period.

3.3 Copula Approach

3.3.1 Introduction to Copulas

As outlined in the Introduction, Copulas are powerful tools to measure dependency structures between random variables. Their main advantage lies in the separate estimation of marginal and joint distributions. Nelsen (2006) provides a comprehensive introduction to copulas. Following this work, a function $C : [0,1]^n \to [0,1]$ is an n-dimensional copula if each of the three properties hold:

1. $\forall u = (u_1, ..., u_n) \in [0,1]^n : \min\{u_1, ..., u_n\} = 0 \Rightarrow C(u) = 0$

2. $C(1, ..., 1, u_i, 1, ..., 1) = u_i \ \forall u_i \in [0,1]$

3. $V_C([a,b]) \geq 0$, where $V_C([a,b])$ represents the C-volume of the hyper-rectangle $[a,b] = \prod_{i=1}^{n} [a_i, b_i], a_i \leq b_i \ \forall i$.

In words, the first property states that if the marginal probability of any outcome is zero, the joint probability of all outcome is zero as well. The second property says that if the marginal probabilities of all but one outcome are known with probability one, the joint probability equals the probability of the remaining uncertain outcome. Finally, the last property claims that the C-volume of any n-dimensional interval is non-negative.

One of the most relevant theorems in the copula framework is 'Sklar's theorem' by Sklar (1959). It states that copulas establish a functional relation between multivariate distribution functions and their marginals. Let $F(x_1, ..., x_n)$ denote any joint distribution function with continuous marginals $F_i(x_i)$, then there is a unique copula function satisfying the above described properties such that:

$$F(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)). \tag{6}$$

3.3.2 Estimation of Marginal Distributions

The estimation procedure of copulas consists of two separate steps: (i) the fitting of marginal distributions and (ii) the estimation of the dependency structure between both random variables. While we cover several different dependency structures in the following subsection, we first discuss our approach of fitting marginal distributions.

There are two different ways to estimate the marginal distribution of a random variable, a parametric and a non-parametric way. Since both approaches inherit certain advantages and disadvantages and might imply different trading outcomes, we employ both ways of fitting marginal distributions in this thesis.

Fitting a non-parametric distribution has been extensively studied by Genest, Ghoudi and Rivest (1995). This method comprises to estimate an empirical distribution of the data on hand. Letting $X_i = \left(X_{i,1}, ..., X_{i,T}\right)'$ be the ith data vector, the corresponding marginal distribution $F_i(x)$ is estimated by

$$\hat{F}_i(x) = \frac{1}{T+1} \sum_{j=1}^{T} \mathbb{1}_{\{X_{i,j} \leq x\}}, \tag{7}$$

where $\mathbb{1}_{\{X_{i,j} \leq x\}}$ is an indicator function that equals one if the statement in curly brackets is true and zero otherwise and T denotes the size of vector X_i. Since the estimator $\hat{F}_i(x)$ almost surely converges to the true marginal distribution $F_i(x)$ according to the law of large numbers, the estimator is considered to be consistent (van der Vaart, 2000). While an empirical distribution is able to capture specific higher moments of the data, it most likely lacks mass in the tails of the distribution due to a finite number of available observations.

Fitting a parametric distribution is done by estimating the respective distribution parameters. Contrary to the non-parametric way, this approach of fitting marginal distributions entails no finite sample problems and thus better handles extreme observations. On the downside, this concept results in two different issues. First, the estimated parameters are subject to estimation error. Second, what parametric distribution does best fit the data? While the estimation error decreases with larger samples, the latter issue remains. In this thesis, we opt for fitting marginal Student-t distributions to the data due to its wide application on financial returns. The Student-t distribution is characterized by its parameter, the degrees of freedom n. Taking the sample estimates $\hat{\mu}$ and $\hat{\sigma}$ of the return series, we first standardize the returns. Thereafter, we gain the degrees of freedom n by using maximum likelihood. Specifically, we opt for that n, which maximizes the log-likelihood function:

$$\log L\left(n; z_1, ..., z_T\right) = \sum_{t=1}^{T} \log\left(\frac{1}{\sqrt{\pi\,(n-2)}}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\left[1+\frac{z_t^2}{n-2}\right]^{-\frac{n+1}{2}}\right), \tag{8}$$

where Γ is the Gamma function.

3.3.3 Copulas applied in this thesis

The literature comprises a large amount of different copulas, each of which representing a certain dependency structure. Nelsen (2006) summarizes the most important copulas in the literature. In this thesis, we follow Liew and Wu (2013) and Xie et al. (2014) and focus on the most frequently used copulas regarding financial assets: the Gaussian, the Student-t, the Gumbel, the Clayton and the Frank copula. Notation-wise, let $\{X, Y\}$ be random variables with marginal distribution $\{F_X, F_Y\}$ and $u = F_X(r^X)$ and $v = F_Y(r^Y)$. Both, u and v lie in the interval $[0,1]$ and represent the value of their respective marginal distribution at the realizations r^X and r^Y.

- **Gaussian Copula:**

  The Gaussian copula is called an implicit copula, since it is implied by the multivariate normal distribution. Meyer (2013) provides a comprehensive study about the bivariate Gaussian copula applied in this thesis. Following this work, let

  $$\phi(x) := \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right), \qquad \Phi(h) := \int_{-\infty}^{h}\phi(x)\,dx$$

  denote the density and distribution function of the standard normal distribution respectively. Moreover, define

  $$\phi_2(x,y;\rho) := \frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left(-\frac{x^2-2\rho xy+y^2}{2(1-\rho^2)}\right), \qquad \Phi_2(h,k;\rho) := \int_{-\infty}^{h}\int_{-\infty}^{k}\phi_2(x,y;\rho)\,dy\,dx$$

  as the density and distribution function of the bivariate standard normal distribution, in which $\rho \in [-1,1]$ represents the correlation coefficient. Then, by Sklar's theorem, the

Gaussian copula reads as

$$C\big(u,v;\rho\big)=\Phi_2\big(\Phi^{-1}(u),\Phi^{-1}(v);\rho\big), \tag{9}$$

where Φ^{-1} is the inverse of the standard normal distribution function. From the copula function in equation (9), we can derive the density by taking first order derivatives:

$$\begin{aligned} c\big(u,v;\rho\big) &= \frac{\partial^2}{\partial u \partial v} C\left(u,v;\rho\right) = \frac{\phi_2\left(\Phi^{-1}(u),\Phi^{-1}(v);\rho\right)}{\phi\left(\Phi^{-1}(u)\right)\phi\left(\Phi^{-1}(v)\right)} \\ &= \frac{1}{\sqrt{1-\rho^2}}\exp\left(\frac{2\rho\Phi^{-1}(u)\Phi^{-1}(v)-\rho^2\left(\Phi^{-1}(u)^2+\Phi^{-1}(v)^2\right)}{2\big(1-\rho^2\big)}\right). \end{aligned} \tag{10}$$

Finally, conditional bivariate copulas can be derived by taking partial derivatives of equation (9):

$$C\big(v|u\big)=\frac{\partial}{\partial u}C\left(u,v;\rho\right)=\Phi\left(\frac{\Phi^{-1}(v)-\rho\Phi^{-1}(u)}{\sqrt{1-\rho^2}}\right), \tag{11}$$

$$C\big(u|v\big)=\frac{\partial}{\partial v}C\left(u,v;\rho\right)=\Phi\left(\frac{\Phi^{-1}(u)-\rho\Phi^{-1}(v)}{\sqrt{1-\rho^2}}\right). \tag{12}$$

The only copula parameter ρ can be calibrated by

$$\rho=\sin\left(\frac{\pi}{2}\tau\right), \tag{13}$$

where τ is the rank correlation coefficient defined by Kendall (1948), also know as 'Kendall's tau'. We illustrate its computation in Section 3.4.3.

- **Student-t Copula:**

  Similar to the Gaussian copula, the Student-t copula belongs to the class of implicit copulas. It differs from the Gaussian copula in the sense that it allows for joint fat tails. Furthermore, joint extreme events are allowed to happen with larger probability. Let the Student-t density and distribution function be denoted by

$$f_n\big(x\big):=\frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}, \qquad t_n\big(h\big):=\int_{-\infty}^{h} f_n\big(x\big)dx,$$

where $n \in (0,\infty)$ represents the degree of freedom and Γ is the Gamma function. Furthermore, the bivariate Student-t density and distribution with correlation coefficient $\rho \in [-1,1]$ are denoted by

$$f_{2,n}\left(x,y;\rho\right) := c\frac{1}{\sqrt{\rho}}\left(1+\frac{x^2}{n\rho}+\frac{y^2}{n\rho}\right)^{-\frac{n+2}{2}}, \qquad t_{2,n}\big(h,k;\rho\big) := \int_{-\infty}^{h}\int_{-\infty}^{k} f_{2,n}\big(x,y;\rho\big)dydx,$$

in which the constant c in the bivariate density is computed according to $c = \Gamma\left(\frac{n}{2}\right)^{-1}\Gamma\left(\frac{n+2}{2}\right)(n\pi)^{-1}$. Again applying Sklar's theorem yields the bivariate Student-t copula

$$C\big(u,v;\rho,n\big) = t_{2,n}\big(t_n^{-1}(u),t_n^{-1}(v)\big). \tag{14}$$

Taking first order derivatives of the bivariate Student-t copula and denoting $\psi = \left(t_n(u)^{-1},t_n(v)^{-1}\right)'$ gives the copula's density (Jondeau, Poon, Rockinger, 2007):

$$\begin{aligned} c\big(u,v;\rho,n\big) &= \frac{\partial^2}{\partial u\partial v}C\left(u,v;\rho\right) \\ &= \frac{1}{\sqrt{\left(1-\rho^2\right)}}\frac{\Gamma\left(\frac{n+2}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\left(\Gamma\left(\frac{n+1}{2}\right)\right)^2}\frac{\left[\left(1+\frac{\psi_1^2}{n}\right)\left(1+\frac{\psi_2^2}{n}\right)\right]^{\frac{n+1}{2}}}{\left[1+\frac{1}{n(1-\rho^2)}\left(\psi_1^2-2\rho\psi_1\psi_2+\psi_2^2\right)\right]^{\frac{n+2}{2}}}. \end{aligned} \tag{15}$$

The conditional bivariate Student-t copulas can be derived by taking partial derivatives of equation (14):

$$C\big(v|u\big) = \frac{\partial}{\partial u}C\left(u,v;\rho,n\right) = t_{(n+1)}\left(\sqrt{\frac{n+1}{n+\left(t_n^{-1}(u)\right)^2}}\times\frac{t_n^{-1}(v)-\rho t_n^{-1}(u)}{\sqrt{1-\rho^2}}\right), \tag{16}$$

$$C\big(u|v\big) = \frac{\partial}{\partial v}C\left(u,v;\rho,n\right) = t_{(n+1)}\left(\sqrt{\frac{n+1}{n+\left(t_n^{-1}(v)\right)^2}}\times\frac{t_n^{-1}(u)-\rho t_n^{-1}(v)}{\sqrt{1-\rho^2}}\right). \tag{17}$$

Contrary to the Gaussian copula, which contains only the correlation coefficient ρ as parameter, the Student-t copula possesses an additional parameter, namely the degrees of freedom n. While ρ can be calibrated using equation (13) as well, a more sophisticated approach is needed to estimate the degree of freedom. Assuming the marginal distributions $F_1(x_1)$ and $F_2(x_2)$ are fitted, we estimate n by following two steps. First, we calibrate ρ by means of equation (13). Thereafter, we numerically opt for that n, which maximises the log-likelihood function

$$\log L(n,\rho;u_1,...,u_T,v_1,...,v_T) = \sum_{t=1}^{T} \log c(u_t,v_t;\rho,n). \tag{18}$$

- **Gumbel Copula:**

  The Gumbel copula is an asymetric copula that belongs to a class called 'Archimedean' copulas. Archimedean copulas are built upon any generator function ψ, satisfying $\psi(1) = 0$ and $\lim_{u\to 0}\psi(u) = \infty$, that is strictly convex and monotonic decreasing. A bivariate Archimedean copula can be written as

$$C\big(u,v\big) = \psi^{-1}\big(\psi(u)+\psi(v)\big), \tag{19}$$

  with density

$$c\big(u,v\big) = \psi^{-1}_{(2)}\big(\psi(u)+\psi(v)\big)\psi^{'}(u)\psi^{'}(v). \tag{20}$$

  In the density above, $\psi^{-1}_{(2)}$ represents the inverse of the second derivative of the generator function and $\psi^{'}$ the first derivative.

  The generator function which yields the Gumbel copula is

$$\psi(u) = -\big(\ln u\big)^{\delta}, \tag{21}$$

  where $\delta \geq 1$ is a parameter, which controls the degree of upper tail dependence $\lambda^u = 2-2^{1/\delta}$ in the copula. Lower tail dependence is not present in the Gumbel copula and therefore equals 0. Taking the inverse of the generator function and making use of equation (19) yields the bivariate distribution of the Gumbel copula (Venter, 2001):

$$C\big(u,v;\delta\big) = \exp\left(-\left[(-\ln u)^{\delta}+(-\ln v)^{\delta}\right]^{\frac{1}{\delta}}\right). \tag{22}$$

  The density of the Gumbel copula can be derived by utilizing equation (20):

$$c\,(u,v;\delta) = C\,(u,v\,;\delta)\times(uv)^{-1}\times A^{-2+2/\delta}\times[(\ln u)\,(\ln v)]^{\delta-1}\times\left[1+(\delta-1)\,A^{-1/\delta}\right], \tag{23}$$

with $A = (-\ln u)^{\delta} + (-\ln v)^{\delta}$. Finally, the bivariate conditional copulas are computed using partial derivatives again:

$$C(v|u) = C(u, v; \delta) \times \left[(-\ln u)^{\delta} + (-\ln v)^{\delta}\right]^{\frac{1-\delta}{\delta}} \times (-\ln u)^{\delta-1} \times \frac{1}{u}, \tag{24}$$

$$C(u|v) = C(u, v; \delta) \times \left[(-\ln u)^{\delta} + (-\ln v)^{\delta}\right]^{\frac{1-\delta}{\delta}} \times (-\ln v)^{\delta-1} \times \frac{1}{v}. \tag{25}$$

We calibrate the copula parameter δ by

$$\delta = (1-\tau)^{-1}, \tag{26}$$

where τ is once again Kendall's correlation coefficient.

- **Clayton Copula:**

Contrary to the Gumbel copula, which replicates upper tail dependence, the Clayton copula possesses lower tail dependence. It belongs to the class of Archimedean copulas as well and is constructed by the following generator function

$$\psi(u) = \alpha^{-1}(u^{-\alpha} - 1), \tag{27}$$

with $\alpha \in (-1, \infty) \setminus \{0\}$ as parameter and lower tail dependence degree $\lambda^{l} = 2^{-\frac{1}{\alpha}}$. From this generator function, the Clayton copula and its density follow directly (Venter, 2001):

$$C(u, v; \alpha) = \left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-\frac{1}{\alpha}}, \tag{28}$$

$$c(u, v; \alpha) = (\alpha + 1) \times \left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-2-\frac{1}{\alpha}} \times u^{-\alpha-1} v^{-\alpha-1}. \tag{29}$$

Moreover, taking partial derivatives of the Copula yields the bivariate conditional copula functions:

$$C(v|u) = u^{-(\alpha+1)} \times \left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-\frac{1}{\alpha}-1}, \tag{30}$$

$$C(u|v) = v^{-(\alpha+1)} \times \left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-\frac{1}{\alpha}-1}. \tag{31}$$

Finally, calibrating the parameter α can be done using Kendall's tau:

$$\alpha = 2\tau\left(1-\tau\right)^{-1}. \tag{32}$$

- **Frank Copula:**

  The Frank copula is yet another Archimedean copula. Contrary to the previous copulas, the Frank copula is not considered to possess as heavy tails. Its generator function is

$$\psi\left(u\right) = -\ln\left(\frac{\exp\left(-\theta u\right)-1}{\exp\left(-\theta\right)-1}\right), \tag{33}$$

  with $\theta \in (-\infty, \infty) \setminus \{0\}$ being the parameter. From the generator function, it follows the Frank copula and its density (Venter, 2001):

$$C\left(u,v;\theta\right) = -\theta^{-1}\ln\left[1+\frac{\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)}{\left(\exp\left(-\theta\right)-1\right)}\right], \tag{34}$$

$$c\left(u,v;\theta\right) = \frac{-\theta\left(\exp\left(-\theta\right)-1\right)\left(\exp\left(-\theta\left(u+v\right)\right)\right)}{\left(\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)+\left(\exp\left(-\theta\right)-1\right)\right)^{2}}. \tag{35}$$

  The bivariate conditional distributions are as follows:

$$C\left(v|u\right) = \frac{\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)+\left(\exp\left(-\theta v\right)-1\right)}{\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)+\left(\exp\left(-\theta\right)-1\right)}, \tag{36}$$

$$C\left(u|v\right) = \frac{\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)+\left(\exp\left(-\theta u\right)-1\right)}{\left(\exp\left(-\theta u\right)-1\right)\left(\exp\left(-\theta v\right)-1\right)+\left(\exp\left(-\theta\right)-1\right)}. \tag{37}$$

  Unfortunately, there exists no closed form solution for the parameter of interest θ. We can, however, use a similar approach as in the degrees of freedom calibration for the Student-t copula. That is, we numerically pick that θ, which maximises the log-likelihood function

$$\log L(\theta; u_1, \ldots, u_T, v_1, \ldots, v_T) = \sum_{t=1}^{T} \log c(u_t, v_t; \theta). \tag{38}$$

3.3.4 Copula Selection

Once all of the described copulas are estimated, the following step is to select the best fitting copula. There are two common ways to determine the fit of a copula. On the one hand, standard Goodness of Fit test-statistics such as the Kolmogorov-Smirnov (KS) and the Anderson-Darling (AD) can be applied, both of which extended to copula application by Kole, Koedijk and Verbeek (2007). On the other hand, information criteria can be used to select the best fitting copula. Similar to Liew and Wu (2013) and Xie et al. (2014), we follow the latter method. Specifically, we choose the copula yielding the lowest Akaike criterion (AIC) (Akaike, 1973):

$$AIC = -2l(\theta) + 2k, \tag{39}$$

where $l(\theta) = \sum_{t=1}^{T} \log c\big(u_t, v_t; \theta\big)$ denotes the optimized log-likelihood function of the copula with parameter set θ and k represents the number of copula parameters.

3.3.5 Copula Algorithm

After the extensive and rather technical introduction to copulas in the previous subsections, we are now able to describe the proposed algorithm by Xie et al. (2014). First, let us denote R_t^X and R_t^Y as the random variables of the high-frequency returns of stocks X and Y with realizations r_t^X and r_t^Y. Furthermore, their marginal distribution functions are denoted by $\{F_X, F_Y\}$ and $u = F_X(r_t^X)$, $v = F_Y(r_t^Y)$. Then, we can define mispricing indexes $MI_t^{X|Y}$ and $MI_t^{Y|X}$ between both stocks as

$$MI_t^{X|Y}\big(r_t^X, r_t^Y\big) = P\big(R_t^X < r_t^X | R_t^Y = r_t^Y\big) = C\big(u|v\big), \tag{40}$$

$$MI_t^{Y|X}\big(r_t^X, r_t^Y\big) = P\big(R_t^Y < r_t^Y | R_t^X = r_t^X\big) = C\big(v|u\big). \tag{41}$$

These mispricing indexes $\in [0,1]$ reflect the degree of mispricing between both stocks with respect to only one observation, namely the returns of both stocks at time t. A value of 0.5 reflects no mispricing, while a value larger than 0.5 indicates a relative overvaluation of the underlying stock conditional on the other stock and vice versa.

To gain an overall impression of mispricing, Xie et al. (2014) suggest to sum up the mispriced values. This leads to two spread series, denoted by $Spread_{X,t}$ and $Spread_{Y,t}$. All in all, the basic algorithm used in this thesis can be summarized in the following steps:

- *Formation period:*

  1. First calculate the high-frequency returns r_t^X and r_t^Y and estimate their marginal distributions.
  2. Fit all five copulas and pick the one with the lowest AIC.
  3. Set $Spread_{X,0} = 0$ and $Spread_{Y,0} = 0$
  4. Compute $Spread_{X,t} = Spread_{X,t-1} + \left(MI_t^{X|Y} - 0.5\right)$ and $Spread_{Y,t} = Spread_{Y,t-1} + \left(MI_t^{Y|X} - 0.5\right)$
  5. Compute the standard deviations σ_X and σ_Y of both spreads

- *Trading period:*

  1. Set $Spread_{X,0} = 0$ and $Spread_{Y,0} = 0$
  2. Compute $Spread_{X,t} = Spread_{X,t-1} + \left(MI_t^{X|Y} - 0.5\right)$ and $Spread_{Y,t} = Spread_{Y,t-1} + \left(MI_t^{Y|X} - 0.5\right)$
  3. Construct long-short positions if one of the spread series deviates by k standard deviations
  4. Close positions if the spread series returns to zero or at the end of the trading period.
  5. If positions are closed, set both spread series equal to zero.

This algorithm is very similar, yet not entirely the same as applied by Xie et al. (2014). We mainly extend their algorithm by introducing steps 3-5 in the formation period. Xie et al. (2014) do not estimate the standard deviation of the spread in the formation period, but rather fix a threshold level D in advance, at which trading signals occur. This technique appears to be too random. Moreover, in order to guarantee a fair comparison of pairs trading frameworks, it is reasonable to apply the same trading threshold methodology in all strategies. In this way, it is possible to fairly determine which spread-construction is superior.

3.4 Pairs selection criteria

All of the pairs trading frameworks can only function on suitable pairs. For this reason, the literature has invented several metrics for pairs selection. In this thesis, we employ conventional linear measures such as the Euclidean distance metric and the ADF test statistic of the spread series as well as the two non-linear correlation measures Kendall's tau and Spearman's rho.

3.4.1 Euclidean Distance Metric

By far the most commonly applied selection criterion is based on the Euclidean distance between asset prices. It is conceived, that the smaller the Euclidean distance between two stocks, the better they are for pairs trading. We compute the Euclidean distance between two cumulative return series $CR_{X,t}$ and $CR_{Y,t}$ as

$$ED = \sqrt{\sum_{t=1}^{T} \left(CR_{X,t} - CR_{Y,t}\right)^2}. \tag{42}$$

3.4.2 ADF test statistic

Another pairs selection criterion is based on the degree of spread mean-reversion, implied by a stock pair. Intuitively, testing for spread mean-reversion makes sense, since profit is only made after the spread returns to zero, hence when it mean-reverts. We make use of the most common test for mean-reversion, the Augmented Dickey-Fuller (ADF) test. The test is constructed by first regressing

$$\Delta Spread_t = \alpha + \gamma Spread_{t-1} + \delta_1 \Delta Spread_{t-1} + ... + \delta_{p-1} \Delta Spread_{t-p+1} + \epsilon, \tag{43}$$

where α is a constant and p is the lag order of the autoregressive process, which can be determined using information criteria. Under the null hypothesis of 'no mean-reversion', the relevant test statistic ADF is then computed by

$$ADF = \frac{\hat{\gamma}}{SE(\hat{\gamma})}, \tag{44}$$

where $SE(\hat{\gamma})$ denotes the standard error of $\hat{\gamma}$. Critical values are different from standard t-test values. Pairs selection can be based on the magnitude of the ADF statistics. The pair resulting in the lowest ADF statistic is linked to the spread containing the highest degree of mean-reversion.

3.4.3 Non-linear correlation measures

Both, the Euclidean distance metric and the ADF test statistic, are subject to certain draw-downs. While the distance metric might not suit the primary objective of a profitable spread series, the ADF test is as any test subject to estimation error. Moreover, constantly changing spread dynamics might imply a decent in-sample, yet a poor out-of-sample performance of the ADF criterion. Hence, we choose to test two alternative selection criteria, namely the correlation measures Spearman's ρ and Kendall's τ. Both correlation measures are closely linked to the copula concept and are thus expected to contain valuable information, especially with respect to the copula spread. A high correlation coefficient suggests a close co-movement of a stock pair.

- *Spearman's ρ:*

  Spearman's $\rho \in [-1, 1]$ is computed by first ranking the return series of stocks X and Y, denoted by $rk(r_t^X)$ and $rk(r_t^Y)$. Defining the difference between the ranks of returns r_t^X and r_t^Y as $d_t = rk(r_t^X) - rk(r_t^Y)$ and their squared sum as $D = \sum_{i=1}^{T} d_i^2$, then Spearman's ρ is given by (Wayne, 1990)

$$\rho = 1 - \frac{6D}{T\left(T^2 - 1\right)}. \tag{45}$$

- *Kendall's τ:*

  Kendall's $\tau \in [-1, 1]$ is based on the concept of concordance. Two pairs of observations on random variables X and Y, denoted by (x_1, y_1) and (x_2, y_2) are said to be 'concordant' if $(x_1 - x_2)$ has the same sign as $y_1 - y_2$. Similarly, they are called 'discordant' if $(x_1 - x_2)$ has the opposite sign as $y_1 - y_2$. By considering all possible observation-pairs in a sample containing T observations, Kendall's τ is computed by first counting all concordant and discordant pairs, denoted by N_c and N_d respectively. Thereafter, Kendall's τ is given by (Kendall, 1948)

$$\tau = \frac{N_c - N_d}{\frac{1}{2}T(T-1)}. \tag{46}$$

4 Application

In this Section, we empirically compare the three pairs trading frameworks by applying them on real data. We commence, in Section 4.1, by introducing the platform in which the algorithms are back-tested. Thereafter, in Section 4.2, we present the Data used for the application. In Section 4.3, we outline the implemented trading strategy. Consequently, in Section 4.4, we illustrate the three pairs trading methodologies by applying them on an example pair. Finally, in Section 4.5, we present and discuss the main results of the application.

4.1 The Platform - Quantconnect

In order to test the three pairs trading methodologies on high-frequency data, we make use of an algorithmic open source backtesting platform called 'Quantconnect'. Quantconnect.com is a free web platform that offers users the possibility to implement and backtest trading algorithms by making use of Quantconnect's data library as well as their servers. Regarding the data, Quantconnect offers high-frequency data for all U.S. equities from January 1998 to the present, ranging from 'tick' to daily frequency. The data are being provided by their partner Quantquote, are free of survivorship bias and are split and dividend adjusted. The data, together with their servers enable retail investors to backtest computationally extensive algorithms in comparatively short time, and most importantly without any cost. Since high-frequency data sources usually are far from being cheap and easily accessible, this platform offers completely new possibilities for research. In this thesis, we code all algorithms in the programming language *C#*.

4.2 Data - U.S. Goldmine Stocks

Since the aim of this thesis is to test the pairs trading frameworks on intraday data, i.e. high-frequency, pairs pre-selection should be based on the degree of intraday co-movement of stocks rather than on their long-term relation. Pairs, for instance, that take multiple days for converging back to their equilibrium are hardly profitable at intraday pairs-trading, yet perfectly suitable for long-term pairs trading strategies.

Assets that most likely fulfill the criterion of intraday co-movement are stocks which are highly dependent on the same commodity prices. For this reason, we choose to apply the pairs trading strategies on U.S. goldmine stocks. Not surprisingly, goldmines severely depend on the traded gold price, making them decent candidates for intraday pairs trading. According to 'Miningfeeds.com', there are currently 16 goldmine stocks listed in the United States of America. However, we narrow the range of tradable goldmines down to 11 by excluding stocks that went public during the sample period, which leaves us with 55 pair combinations. The sample period ranges from January 1998 to April 2016 and the data comprise minute resolution.

Table 1 tabulates the remaining 11 goldmine stocks used in this thesis, including a short description of their business model and their stock symbols. While all of the firms have a gold production in common, some differ in terms of additional mining operations. Hence, depending on their complete business model, their stock prices are expected to be either moderately or very strongly correlated.

In order to gain a sense of the stock price development and co-movement of the goldmine stocks throughout the sample period, we compute the daily cumulative return series of all stocks plus the gold price and provide their plots, cross-correlations and Euclidean distance ranking. The data are downloaded from Yahoo Finance.

Figure 1 plots all cumulative return series. From this plot, we can observe that apart from RGLD and AEM, all stocks eventually underperform the gold price during the sample period. Moreover, some stock pairs seem to be strongly co-moving.

Table 2 summarizes the cross-correlations between all goldmine stocks and the gold price. Generally, it shows that most pairs are moderately to strongly correlated. However, interestingly, there are two negative correlations present. HMY is negatively correlated with the gold price and RGLD. A reason could be larger alternate mining operations of HMY.

Table 3 displays the ranking implied by computing the Euclidean distances between all pair combinations. It seems that especially ABX, NEM and GFI closely move together. On the other hand, VGZ is furthest apart from all other stocks.

All in all, we can conclude that there appear to be some suitable pairs for pairs trading among the 11 pre-selected stocks. Whether pairs prove to be profitable on high-frequency after all, is discussed in Section 4.5.

Table 1: Description of utilized U.S. Goldmine stocks

Company	Description	Symbol
Barrick Gold Corp.	Largest Market-cap. They produce Gold and Copper	ABX
Agnico-Eagle Mines	Produce gold, silver, zinc and copper	AEM
Gold Fields Ltd.	Produce and process gold and copper	GFI
Goldcorp Inc.	Produce gold, silver, lead, zinc and copper	GG
Harmony Gold	Produce gold, silver, copper and uranium	HMY
Kinross Gold Corp.	Produce gold, silver and copper	KGC
McEwen Mining Inc.	Produce gold, silver and copper	MUX
Newmont Mining Corp.	Produce gold, silver and copper	NEM
Royal Gold Inc.	Acquire and manage precious metals royalties	RGLD
Richmont Mines Inc.	Produce gold	RIC
Vista Gold Corp.	Produce mainly gold	VGZ

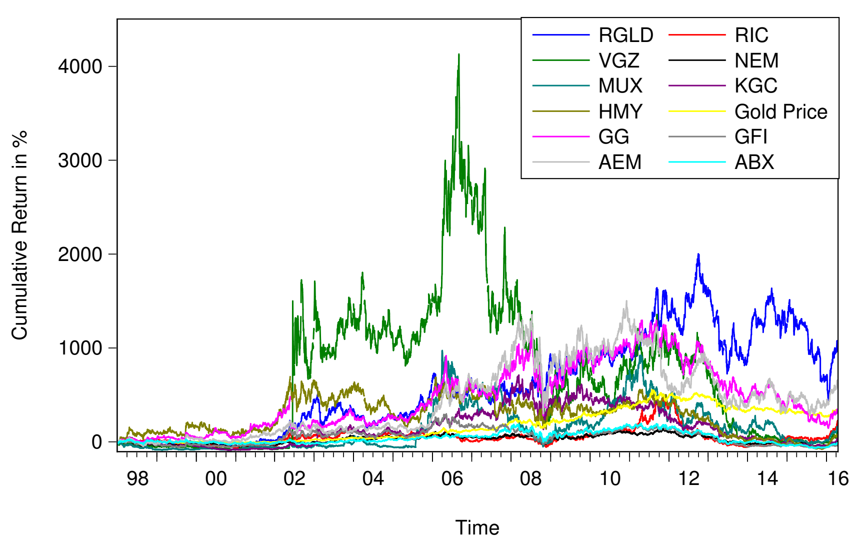


Figure 1: Cumulative return plots of all included U.S. goldmine stocks plus the gold price in daily frequency from January 1998 to April 2016. Cumulative returns are computed using the daily close prices of the stocks.

Table 2: Cross-correlations of Goldmine Stocks and Gold price

	ABX	AEM	GFI	GG	HMY	KGC	MUX	NEM	RGLD	RIC	VGZ
AEM	0.81										
GFI	0.71	0.51									
GG	0.87	0.92	0.60								
HMY	0.55	0.31	0.89	0.39							
KGC	0.88	0.87	0.70	0.82	0.55						
MUX	0.72	0.71	0.73	0.77	0.47	0.65					
NEM	0.88	0.71	0.87	0.81	0.70	0.81	0.74				
RGLD	0.44	0.69	0.22	0.77	-0.02	0.41	0.53	0.46			
RIC	0.63	0.42	0.56	0.60	0.47	0.43	0.48	0.71	0.46		
VGZ	0.45	0.31	0.86	0.37	0.77	0.52	0.59	0.66	0.10	0.34	
Gold	0.53	0.75	0.19	0.82	-0.06	0.46	0.57	0.48	0.95	0.50	0.03

Table 3: Ranking according to Euclidean Distance metric

	ABX	AEM	GFI	GG	HMY	KGC	MUX	NEM	RGLD	RIC
AEM	36									
GFI	3	35								
GG	33	7	32							
HMY	21	31	18	24						
KGC	9	26	8	27	15					
MUX	14	30	13	28	19	10				
NEM	1	38	2	25	22	12	17			
RGLD	43	29	46	23	40	41	39	45		
RIC	4	37	6	34	20	11	16	5	42	
VGZ	54	48	52	44	47	51	49	55	50	53

4.3 The Trading Strategy

The trading strategy applied by an investor is of upmost importance. It might decide whether a framework is profitable or not. Regarding the trading strategy in this thesis, there are five main decisions to take: (i) what data frequency to use; (ii) the length of formation and trading periods; (iii) appropriate threshold values for k that trigger trades; (iv) when to close positions; (v) whether and what stop-loss threshold to implement. All remaining decisions concern the frameworks themselves.

Varying in one of these decisions might lead to different trading outcomes. In this thesis, however, the aim is not to find the optimal combination of input parameters, but rather to gain a general impression of what pairs trading framework performs best.

We aim to trade by making the following decisions: (i) we use minutes as data frequency. (ii) Since price dynamics and thus spread dynamics constantly change, we opt for multiple formation periods throughout the sample instead of relying on only one formation period in the beginning. Specifically, the main idea is to trade intraday and use the time from market opening at 9:30 to 11:00 as formation period and the time from 11:00 until market closing at 16:00 as trading period. In this way we rely on latest parameter estimations and avoid over-night risks. (iii) We follow, among others, Gatev et. al (2006) and set $k = 2$. (iv) We follow the main stream of literature and exit trades once the spread reverts to zero or at the end of the trading period, i.e. when the market closes. (v) We do not implement stop-loss positions since the risk of pair-divergence is limited intraday.

Using the trading rules outlined above, we gain a stream of returns by selecting (in every formation period) the top stock pair suggested by a certain selection criterion. From these returns, we compute several performance metrics, which make the trading frameworks comparable.

4.4 Illustration at an Example

In order to illustrate the functionality of the three algorithms and of the trading strategy described in the previous subsection, we consider an example. Specifically, we analyze the performance of the algorithms on June 14, 2007. This date serves well as it illustrates different trading outcomes. By choosing $k = 2$, we can define $delta = \pm 2\sigma$ as the threshold to enter positions.

For simplicity, we trade by selecting one stock pair according to the highest Spearman's rho criterion. During the formation period from 9:30 to 11:00, ABX and AEM are selected for trading, with a Spearman's rho of 0.44. Figure 2 plots the intraday cumulative returns of both stocks on the chosen trading day, in minute resolution. It shows that there are phases of co-movement, divergence and convergence present. In all instances, we trade by making use of 20.000$ capital and by ignoring transaction costs.

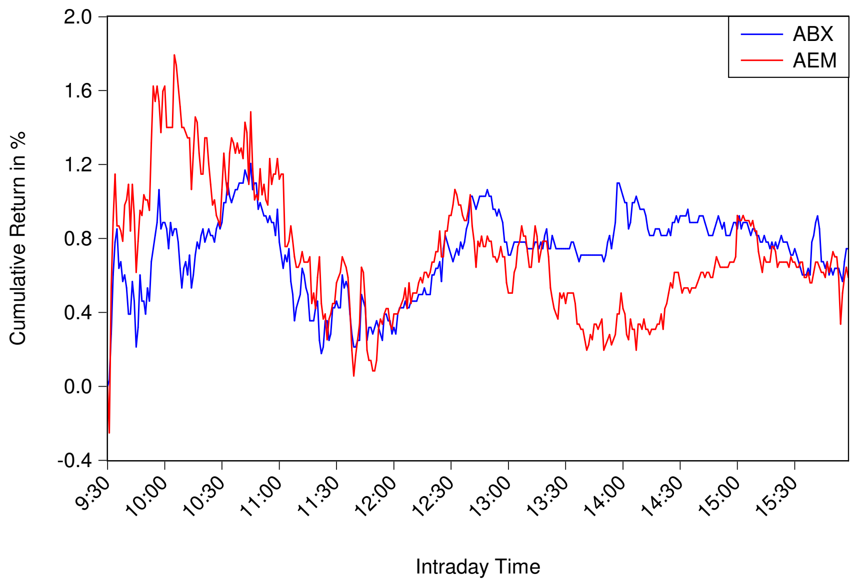
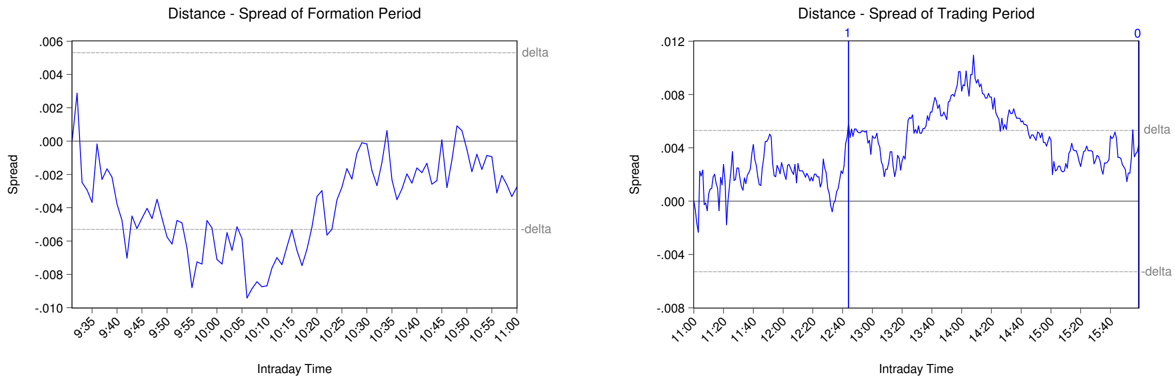


Figure 2: Cumulative intraday return plots of ABX and AEM on June 15, 2007.

4.4.1 Distance Approach

Figure 3 shows plots of the spread constructed by the distance approach, both in formation and trading period. In the formation period, a trade would have been successful. In the trading period, positions are entered, but could not be closed due to a non-reverting spread. Hence, the trading rules applied and the positions are closed at the end of the trading day. Since the spread happened to end between zero and $delta$ at the time of closing, a positive return of 0.15% is generated. The trading summary of the distance approach is tabulated in Table 4.



(a) (b)

Figure 3: The Distance Spread in the Formation Period (a) and in the Trading Period (b). In (b), the vertical lines represent times in which positions are opened (1) and closed (0).

Table 4: Distance approach - Trading summary

Position	Symbol	Price $	Quantity	Time
Open	ABX	25.359	-788	12:44
Open	AEM	32.794	609	12:44
Close	ABX	25.305	788	15:59
Close	AEM	32.775	-609	15:59
Profit:	**30.98$**		**Return:**	**0.15%**

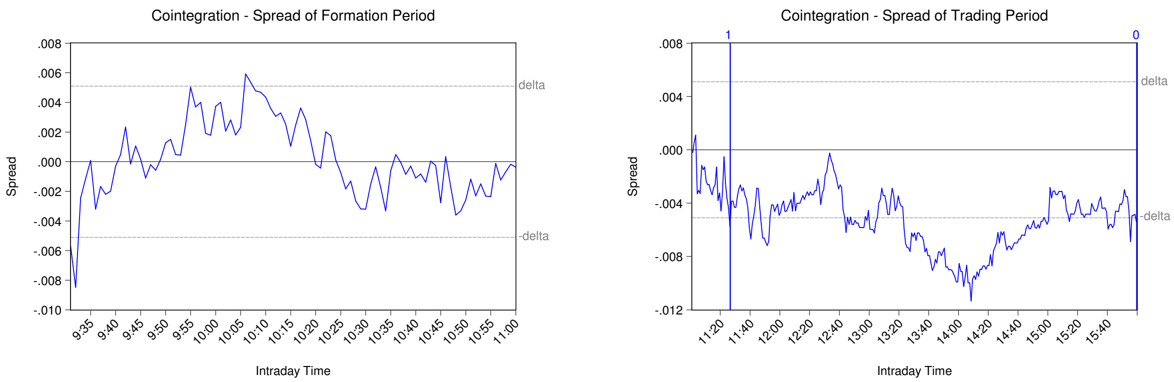
Notes: The invested capital is set to 20.000$.

4.4.2 Cointegration Approach

Figure 4 shows the plots of the resulting cointegration spread. The spread seems to be similar to an inverted version of the distance spread. It is constructed by

$$Spread_t = \log AEM_t - 1.15 - 0.73 \times \log ABX_t. \tag{47}$$

Similarly to the distance approach, positions are entered, but could not be successfully closed. In the end, the trades result in a small loss of -0.06%. The corresponding trading summary is depicted in Table 5.



(a) (b)

Figure 4: The Cointegration Spread in the Formation Period (a) and in the Trading Period (b). In (b), the vertical lines represent times in which positions are opened (1) and closed (0).

Table 5: Cointegration approach - Trading summary

Position	Symbol	Price $	Quantity	Time
Open	ABX	25.207	-793	11:26
Open	AEM	32.666	612	11:26
Close	ABX	25.305	793	15:59
Close	AEM	32.775	-612	15:59
Profit:	**-11.01$**		**Return:**	**-0.06%**

Notes: The invested capital is set to 20.000$.

4.4.3 Copula Approach

Regarding the copula approach in this example, we estimate the marginal distributions of ABX and AEM non-parametrically, meaning we compute their empirical distribution as outlined in Section 3.3.2. Moreover, we do rely on a constant copula parameter. Figure 5 provides scatter plots of the marginal distribution realizations during the formation period and the corresponding theoretical copulas. In (a), it can be observed that both stocks are moderately correlated with a slightly larger degree of upper than lower tail dependence. Comparing (a) to the theoretical copulas in (b)-(f), it becomes apparent that both, the Gumbel and the Clayton copula, capture only one side of tail dependence, meaning they are not entirely able to capture the dependence structure between ABX and AEM. Among the remaining three copulas, it seems that the Student-t copula is best able to capture the dependence structure of both variables, because of its ability to replicate both, upper and lower tail dependence.

Table 6 displays the resulting AIC for every copula together with their parameter. Clearly, the AIC is lowest for the Student-t copula, suggesting that it fits the data best.

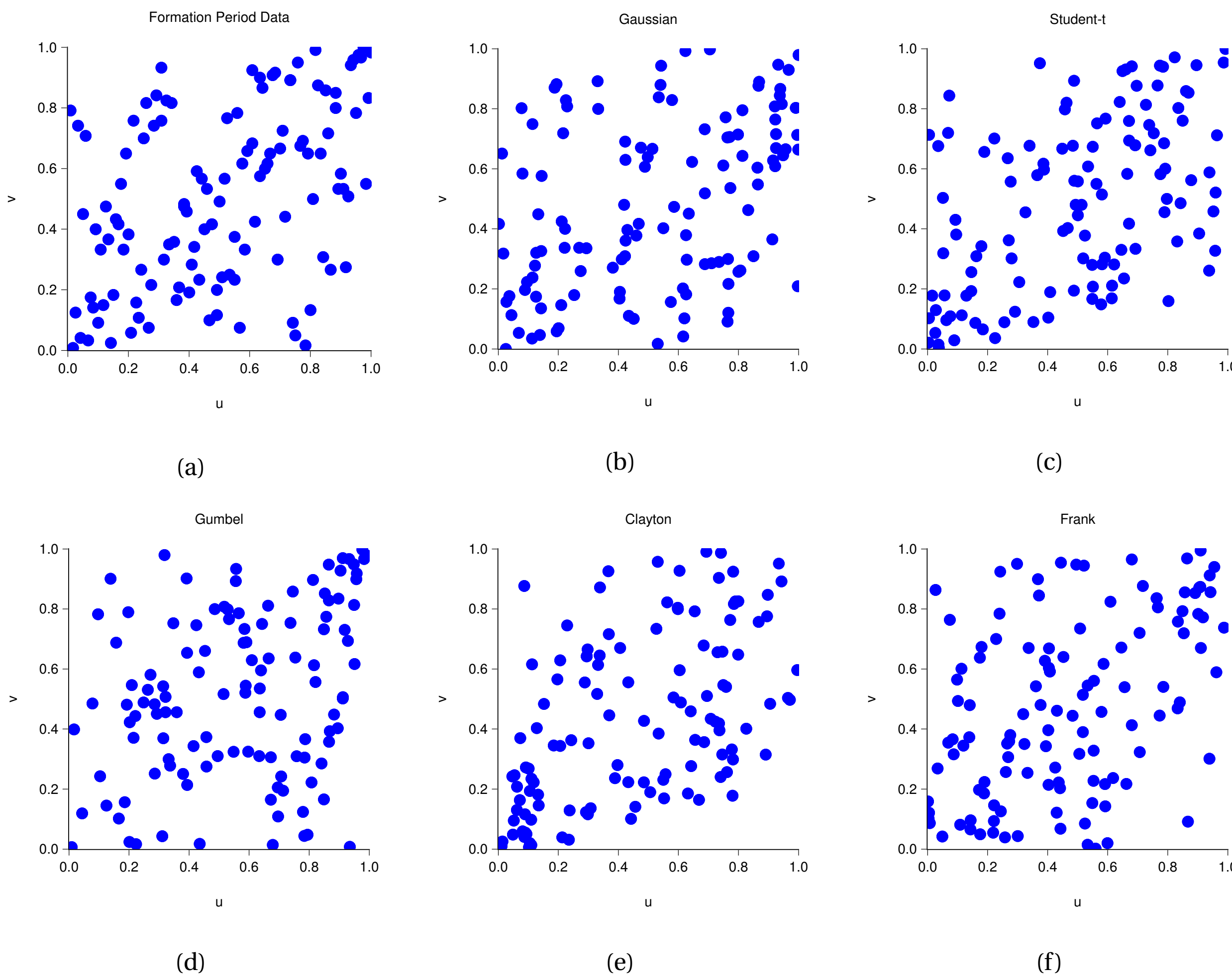
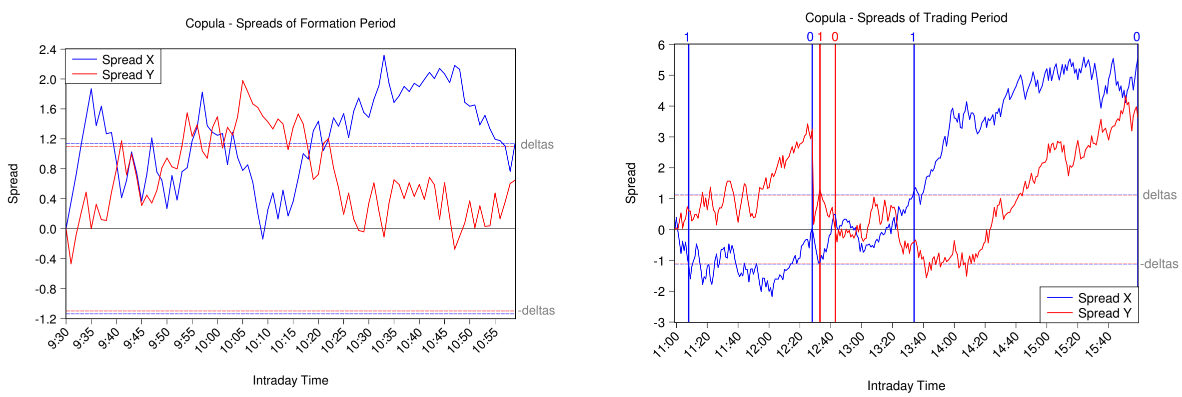


Figure 5: Scatter plots of the marginal distribution realizations of ABX and AEM during the Formation Period in (a) and the corresponding theoretical Copulas in (b)-(f).

Table 6: AIC and Copula Parameter

Copula	AIC	Parameter
Student-t	-48.99	$\rho = 0.39$; $n = 4.51$
Gumbel	-22.87	$\delta = 1.33$
Gaussian	-20.55	$\rho = 0.39$
Frank	-17.00	$\theta = 2.98$
Clayton	-14.52	$\alpha = 0.67$

Figure 6 shows the plots of the Student-t copula spreads in formation and trading period. Contrary to the previous two approaches, the copula method is able to detect more trading opportunity. Twice, positions are opened according to $Spread_X$ and once by $Spread_Y$. Moreover, in two instances the spread reverted back to zero, so that positions are successfully closed. All in all, the copula algorithm generated a remarkable daily return of 0.72%. The trading summary can be found in Table 7.



(a) (b)

Figure 6: The Student-t Copula Spreads in the Formation Period (a) and in the Trading Period (b). In (b), the vertical lines represent times in which positions are opened (1) and closed (0).

Table 7: Copula approach - Trading summary

Position	Symbol	Price $	Quantity	Time	Spread
Open	AEM	32.839	-609	11:08	X
Open	ABX	25.243	792	11:08	X
Close	AEM	32.930	609	12:28	X
Close	ABX	25.323	-792	12:28	X
Open	AEM	32.930	-608	12:33	Y
Open	ABX	25.296	792	12:33	Y
Close	AEM	32.839	608	12:43	Y
Close	ABX	25.368	-792	12:43	Y
Open	AEM	32.748	615	13:34	X
Open	ABX	25.314	-796	13:34	X
Close	AEM	32.775	-615	15:59	X
Close	ABX	25.305	796	15:59	X
Profit:	**144.06$**			**Return:**	**0.72%**

Notes: The invested capital is set to 20.000$.

4.5 Main Results

In this Section, we evaluate the performance of $5 \times 4 = 20$ pairs trading variants based on several criteria. The variants consist of five different algorithms (Distance, Cointegration, constant Copula with empirical marginals, constant Copula with parametric marginals, time-varying Copula with parametric marginals) tested on four pairs selection criteria (Euclidean Distance, ADF - Statistic, Kendall's τ, Spearman's ρ).

The time-varying copula with parametric marginals differs compared to its constant counterpart to the extent, that the copula parameter is re-calibrated every 30 minutes using an expanding window. Contrary to a moving window of a fixed size, the expanding window approach guarantees more data points and hence more reliable copula estimates the more time passes. Unfortunately, an even higher updating frequency, i.e. every minute, seemed impossible due to its immense computation time (it took around 24 hours to gain the return series for the time-varying copula variant already).

We structure this Section as follows. At first, in Section 4.5.1, we present the excess return distribution and the main trading statistics and of the algorithms. Thereafter, in Section 4.5.2, we analyze several risk-return adjusted characteristics and compute drawdown measures. In Section 4.5.3, we investigate differences over time. In Section 4.5.4, we perform a common risk factor analysis. Finally, in Section 4.5.5, we check to what extent transaction costs affect profitability.

4.5.1 Excess Return Distribution and Trading Statistics

Table 8 depicts the daily return distribution in excess of the 1 month U.S. treasury bill rate resulting from the return series of all applied pairs trading variants, excluding transaction costs. Since there are trading days in the sample, in which the selected stock pair did not diverge by more than two spread standard deviations and thus no trades occurred, we can either disregard these trading days in our daily return computation or mark their returns as zero. While the first method can be denoted as the return on 'Employed Capital', the latter method is a more conservative approach and can be described as return on 'Committed Capital'. As Gatev et al. (2006) note, this method accounts for the opportunity cost of committing capital in spite of no generated trades of the strategy.

The table shows that all strategies produce both statistically and economically significant average daily returns with Newey-West (NW) t-statistics all larger than 7.08. Not surprisingly, the committed capital mean returns are lower than their corresponding employed capital returns, yet not less statistically significant (due to the larger return series of committed capital). Generally, with up to 1.07%, we observe that the ADF criterion implies highest average daily returns, followed by Kendall's τ and roughly similar Euclidean distance and Spearman's ρ criteria. In Appendix A, we illustrate the five most frequently selected pairs for either selection criterion. Regarding the different algorithms, we find that the distance approach generates highest average returns, followed by the cointegration method. Among the copula variants, we find considerably lower returns when fitting marginal distributions empirically than parametrically. Moreover, the difference between constant and time-varying copula parameter seems to be negligible. In total, it can be noted that higher average returns come with greater risk. For example, the standard deviation of the distance approach variants is almost twice as large as the corresponding empirical copula variants. Due to the -from investors favorable- positively skewed distributions, the median returns are smaller than the average returns and even turn negative in the copula with empirical marginals case (for committed capital). All variants posses high excess kurtosis, indicating non-normal return distributions. Looking at the minima and maxima of the return series we obviously find no differences between committed and employed capital. Keeping in mind that the table depicts daily returns, both minima and maxima appear to be large in their magnitude and range from -27.5% to 30.6%. The fact, that the sample period covers more than 18 years including many high volatility states puts these magnitudes into perspective, however. These magnitudes just show that even though pairs trading is a market-neutral strategy, it still entails the idiosyncratic risks of both stock pair constituents. The percentage of daily excess returns below zero indicates that all strategies produce more positive than negative daily returns. Since the deduction of the interest rate implies negative returns for all 'zero' returns on committed capital, we find considerably more negative committed capital returns. Finally, the empirical Value at Risk (VaR) and the Expected Shortfall (ES) at 1% reveal information about the tail of the return distributions. Interestingly, the commonly used pairs selection criteria, the Euclidean distance and the ADF statistic, both show considerably larger VaR and ES than the nonlinear correlation measures. Especially in the cointegration approach, the differences appear to be huge, with ES and VaR almost twice as large.

Table 9 summarizes the main trading statistics of the algorithms. Not surprisingly, we observe that pairs selected according to the highest spread mean reversion imply the highest number of transactions. The difference to the other selection criteria is most extreme in the empirical copula case, where the ADF statistic generated over 60% more trades. Regarding the annualized returns and the percentage of positive trades (winrate), a clear picture is drawn: Even though all variants are highly profitable, the ADF statistic once again stands out and provides best annualized returns and winrates (all above 60%). Furthermore, the distance approach is the best algorithm performer. By far the most profitable variant is the 'distance approach - ADF statistic', yielding remarkable 1117% annually before transaction costs. Interestingly, the ADF statistic variants yield highest annual return rates despite the fact that they possess the lowest average negative returns and not the highest average positive returns among all selection criteria. This fact suggests that the main return driver is the winrate of an algorithm. Several relevant statistics in the pairs trading literature concern the so called round-trips (RT) of a strategy, i.e. successfully mean-reverted spread series before market closing at 16:00 o'clock. In two of these statistics, the (parametric marginal) copula methods are superior compared to the distance and the cointegration methods. For instance, remarkable 68% of all trades appeared to be RT trades for the parametric copula - ADF variants, compared to 65% of the respective distance and cointegration variants. Moreover, in terms of average RT trades per day, both copula methods outperform in three out of four pairs selection criteria, generating up to 1.81 RT trades per day on average. In that statistic, the Spearman's ρ criterion clearly falls behind with only around 1 RT per day, suggesting little mean-reverting spread properties. So while it seems that copulas generate more frequent RT trades than the distance and cointegration methods, they achieve lower average RT trade returns on the downside. That might be due to the surprising fact that up to 15% of the copula RT trades are negative, compared to none negative RT trade in the distance approach and less than 1% negative RT trades in the cointegration approach. While the distance approach forbids negative RT trades by definition, they might occur in the other two algorithms due to falsely estimated parameter. In other words, at times, the estimation error of the parameter might have been too large to adequately specify the spread, which in turn led to false transaction signals and thus negative RT trades (see Appendix B for illustration). Especially the estimation of the empirical marginals turns out to be not accurate enough. It implies around 3% more negative RT trades than a parametric marginal estimation. Obviously, the 90 data points during the formation period do not suffice to specify a marginal distribution. Interestingly, the time-varying copula produces slightly more positive RT trades than the constant copula, which confirms that estimation errors decrease with an increasing amount of observations. Finally, the last column of the table depicts the average holding time until closing of positions, which matter with respect to borrowing costs for short selling. On average, an investor would hold a position for only around 86 to 114 minutes, again with shortest holding times in the ADF statistic case.

Compared to the existing pairs trading literature we find similar results as Rad et al. (2015). They also find the distance and cointegration approaches considerably more profitable than the copula approach. On the other hand, our results contradict the findings of Xie et al. (2014), who conclude the copula approach to be superior. Only with respect to more generated RT trades of the copula approach, our findings are consistent with Xie et al. (2014).

Table 8: Daily Excess return Distribution

	Average (%)	t-stat (NW)	Median (%)	Std. (%)	Skew.	Kurt.	Min. (%)	Max. (%)	Ret. < 0 (%)	VaR (1%)	ES (1%)
COMMITTED CAPITAL											
(A) Distance Approach											
Dist	0.72	11.44	0.41	2.36	2.26	18.53	-17.8	26.9	35.8	-4.44	-6.94
ADF	1.04	16.12	0.66	2.67	2.01	12.59	-16.3	26.9	33.2	-4.73	-6.70
τ	0.71	9.13	0.39	2.07	2.42	30.94	-27.5	26.9	35.4	-3.17	-5.16
ρ	0.59	7.34	0.13	2.08	3.48	28.33	-16.2	26.9	39.7	-2.98	-5.29
(B) Cointegration Approach											
Dist	0.54	12.47	0.40	2.48	0.75	26.32	-28.8	32.1	38.0	-6.24	-10.58
ADF	0.85	16.48	0.55	2.70	1.57	15.69	-19.2	32.0	36.3	-5.61	-8.75
τ	0.79	10.78	0.51	2.18	2.58	21.89	-19.2	26.7	33.3	-3.44	-5.55
ρ	0.53	9.09	0.27	1.97	2.35	16.52	-12.9	20.4	39.0	-3.66	-5.69
(C) Copula Approach - Constant with Empirical Marginals											
Dist	0.13	7.44	-0.01	1.25	2.12	47.29	-11.1	24.5	53.5	-3.35	-5.31
ADF	0.42	12.40	0.27	2.22	1.59	20.32	-16.1	28.5	42.4	-5.33	-7.98
τ	0.15	9.39	-0.01	1.07	0.58	12.00	-10.5	10.3	53.9	-2.59	-3.93
ρ	0.14	8.08	-0.01	1.20	0.72	42.70	-20.0	17.6	53.3	-2.87	-4.50
(D) Copula Approach - Constant with Parametric Marginals											
Dist	0.31	9.94	0.21	1.75	0.74	19.12	-17.7	20.6	42.3	-4.47	-7.16
ADF	0.66	16.30	0.47	2.35	1.16	10.95	-14.5	26.3	37.6	-5.09	-7.57
τ	0.40	11.26	0.24	1.60	1.50	20.27	-13.9	23.1	40.5	-3.26	-5.04
ρ	0.29	9.48	0.12	1.55	3.14	45.83	-12.2	30.6	44.6	-3.35	-4.90
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	0.32	8.92	0.20	1.82	1.58	29.58	-17.7	28.5	42.6	-4.38	-7.17
ADF	0.63	15.98	0.46	2.29	1.25	11.83	-14.5	27.0	38.3	-4.94	-7.11
τ	0.34	11.46	0.21	1.49	0.18	24.92	-22.9	17.0	41.7	-3.16	-5.15
ρ	0.29	9.83	0.12	1.51	3.06	48.25	-12.2	30.6	44.5	-3.21	-4.81
EMPLOYED CAPITAL											
(A) Distance Approach											
Dist	0.76	10.62	0.44	2.51	2.64	21.96	-17.8	26.9	34.5	-4.63	-7.08
ADF	1.07	15.42	0.71	2.71	1.96	12.15	-16.3	26.9	31.6	-4.79	-6.74
τ	0.78	8.57	0.49	2.14	2.33	28.89	-27.5	26.8	32.2	-3.26	-5.14
ρ	0.70	7.28	0.38	2.27	3.20	24.13	-16.2	26.8	35.0	-3.51	-5.64
(B) Cointegration Approach											
Dist	0.54	11.74	0.42	2.52	0.69	26.59	-16.4	32.1	37.2	-6.26	-10.80
ADF	0.89	15.48	0.62	2.74	1.53	15.13	-19.2	32.0	34.4	-5.65	-8.65
τ	0.80	10.28	0.53	2.18	2.59	21.87	-19.2	26.7	32.4	-3.43	-5.40
ρ	0.56	9.31	0.34	2.01	2.24	15.53	-12.9	20.4	37.3	-3.74	-5.69
(C) Copula Approach - Constant with Empirical Marginals											
Dist	0.17	8.13	0.16	1.44	1.75	34.91	-11.1	24.5	42.3	-3.70	-5.40
ADF	0.44	12.23	0.31	2.26	1.66	20.39	-16.1	28.5	40.7	-5.34	-7.94
τ	0.20	7.67	0.16	1.22	0.38	8.45	-10.5	10.3	42.0	-2.93	-3.96
ρ	0.18	7.08	0.17	1.32	1.35	23.18	-12.1	17.6	42.0	-3.14	-4.35
(D) Copula Approach - Constant with Parametric Marginals											
Dist	0.33	8.61	0.29	1.79	0.69	18.04	-17.7	20.6	39.3	-4.55	-7.16
ADF	0.67	15.22	0.49	2.35	1.16	10.86	-14.5	26.3	37.3	-5.09	-7.46
τ	0.41	10.33	0.27	1.62	1.46	19.59	-13.9	23.1	39.0	-3.29	-5.04
ρ	0.32	7.67	0.22	1.61	2.93	42.40	-12.2	30.6	39.9	-3.40	-4.90
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	0.34	8.46	0.28	1.87	1.51	27.87	-17.7	28.5	39.7	-4.52	-7.29
ADF	0.64	15.88	0.48	2.30	1.24	11.72	-14.5	27.0	37.9	-4.96	-7.11
τ	0.36	9.36	0.27	1.53	0.14	23.64	-22.9	17.0	39.1	-3.17	-5.24
ρ	0.32	8.19	0.22	1.58	2.89	44.17	-12.2	30.6	39.9	-3.29	-4.92

Notes: This table depicts the daily excess return distributions of all pairs trading variants for both committed and employed capital return streams, before transaction costs. The committed capital returns extend employed capital returns by the trading days in which a strategy did not generate trades. These trading days are captured with 'zero' return. The t-statistics are computed with Newey-West (NW) standard errors. The number of lags included are determined subject to the lowest AIC.

Table 9: Summary of Trading Statistics

	Number of Trades	Annualized Ret. (%)	Winrate (%)	Average (Ret>0) (%)	Average (Ret<0) (%)	% of RT Trades	Average No. of RT per day	Average RT Ret. (%)	% of RT (Ret<0)	Average holding Time (min)
A) Distance Approach										
Dist	38280	512	64	0.73	-0.87	63	1.35	0.75	0.00	94
ADF	39052	1117	66	0.84	-0.93	65	1.43	0.99	0.00	89
τ	33012	471	64	0.79	-0.85	61	1.19	0.86	0.00	96
ρ	25038	318	62	0.89	-0.89	53	0.86	1.01	0.00	109
B) Cointegration Approach										
Dist	37842	256	62	0.74	-0.88	61	1.28	0.64	0.12	106
ADF	39834	668	65	0.85	-0.99	65	1.46	0.76	0.76	92
τ	41046	580	64	0.76	-0.85	65	1.48	0.72	0.44	93
ρ	31029	262	60	0.86	-0.91	57	1.01	0.73	0.84	114
C) Copula Approach - Constant with Empirical Marginals										
Dist	25440	38	56	0.71	-0.79	55	1.02	0.35	15.3	106
ADF	41846	177	60	0.76	-0.89	63	1.50	0.45	15.4	96
τ	25024	46	55	0.75	-0.80	54	0.96	0.34	15.1	112
ρ	26024	43	56	0.72	-0.80	53	0.97	0.35	14.7	113
D) Copula Approach - Constant with Parametric Marginals										
Dist	38516	112	59	0.68	-0.79	63	1.39	0.37	13.2	95
ADF	48072	390	63	0.74	-0.87	68	1.79	0.54	12.6	87
τ	36720	166	59	0.79	-0.86	58	1.20	0.49	12.4	110
ρ	30876	103	58	0.75	-0.82	55	1.01	0.46	11.7	113
E) Copula Approach - Time-varying with Parametric Marginals										
Dist	37644	116	59	0.69	-0.78	63	1.36	0.39	12.7	94
ADF	48490	357	63	0.73	-0.87	68	1.81	0.51	11.9	86
τ	33472	133	58	0.77	-0.83	57	1.10	0.45	12.0	107
ρ	31118	106	58	0.76	-0.81	56	1.03	0.45	10.7	111

Notes: This table depicts the main trading statistics of all pairs trading variants. Round-trip trades, i.e. successfully closed trades before market closing at 16:00 o'clock are abbreviated with 'RT'.

4.5.2 Risk-adjusted Return Characteristics and Drawdown Measures

In table 10 we present the main annualized risk-adjusted performance characteristics as well as several drawdown measures of the algorithms. Their computation is illustrated in Appendix C. We focus on the committed capital return streams. Starting with the most commonly used performance metric -the Sharpe ratio- we can carefully conclude the following ranking among the selection criteria: (1) ADF, (2) Kendall's τ, (3) Spearman's ρ, (4) Euclidean Distance. Only the cointegration approach slightly distorts this ranking, with Kendall's τ as best performing criterion. Similarly, we can rank the algorithms as: (1) Distance approach, (2) Cointegration approach, (3) Constant / Time-varying Copula with parametric marginals, (4) Constant Copula with empirical marginals. All in all, most variants possess 'excellent' Sharpe ratios according to Maverick (2015), with annual Sharpe ratios above 3. In contrast to the Sharpe ratio, which relates annual excess return to total risk as standard deviation, Sortino ratio, Omega and Upside potential are classified as 'lower partial moment' characteristics, meaning they measure risk as downside deviation. This is a valuable feature as the Sharpe ratio potentially underestimates risk (Eling, 2008). The Sortino ratio relates annual excess return to downside deviation, Omega forms a ratio of positive to negative returns and Upside potential relates positive returns to downside deviation. All these measures show impressive risk-return ratios for investors, confirming the above 'ranking' of selection criteria and pairs trading methodologies.

The second pillar in table 10 depicts drawdown measures. These measures are connected to the equity curves resulting from the algorithms rather than to their returns. The tabulated maximum drawdowns appear relatively large, ranging from 16% to 34%. However, keeping in mind the rather extreme minima of the pairs trading return distributions, these values are put into perspective. Subtracting the minima from the maximum drawdowns suggests that the portfolios do not loose considerably more before reaching new highs. In total, Spearman's ρ, poorly ranked in terms of risk-return characteristics, seems to be safest regarding maximum drawdowns. The Calmar ratio relates the annualized returns to the maximum drawdowns. Clearly, all strategies tend to recover easily within one year after a maximum drawdown with Calmar ratios well above 1. The remaining tabulated drawdown measures are the Sterling ratio, the Burke ratio, the Ulcer index and the Martin Ratio. The Sterling ratio relates annual excess return to the average yearly maximum drawdown. The Burke ratio is defined as annual excess return to the square root of sum of the squared N largest drawdowns [2]. The Ulcer Index takes both the degree as well as the time of drawdowns into account. Finally, the Martin ratio divides the annualized excess return by the Ulcer Index. All of these ratios reveal the clear underperformance of the empirical copula framework. Furthermore, mainly due to their impressive annualized returns, the Distance approach can be viewed as best risk-rewarded.

The fact that Calmar, Sterling and Burke all relate drawdowns to annual excess return makes it hard to examine the standalone drawdown properties of a strategy, though. The large differences in annual excess returns between the variants considerably cloud differences in drawdown statistics. Therefore, we compute the five largest drawdowns for either variant and depict them in form of bar charts in figure 7. Generally, we observe that the non-linear correlation measures induce smaller drawdowns than the conventional selection criteria, a further indication of their lower risk profile (next to smaller VaR, ES and standard deviation of returns). Furthermore, the distance approach seems to generate smaller drawdowns than the other frameworks, confirming its overall superiority.

In order to determine statistically significant differences in the reported Sharpe ratios of table 10, we additionally perform the by Memmel (2003) extended version of the test of Jobson and Korkie (1981). The technical details of this test can be found in Appendix D. Table 11 tabulates the z-scores resulting from every Sharpe ratio comparison. Positive $z-$scores indicate higher Sharpe ratios in favor of the vertically reported pairs trading variants and negative values in favor of the horizontally reported ones. Significance can be determined at the usual thresholds, i.e. ± 1.96 for 95% confidence and ± 2.575 for 99% confidence. We find that the best performing variant 'Distance approach - ADF statistic' generates a significantly higher Sharpe ratio compared to all variants except 'Cointegration approach - Kendall's τ'. Furthermore, in all but 'Constant parametric Copula - Kendall's τ' and 'Cointegration approach - Kendall's τ', the ADF statistic proves to generate significantly higher Sharpe ratios than the other pairs selection criteria. Finally, comparing the Sharpe ratios of the constant parametric copula to the time-varying counterpart, we find no significant differences between same pairs selection criteria, questioning the sense of sacrificing computation time in favor of a time-varying copula model.

[2] In our computation we used $N = 10$.

Table 10: Annualized Risk-adjusted Performance

	Risk-Return characteristic				*Drawdown Measures*					
	Sharpe Ratio	Sortino Ratio	Omega	Upside Pot.	Max Drawd. (%)	Calmar Ratio	Sterling Ratio	Burke Ratio	Ulcer Index	Martin Ratio
(A) Distance Approach										
Dist	4.88	11.89	50.13	17.40	23.83	21.48	53.04	11.67	3.10	1.65
ADF	6.28	16.24	60.01	22.08	21.40	52.18	100.91	23.60	2.55	4.37
τ	5.78	14.69	59.52	20.04	27.51	17.14	56.00	11.55	2.18	2.16
ρ	4.91	13.62	54.38	19.24	19.15	16.60	39.38	8.73	2.15	1.48
(B) Cointegration Approach										
Dist	3.55	6.00	34.99	10.99	34.00	7.54	21.19	4.22	5.51	0.47
ADF	5.14	11.00	46.57	16.65	29.31	22.78	52.34	12.06	4.51	1.48
τ	5.85	15.30	57.98	21.05	21.09	27.49	73.90	16.35	2.08	2.79
ρ	4.48	10.40	43.45	16.32	17.07	15.38	28.88	6.43	3.45	0.76
(C) Copula Approach - Constant with Empirical Marginals										
Dist	1.95	3.26	23.96	9.67	30.99	1.22	3.41	0.70	5.46	0.07
ADF	3.10	5.75	30.20	12.13	30.21	5.86	12.53	2.93	5.78	0.31
τ	2.65	4.50	26.15	11.45	22.63	2.05	4.54	1.06	4.40	0.11
ρ	2.24	3.89	25.26	10.48	23.59	1.82	4.04	0.87	5.35	0.08
(D) Copula Approach - Constant with Parametric Marginals										
Dist	2.94	5.03	29.54	10.88	23.31	4.83	9.73	2.13	4.75	0.24
ADF	4.51	8.99	38.68	15.25	23.72	16.43	33.23	7.97	3.67	1.06
τ	4.06	8.03	35.87	14.41	19.59	8.47	15.29	3.61	3.77	0.44
ρ	3.17	6.38	31.62	12.82	20.95	4.94	10.39	2.28	4.41	0.23
(E) Copula Approach - Time-varying with Parametric Marginals										
Dist	2.94	5.18	29.97	11.01	23.52	4.94	10.37	2.26	4.43	0.26
ADF	4.45	8.77	37.57	15.18	25.71	13.87	27.86	6.67	3.79	0.94
τ	3.86	6.84	33.92	12.85	22.90	5.81	13.05	3.10	3.54	0.38
ρ	3.31	6.53	32.10	12.93	16.56	6.37	11.75	2.78	3.44	0.31

Notes: This table displays annualized risk-return performance metrics, as well as drawdown measures for every pairs trading variant. Formulae of their computation are attached to Appendix C.

Other high-frequency pairs trading applications on equities report high Sharpe ratios before transaction costs as well. Miao (2014) proposes a cointegration-based strategy which yields an annual Sharpe ratio of 9.25, applied on Gas and Oil stocks at 15-minute frequency. Moreover, Dunis et al. (2010) test cointegration variants on the constituents of the Eurostoxx 50 index, at 5-minute to daily frequencies. On their five months sample, they report Information ratios[3] (IR) of up to 12.05 for a 'ADF-like' cointegration variant at 5 minute frequency. Interestingly, the reported high-frequency IRs are considerably higher than the daily IRs, confirming the findings of Aldridge (2009).

[3]They define the Information ratio as the annual return over annual standard deviation.

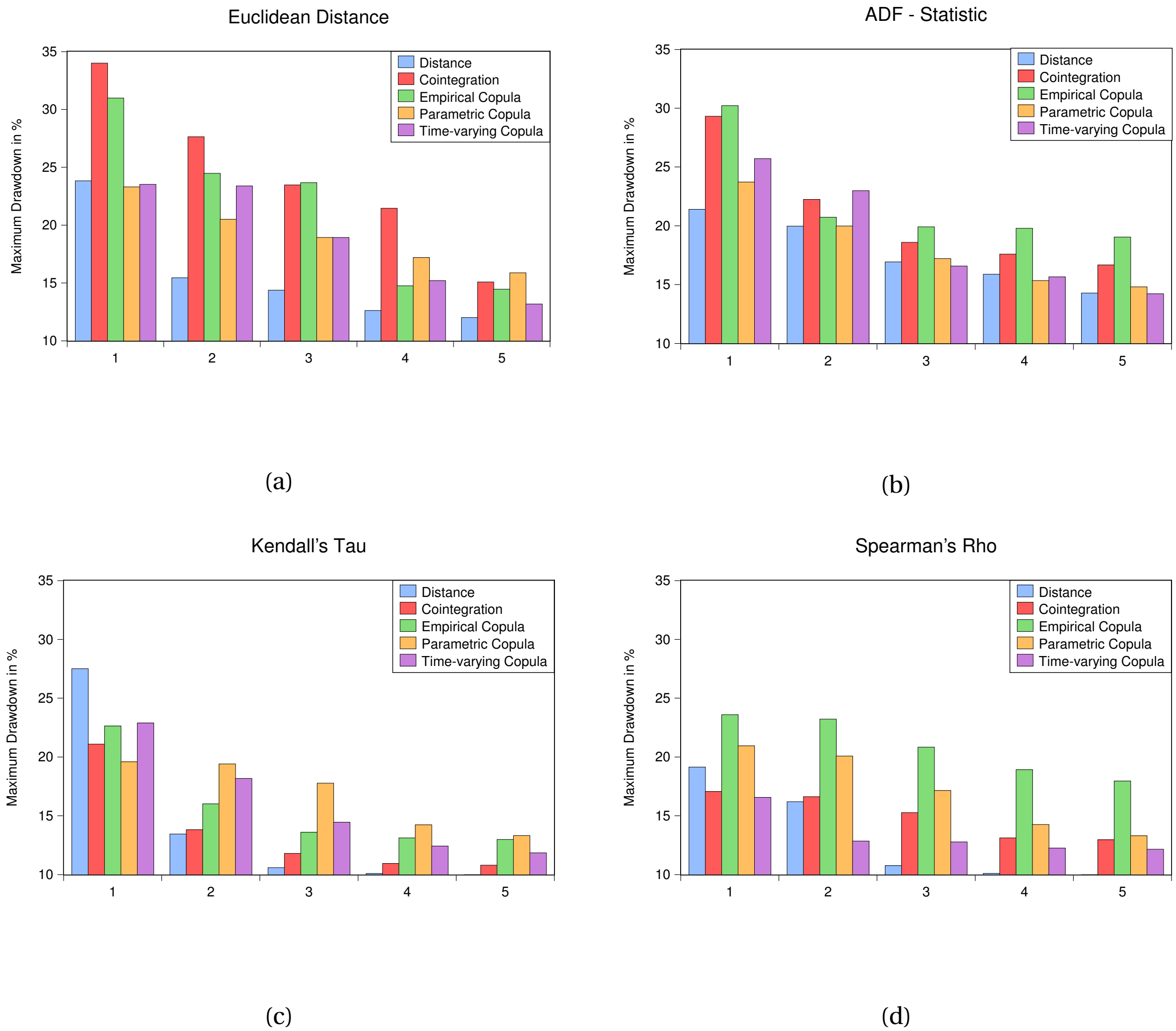


Figure 7: This Figure depicts the five largest Maximum Drawdowns of either pairs trading variant.

Table 11: Jobson-Korkie Sharpe Ratio Comparison Test Results

		Distance				Cointegration				Cop. Emp.				Cop. Par.				Cop. T-Var.		
		Dist	*ADF*	*τ*	*ρ*	*Dist*	*ADF*	*τ*	*ρ*	*Dist*	*ADF*	*τ*	*ρ*	*Dist*	*ADF*	*τ*	*ρ*	*Dist*	*ADF*	*τ*
Distance	*ADF*	5.54																		
	τ	**2.14**	**-2.17**																	
	ρ	-1.06	**-5.27**	**-3.40**																
Coint.	*Dist*	**-5.04**	**-9.11**	**-6.43**	**-3.32**															
	ADF	0.55	**-3.68**	-1.52	1.58	**4.88**														
	τ	**2.97**	-1.28	1.36	**4.18**	**7.37**	**2.48**													
	ρ	-1.65	**-5.78**	**-3.83**	-0.81	**2.74**	**-2.18**	**-5.00**												
Cop. Emp.	*Dist*	**-10.26**	**-14.05**	**-11.63**	**-8.49**	**-5.55**	**-10.00**	**-12.49**	**-7.98**											
	ADF	**-5.42**	**-9.48**	**-7.38**	**-4.51**	-1.24	**-5.97**	**-8.42**	**-3.99**	**4.24**										
	τ	**-7.61**	**-11.60**	**-10.08**	**-6.54**	**-3.45**	**-8.16**	**-11.01**	**-5.99**	**2.00**	**-2.32**									
	ρ	**-8.66**	**-12.74**	**-10.78**	**-8.25**	**-4.61**	**-9.32**	**-11.72**	**-7.72**	0.78	**-3.72**	-1.24								
Cop. Par.	*Dist*	**-6.76**	**-10.66**	**-8.17**	**-5.03**	-1.94	**-6.43**	**-9.24**	**-4.51**	**5.07**	-0.55	1.73	**2.91**							
	ADF	-1.12	**-5.37**	**-3.17**	-0.11	**3.15**	-1.70	**-4.05**	0.47	**8.51**	**4.53**	**6.62**	**7.64**	**5.07**						
	τ	**-2.60**	**-6.67**	**-5.41**	-1.60	1.68	**-3.10**	**-6.35**	-0.99	**7.09**	**2.92**	**6.03**	**6.44**	**3.65**	-1.47					
	ρ	**-5.48**	**-9.63**	**-7.60**	**-4.97**	-1.22	**-6.03**	**-8.64**	**-4.20**	**4.21**	0.02	**2.34**	**4.70**	0.59	**-4.37**	**-3.10**				
Cop. T-Var.	*Dist*	**-7.20**	**-11.10**	**-8.34**	**-5.17**	**-2.09**	**-6.59**	**-9.37**	**-4.61**	**4.76**	-0.69	1.59	**2.77**	-0.30	**-5.21**	**-3.75**	-0.74			
	ADF	-1.37	**-5.52**	**-3.40**	-0.34	**2.91**	-1.91	**-4.28**	0.24	**8.27**	**4.32**	**6.38**	**7.39**	**4.80**	-0.62	1.23	**4.14**	**4.96**		
	τ	**-3.44**	**-7.49**	**-6.27**	**-2.41**	0.82	**-3.90**	**-7.31**	-1.83	**6.31**	**2.04**	**5.80**	**5.61**	**2.79**	**-2.39**	-1.46	**2.18**	**2.94**	**-2.15**	
	ρ	**-5.25**	**-9.36**	**-7.32**	**-4.71**	-0.97	**-5.73**	**-8.38**	**-3.96**	**4.46**	0.28	**2.58**	**5.08**	0.84	**-4.08**	**-2.82**	0.73	1.01	**-3.88**	-1.92

Notes: This table reports the test results of the by Memmel (2003) extended Jobson and Korkie (1981) Sharpe ratio test. This test determines whether two Sharpe ratios are significantly different from each other. Positive reported values indicate higher Sharpe ratios in favor of the vertically reported pairs trading variants and negative values in favor of the horizontally reported ones. All reported values are z-scores. At 95% significant z-scores are printed bold. Technical details of this test can be found in Appendix D.

4.5.3 Sub-period Analysis

Due to the long sample horizon of over 18 years, a sub-period analysis is inevitable to illustrate possible differences over time. Figure 8 shows how an investment would have evolved by following either of the pairs trading variants. It is important to note, that the figure depicts the log of cumulative returns. Due to the exponential growth of cumulative returns, other than plotting logs would have resulted in an exponential curve, which does not serve the purpose of illustrating time differences. Generally, we find that all pairs trading variants show steady growth over the whole sample period, without major drawdowns. Moreover, it seems that most log-cumulative return curves are sightly concave, which is a sign of decreasing profitability. Especially from around 2002 onwards, the curves appear to be less steep. This is in line with the findings of Do and Faff (2010), who find a decline in pairs trading profitability as well. All in all, this figure confirms what we have found before: The distance approach is on top; Empirical marginals are no substitute for parametric marginals; there is no considerable difference between constant and time-varying copula approach.

A further sub-period analysis is conducted in Figure 9. This figures depicts bar charts of annualized Sharpe ratios in two year intervals. In all subfigures (a) - (d), we can observe positive 2-year Sharpe ratios, which is evidence for no negative average returns in these periods. In total, Sharpe ratios are higher in the beginning of the sample horizon, which is in line with our observation of steep cumulative return curves during that time. Particularly the Sharpe ratios belonging to the selection criteria Kendall's τ (c) and Spearman's ρ (d) seem to steadily decline. On the other hand, the ADF statistic (b) ended up with higher Sharpe ratios, after a weaker period from around 2006 - 2012. Comparing all algorithms, we find that the Sharpe ratio differences between the distance/ cointegration and the copula approaches become smaller after time. In some years a copula approach manages to even outperform both (ADF- statistic in 2014), or at least the cointegration approach (Euclidean distance in 2002; ADF in 2006).

In Appendix E, we additionally provide similar bar charts as in Figure 9, depicting the evolution of daily average returns. Observing these bar charts confirms both a drop in pairs trading profitability as well as diminishing differences between the copula and the distance/ cointegration methods. Furthermore, it suggests that the deterioration in Sharpe ratios can be attributed to a decline in average returns rather than to an increase in their volatility.

All in all, it is hard to find reasons for the sharp fall in pairs trading profitability and the overall time-variation of Sharpe ratios. Before 2002, impressive gains could have been made by applying one of these intraday pairs trading strategies, an indication for an inefficient market. After 2002, more and more traders seemed to become aware of these trading opportunities, which might have led to an exploitation of the existing 'arbitrage'. Additionally, the advance of electronic and high-frequency trading might have contributed to ever decreasing profits as well. An overall variation of Sharpe ratios could be explained by changing dependency structures or different volatility states among the goldmine stocks.

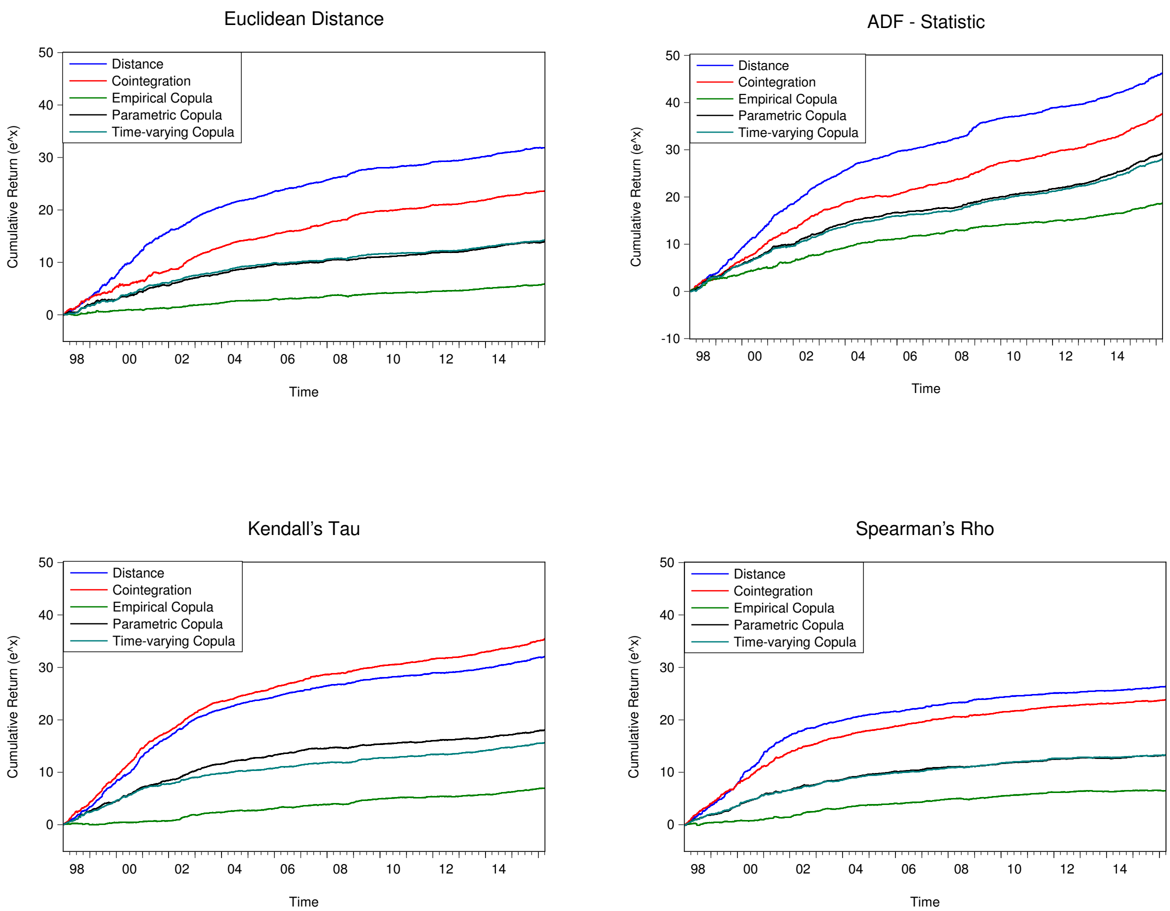
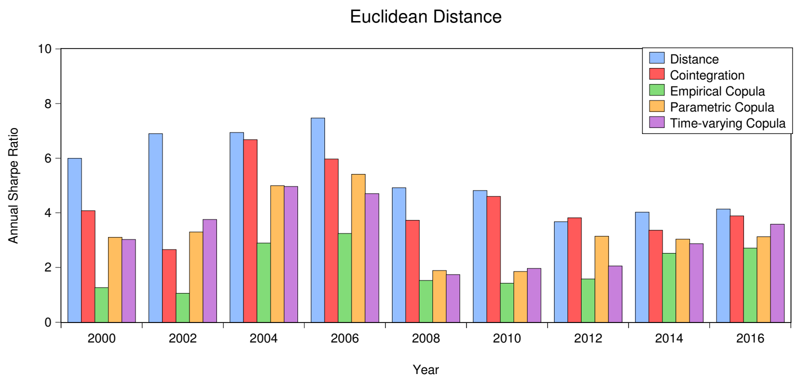
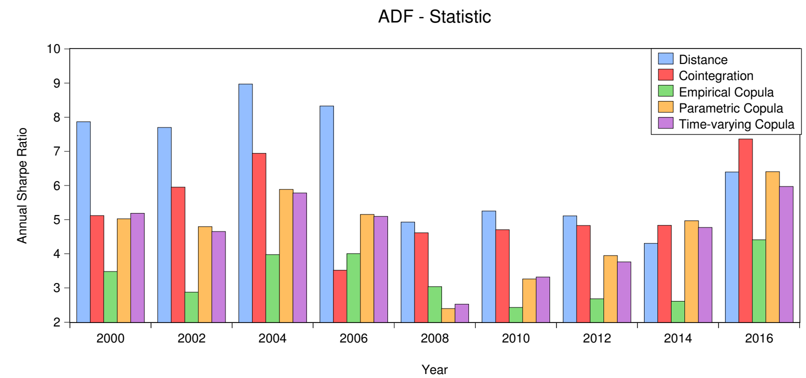


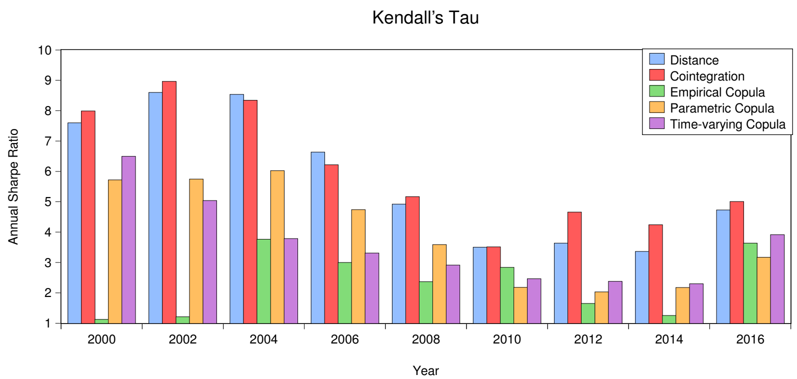
Figure 8: Plots of the Log - Cumulative Returns. Taking the exp of the Y - Axis yields real Portfolio Cumulative Returns.



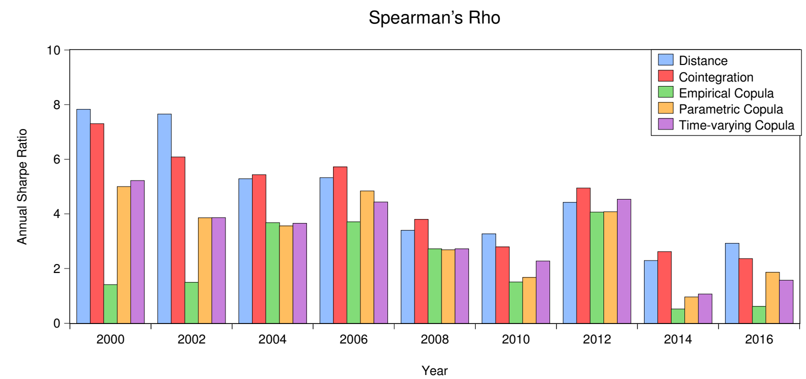
(a)



(b)



(c)



(d)

Figure 9: Annualized Sharpe Ratio Bar Charts over two-year horizons.

4.5.4 Exposure to common Risk Factors

Another important feature of trading strategy performance-analysis is to reveal the strategies' exposure to common risk factors. We do this by regressing both, the daily and monthly committed capital return streams, on the so called Carhart (1997) four-factor model. This model extends the famous Fama and French (1993) three-factor model (Market excess return, Size (Small minus Big), Value (High minus Low)) by the momentum factor of Jegadeesh and Titman (1993) [4]. Table 12 summarizes the resulting regression coefficients, their Newey-West (NW) t-statistics as well as the corresponding regression R^2.

The upper half of the table depicts the daily regression outcome. Generally, we can observe very low R^2 values of maximum 0.007 in all instances, suggesting that the chosen risk factors are not able to explain the large variance in daily pairs trading returns. Furthermore, all strategy variants generate statistically and economically positive alphas with t-statistics all above 6.89. Observing the factor loadings, we find most of the pairs trading variants to be market neutral, a typical pairs trading characteristic due to its long-short strategy. Surprisingly, some variants are significantly exposed to the size factor, even though none of the goldmine stocks can be categorized as 'small'. This paradox was also observed in the studies by Chan, Chen and Lakonishok (2002) concerning large cap mutual funds and more recently by Rad et al. (2015) and Krauss and Stübinger (2015) with respect to pairs trading. Regarding the value factor, we only find the Euclidean distance criterion of the copula approaches slightly exposed and apart from one instance no exposure to the momentum factor of any variant.

The monthly returns show less variation and thus result in higher regression R^2. Still, all alphas are economically and statistically significant. Compared to the daily regression, we observe more variants to be dependent on the market factor. On the other hand, none variant is exposed to the size factor anymore. Counting the significant dependencies of both daily and monthly regressions determines the Euclidean distance criterion to be mostly exposed to systematic risk (with 9 significant exposures) compared to Spearman's ρ (3), Kendall's τ (2) and ADF (2).

[4] We downloaded all factors from the data library of Kenneth French's website.

Table 12: Exposure to common Risk Factors

	Intercept		Market Exc. Return		Small minus Big		High minus Low		Momentum		
	Alpha	t-stat	Factor	t-stat	Factor	t-stat	Factor	t-stat	Factor	t-stat	R^2
DAILY DEPENDENCE											
A) Distance Approach											
Dist	0.723	**11.510**	-0.059	**-2.259**	0.191	**2.440**	0.044	0.564	-0.002	-0.039	0.003
ADF	1.048	**15.533**	-0.073	-1.735	0.072	0.822	-0.018	-0.174	-0.029	-0.426	0.001
τ	0.722	**8.990**	-0.050	-1.344	-0.006	-0.103	0.055	0.802	0.026	0.570	0.002
ρ	0.596	**7.440**	-0.005	-0.177	-0.009	-0.090	0.098	1.290	-0.005	-0.125	0.001
B) Cointegration Approach											
Dist	0.543	**13.790**	-0.023	-0.659	0.187	**2.019**	-0.020	-0.221	0.054	1.045	0.003
ADF	0.858	**16.550**	-0.015	-0.515	-0.006	-0.094	-0.007	-0.089	0.033	0.614	0.000
τ	0.798	**10.790**	-0.014	-0.280	-0.032	-0.516	0.095	1.397	0.053	1.350	0.001
ρ	0.540	**9.016**	-0.033	-1.030	-0.004	-0.084	0.072	1.138	-0.025	-0.633	0.001
C) Copula Approach - Constant with Empirical Marginals											
Dist	0.136	**7.597**	0.019	1.069	-0.037	-1.053	-0.077	**-2.260**	0.043	**2.300**	0.004
ADF	0.433	**12.207**	-0.015	-0.358	0.014	0.290	-0.041	-0.588	-0.032	-0.767	0.000
τ	0.159	**9.319**	-0.007	-0.517	0.010	0.337	0.008	0.226	-0.026	-1.087	0.001
ρ	0.150	**6.890**	-0.037	-1.820	0.049	1.450	0.045	1.340	-0.040	-1.610	0.003
D) Copula Approach - Constant with Parametric Marginals											
Dist	0.321	**10.260**	-0.051	-1.770	0.039	0.743	-0.114	**-2.076**	-0.009	-0.220	0.003
ADF	0.667	**15.844**	-0.045	-1.000	0.066	0.875	-0.003	-0.004	-0.042	-0.949	0.001
τ	0.406	**11.780**	-0.010	-0.250	-0.021	-0.359	0.024	0.417	0.014	0.360	0.000
ρ	0.302	**9.250**	-0.041	-1.530	0.081	**2.060**	0.031	0.620	-0.041	-1.140	0.002
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	0.328	**9.000**	-0.044	-1.260	0.181	1.780	-0.095	-1.370	0.013	0.270	0.007
ADF	0.642	**14.170**	-0.075	-1.410	0.056	0.760	-0.007	-0.110	-0.043	-1.090	0.002
τ	0.353	**11.580**	-0.020	-0.460	0.034	0.700	0.024	0.380	0.002	0.050	0.001
ρ	0.303	**10.190**	-0.059	**-2.450**	0.121	**2.760**	0.032	0.740	-0.050	-1.570	0.005
MONTHLY DEPENDENCE											
A) Distance Approach											
Dist	17.44	**2.330**	-0.639	**-2.121**	0.586	1.567	-0.116	-0.254	-0.112	-0.307	0.019
ADF	25.93	**4.220**	-1.102	-1.929	0.253	0.407	-0.232	-0.256	0.009	0.025	0.042
τ	17.42	**1.966**	-0.699	-1.234	0.310	0.604	0.671	0.640	-0.061	-0.113	0.037
ρ	14.02	**2.047**	-0.100	-0.211	0.604	1.134	0.862	0.880	0.672	1.514	0.042
B) Cointegration Approach											
Dist	12.21	**5.372**	-0.322	**-1.999**	0.050	0.123	-0.236	-0.469	0.393	**2.318**	0.046
ADF	19.97	**6.940**	-0.419	**-2.532**	0.659	1.840	-0.043	-0.107	0.113	0.542	0.025
τ	19.09	**2.606**	-0.308	-0.484	0.072	0.181	0.615	0.641	0.199	0.404	0.019
ρ	12.19	**4.274**	-0.172	-0.708	0.767	1.614	0.333	0.382	0.391	1.532	0.048
C) Copula Approach - Constant with Empirical Marginals											
Dist	2.99	**8.416**	-0.159	-1.280	0.010	0.113	-0.151	-1.385	-0.126	-1.792	0.027
ADF	9.65	**8.031**	-0.278	-0.975	0.103	0.290	-0.220	-1.022	0.077	0.611	0.016
τ	3.62	**7.161**	-0.296	**-2.384**	0.210	1.442	-0.240	-1.531	-0.173	**-2.134**	0.056
ρ	3.30	**7.024**	-0.176	-1.324	0.172	1.420	-0.053	-0.418	-0.100	-1.003	0.018
D) Copula Approach - Constant with Parametric Marginals											
Dist	7.14	**5.983**	-0.223	-0.892	-0.102	-0.404	-0.006	-0.030	-0.096	-0.491	0.015
ADF	14.88	**6.584**	-0.089	-0.309	0.632	1.274	0.949	**2.352**	0.151	0.789	0.045
τ	9.24	**3.958**	-0.063	-0.352	-0.422	-1.433	-0.322	-1.548	0.078	0.489	0.022
ρ	6.61	**4.134**	-0.080	-0.413	0.328	1.209	-0.015	-0.066	-0.028	-0.139	0.014
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	7.47	**4.702**	-0.429	-1.453	0.287	0.986	-0.279	-1.247	-0.088	-0.348	0.031
ADF	14.27	**6.724**	-0.136	-0.730	0.498	1.090	0.796	**2.283**	0.184	0.970	0.037
τ	7.92	**4.120**	-0.018	-0.112	-0.192	-0.840	-0.215	-1.238	0.044	0.245	0.007
ρ	6.62	**5.695**	-0.176	-0.970	0.389	1.412	-0.047	-0.192	0.043	0.225	0.027

Notes: This table depicts the exposure of all pairs trading variants to the four risk factors of Carhart (1997). Specifically, the table reports the coefficients, Newey-West t-statistics and R^2 resulting from the regression of daily and monthly pairs trading returns on the factors (i) excess market returns, (ii) size (Small minus Big) , (iii) value (High minus Low) and (iv) momentum. Newey-West standard errors are computed by including the optimal number of lags subject to the lowest AIC. At 95% significant values are printed bold.

4.5.5 Transaction Costs

The previous sections have shown that all pairs trading variants are highly profitable before transaction costs. Since we are dealing with high-frequency data and some pairs trading variants generated up to around 48000 trades within 18 years, including transaction costs in our analysis plays a major role in determining whether these strategies are actually trade-able in reality. Do and Faff (2012) have extensively discussed trading costs that arise within pairs trading frameworks. They estimate commissions per trade for institutional traders to decline from 10 basis points (bps) in 1998 to 7-9 bps in 2007-2009. Retail traders trade at around 10 bps according to Bogomolov (2013). In addition to these broker commissions, other transaction costs are market impact and short selling costs. Concerning both, we follow Krauss and Stübinger (2015) and estimate the bid-ask spread to be 5 bps and short selling costs to be negligible. All in all, assuming 10 bps per transaction, this yields conservative $4 \times 10 + 2 \times 5 = 50$ bps per 'round-trip' trade (four trades and two bid-ask spread crossings). The upper half of table 13 tabulates the daily excess return distribution (on committed capital) resulting from this conservative approach. It becomes clear that all of the pairs trading variants loose their profitability under this transaction cost scheme -a finding in line with Do and Faff (2012). Except for the variants 'Distance approach - ADF' and 'Distance approach - Spearman's ρ', all pairs trading frameworks show significantly negative average excess returns, with up to 73% negative daily returns. No investor would trade under these circumstances.

We believe, however, that the above used transaction cost scheme is too conservative and not representative considering average transaction costs in 2016. For example, as one of the largest brokers, 'Interactive Brokers' offers commission fees of 0.005 USD per U.S. equity share (with a maximum of 0.5% of transaction volume), which will yield a more moderate cost structure compared to above -unless one would trade with pennystocks[5]. Taking this cost structure as representative, together with the 10 bps per RT trade for two bid-ask spread crossings, we now re-evaluate the daily excess return distribution by investing a fixed amount of 100,000$ daily. Short selling costs are still negligible. The lower half of table 13 reports the corresponding daily excess return distribution. We can observe a completely different picture compared to before. While all copula variants remain slightly unprofitable, the distance and cointegration approaches be-

[5]The exact cost structure can be found here: https://www.interactivebrokers.com/en/index.php?f=1590p=stocks1.

come successful. In both frameworks, we find positive daily average excess returns, significantly different from zero in six out of eight cases. Interestingly, the best performing selection criterion without transaction costs, the ADF statistic, does not remain superior to the remaining criteria after accounting for commissions. On the contrary, in all instances the Kendall's τ criterion shows largest mean excess returns, followed by the Spearman's ρ criterion. We find two explainations for the change in profitability under this transaction cost scheme. First, as described in Section 4.5.1, the ADF criterion generated considerably more trades than the other criteria, which is now penalized by transaction costs. Second, and more interestingly, the ADF criterion implies an average of 35 bps per RT, compared to much lower 26 bps per RT for the Euclidean distance criterion and 25 bps per RT for both non-linear correlation measures. This significantly higher cost per RT trade suggests that the ADF criterion more often tends to select stocks with lower share prices. The reason for this tendency could be that lower priced stocks, i.e. pennystocks, are known to be rather volatile and thus more likely to generate higher degrees of spread mean-reversion.

In light of our findings in Section 4.5.3, however, the question arises whether the reported daily excess return distributions of table 13 are representative for *future* pairs trading returns. To answer this question, we plot the cumulative returns of the four most lucrative variants after IB transaction costs in figure 10. This figure clearly confirms our suspicion that the reported daily excess return distribution is biased towards the impressive average returns in the beginning of the sample. It appears, that the sample is dividable into three phases: (1) From 1998 - 2004, we experience large profits. (2) From 2004 - 2008, small but still positive profits are generated. (3) From 2008 - 2016, all strategies become unprofitable with negative average returns.

Overall, we can conclude that transaction costs considerably affect the profitability of high-frequency pairs trading strategies and the final performance of selection criteria. Even though some variants show positive daily average returns considering the whole data sample, the recent decline in pairs trading profitability suggests that neither pairs trading variant will generate positive returns in the near future. The only 'hope' for future high-frequency pairs trading is that the strategies benefit from the ever decreasing transaction costs in the industry. In the meantime, researchers and traders should refrain from utilizing the Euclidean distance between assets as selection criterion. We find both non-linear correlation measures to be superior.

Table 13: Daily Excess return Distribution on Committed Capital after Transaction Costs

	Average (%)	t-stat (NW)	Median (%)	Std. (%)	Skew.	Kurt.	Min. (%)	Max. (%)	Ret. < 0 (%)	VaR (1%)	ES (1%)
Conservative approach: 0.5% for 1 RT (4 trades)											
(A) Distance Approach											
Dist	-0.34	**-6.50**	-0.43	1.73	0.54	17.63	-18.3	19.7	66.4	-5.19	-7.91
ADF	-0.03	-0.51	-0.16	2.10	1.40	12.23	-16.8	19.7	54.9	-5.23	-7.68
τ	-0.16	**-2.08**	-0.29	1.70	1.60	39.53	-28.0	19.7	60.4	-3.72	-5.59
ρ	-0.09	-1.14	-0.16	1.67	2.05	17.95	-16.7	19.7	55.0	-3.74	-5.90
(B) Cointegration Approach											
Dist	-0.45	**-10.83**	-0.52	1.87	1.68	37.79	-29.3	29.2	69.8	-5.75	-11.12
ADF	-0.18	**-3.86**	-0.29	2.22	1.25	20.52	-19.7	27.3	58.1	-5.84	-9.24
τ	-0.29	**-3.66**	-0.50	1.80	2.93	42.02	-19.7	29.2	67.7	-4.12	-6.06
ρ	-0.29	**-4.58**	-0.43	1.72	2.88	39.83	-13.4	29.2	66.3	-4.16	-6.31
(C) Copula Approach - Constant with Empirical Marginals											
Dist	-0.53	**-13.14**	-0.37	1.26	1.82	43.10	-11.6	21.5	72.9	-3.90	-5.85
ADF	-0.63	**-12.05**	-0.76	2.12	1.46	17.97	-16.6	25.5	71.3	-6.21	-8.72
τ	-0.51	**-12.60**	-0.34	1.08	0.00	10.04	-11.0	9.8	72.4	-3.47	-4.63
ρ	-0.55	**-13.85**	-0.45	1.18	1.31	28.63	-20.5	17.1	74.1	-3.50	-4.96
(D) Copula Approach - Constant with Parametric Marginals											
Dist	-0.68	**-14.51**	-0.71	1.63	0.69	21.95	-18.3	20.1	73.8	-5.02	-7.91
ADF	-0.55	**-9.83**	-0.72	2.12	1.05	9.83	-15.0	21.3	69.4	-5.72	-8.46
τ	-0.55	**-9.75**	-0.66	3.37	1.49	23.54	-15.9	22.6	72.4	-4.04	-5.71
ρ	-0.51	**-9.68**	-0.56	1.49	3.34	53.17	-12.7	29.6	71.2	-3.91	-5.48
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	-0.65	**-13.69**	-0.67	1.69	1.60	35.02	-18.3	26.5	73.9	-4.95	-7.89
ADF	-0.59	**-10.16**	-0.74	2.10	1.29	13.10	-15.0	23.0	69.4	-5.73	-8.48
τ	-0.52	**-10.84**	-0.58	1.43	-0.12	29.81	-24.9	14.0	70.6	-3.81	-5.71
ρ	-0.51	**-9.96**	-0.56	1.48	3.31	54.03	-12.7	28.6	71.3	-3.85	-5.41
Interactive Brokers approach: 0.10% per RT plus 0.005$ per share											
(A) Distance Approach											
Dist	0.10	**2.43**	0.04	1.77	-0.08	15.63	-18.2	18.6	45.2	-5.18	-7.83
ADF	0.21	**3.64**	0.13	2.08	0.73	9.72	-28.0	18.6	42.8	-5.18	-7.52
τ	0.26	**4.19**	0.07	1.70	1.08	36.64	-28.1	18.6	40.5	-3.43	-5.43
ρ	0.22	**3.64**	0.05	1.64	1.62	17.37	-16.9	18.6	38.6	-3.50	-5.72
(B) Cointegration Approach											
Dist	0.01	0.22	0.01	1.92	1.02	32.49	-29.2	32.6	47.8	-5.67	-11.08
ADF	0.08	1.81	0.02	2.24	0.53	16.69	-19.5	27.3	45.7	-5.65	-9.17
τ	0.24	**3.99**	0.11	1.83	2.32	37.43	-19.5	32.6	44.9	-3.92	-6.04
ρ	0.11	**2.67**	0.05	1.70	2.36	38.65	-13.3	32.6	46.4	-4.13	-6.28
(C) Copula Approach - Constant with Empirical Marginals											
Dist	-0.16	**-9.63**	-0.20	1.21	1.55	49.78	-11.5	23.5	57.6	-3.69	-5.83
ADF	-0.20	**-5.52**	-0.21	2.10	0.97	17.51	-16.6	26.0	56.2	-6.02	-8.69
τ	-0.13	**-7.53**	-0.12	1.03	-0.02	13.31	-11.0	10.0	55.4	-3.01	-4.35
ρ	-0.14	**-7.64**	-0.16	1.12	0.96	34.12	-20.4	17.0	56.9	-3.21	-4.77
(D) Copula Approach - Constant with Parametric Marginals											
Dist	-0.18	**-5.66**	-0.10	1.62	-0.05	22.16	-18.2	20.1	53.7	-5.02	-7.86
ADF	-0.18	**-3.81**	-0.20	2.08	0.46	9.39	-14.9	21.4	56.1	-5.67	-8.36
τ	-0.04	-1.05	-0.10	1.50	1.10	22.46	-15.2	22.9	54.4	-3.78	-5.57
ρ	-0.07	-1.93	-0.04	1.45	2.84	50.51	-12.6	29.8	51.9	-3.77	-5.41
E) Copula Approach - Time-varying with Parametric Marginals											
Dist	-0.16	**-4.96**	-0.10	1.67	0.86	33.16	-18.2	27.0	54.0	-4.92	-7.81
ADF	-0.21	**-4.12**	-0.22	2.07	0.52	10.62	-14.9	22.4	56.4	-5.60	-8.31
τ	-0.06	-1.61	-0.06	1.41	-0.58	33.22	-24.2	14.7	52.6	-3.55	-5.53
ρ	-0.06	-1.84	-0.04	1.43	2.83	52.11	-12.6	29.1	52.1	-3.55	-5.33

Notes: This table displays the daily excess return distributions (on committed capital) under the light of two transaction cost schemes. The t-statistics are computed with Newey-West (NW) standard errors. The number of lags included are determined subject to the lowest AIC. Statistically significant values (at 95%) are printed bold.

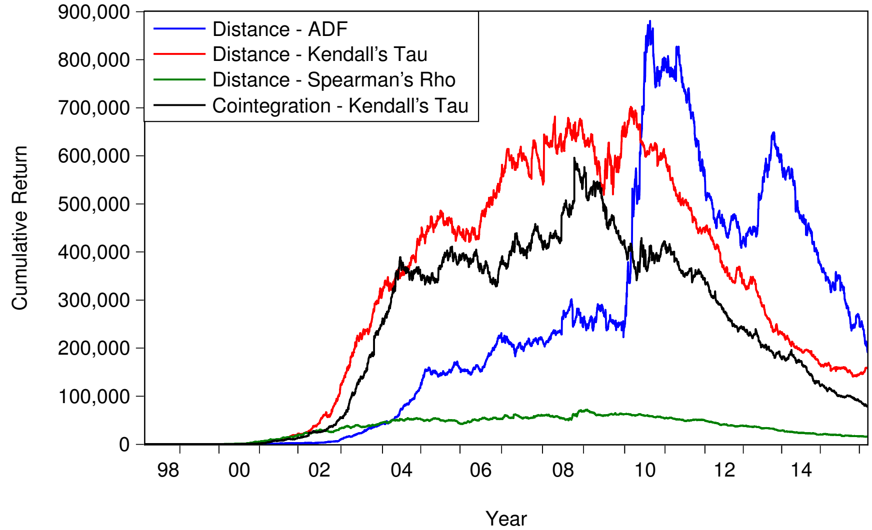


Figure 10: Cumulative return plots of the four best performing pairs trading variants after transaction costs (Interactive Brokers approach).

5 Conclusion

This thesis is about the empirical evaluation of three pairs trading strategies (distance, cointegration, copula variants) and four pairs selection criteria (Euclidean distance, degree of spread mean-reversion, Kendall's τ, Spearman's ρ). In particular, we compare the performance of either strategy and selection criterion, by means of a high-frequency trading strategy.

Our contribution to the literature is two-fold. The first contribution is entirely methodological. To the best of our knowledge, we are the first to utilize a time-varying copula-based pairs trading algorithm. We update the copula parameter throughout the trading period by using an expanding window size. Moreover, we examine the benefit of estimating empirical marginal distributions, next to the common way of fitting parametric marginals. Finally, we introduce the use of two non-linear correlation measures as pairs selection criteria, Kendall's τ and Spearman's ρ. In the recent pairs trading literature, most authors selected pairs according to their Euclidean distance.

The second contribution is entirely empirical. We develop an intraday strategy and apply all pairs trading variants on U.S. goldmine stocks. The data are in minute resolution and cover the time from January 1998 to April 2016. In terms of time-span, this dataset can be regarded as the longest of all high-frequency pairs trading applications in the literature so far. Concerning copula-based pairs trading, this thesis represents the first high-frequency application. Before transaction costs, we find all pairs trading variants to be highly profitable with average daily excess returns of 13 - 104 bps. In total, the distance approach generates highest average returns, followed by the cointegration method. The copula-based framework performs comparably poor. Among their variants, we find considerably higher returns by fitting parametric marginal distributions. Furthermore, we do not observe significant improvements by varying the copula parameter. Even though the copula-based method tends to generate more round-trip trades per day, falsely estimated parameter considerably decrease profitability by inducing wrong transaction signals. Among the pairs selection criteria, the degree of spread mean-reversion proves to be most profitable, followed by Kendall's τ and very similar Spearman's ρ and Euclidean distance. Decisive for the superiority of both criteria are more generated (round-trip) trades and winrates of up to 66%. Highly appealing are the risk-adjusted return characteristics of the pairs trading strategies. With annual Sharpe ratios of up to 6.25, the distance approach proves to be the best risk-rewarded strategy. Over time, we find a sharp decline in pairs trading profitability. Moreover, we recognize that differences in Sharpe ratios among the three trading frameworks become smaller. A common risk factor analysis shows that neither of the pairs trading variants is greatly exposed to systematic risk, leading to statistically and economically significant alphas. Transaction costs greatly affect the profitability of high-frequency pairs trading. While the distance and cointegration approaches still manage to achieve positive (significant) average daily returns of up to 26 bps over the *whole* sample, the declining returns in recent years suggest that neither of the pairs trading methods will be lucrative anymore. The only hope for high-frequency pairs trading is provided by the ever decreasing transaction costs in the industry. Under lower commission fees, traders should then refrain from selecting pairs according to the conventional selection criteria. Both non-linear correlation measures, Kendall's τ and Spearman's ρ, provide lower risk and higher average returns after transaction costs.

Related to the existing pairs trading literature, our findings show the following properties. First, regarding the superiority of pairs trading frameworks, we find similar results as Rad et al. (2015), yet contradicting findings compared to Liew and Wu (2013) and Xie et al. (2014). While the two latter papers conclude that the copula-based method is superior, we find the distance and cointegration approaches to be more profitable. In terms of generated trading opportunities, however, our findings are in line with Xie et al. (2014) - the copula method indeed seems to generate more round-trip trades per day. Second, concerning a decline in pairs trading profitability our findings confirm Do and Faff (2010). Finally, after accounting for transaction costs we examine an unprofitable copula-based strategy, in a similar manner as Stander et al. (2013). Moreover, we also examine high-frequency pairs trading to be highly sensitive to transaction costs, similarly as Kishore (2012).

Based on our findings, we can conclude that the simplest form of pairs trading, the distance approach, is hard to beat. The mere challenge in advancing the copula-based framework lies in the correct estimation of the parameter. Only then, the advantages inherent with the use of copulas may be fully observed. Further research could thus be aimed at estimating more sophisticated time-varying copula models. Inspiration can be found in Manner and Reznikova (2012). Furthermore, we conclude that it is not optimal to select pairs according to the most conventional criterion - the minimum Euclidean distance between pair constituents. We rather suggest to select pairs subject to their highest Kendall's τ correlation coefficient. However, there is still great room for further research, with several unexamined correlation measures to choose from. As a relatively new concept, for instance, one could investigate the use of the 'randomized dependency coefficient' by Lopez-Paz, Henning and Schölkopf (2013). This concept appears to be promising as it is closely linked to copulas. Finally, due to their tendency to generate many small returns at high-frequency, we observe great difficulty for the pairs trading strategies to succeed after transaction costs. Therefore, further research could be aimed at exploring ways which generate fewer, yet larger returns at high-frequency.

References

Akaike, H. (1973). Maximum likelihood identification of gaussian autoregressive moving average models. *Biometrika*, 60(2):255–265.

Aldridge, I. (2009). *High-frequency trading: a practical guide to algorithmic strategies and trading systems*, volume 459. John Wiley and Sons.

Aussenegg, W. and Cech, C. (2008). Simple time-varying copula estimation. *Available at SSRN 1313714*.

Bianchi, R., Drew, M., and Zhu, R. (2009). Pairs trading profits in commodity futures markets. In *Proceedings of Asian Finance Association 2009 International Conference*, pages 1–26. University of Queensland.

Bogomolov, T. (2011). Pairs trading in the land down under. In *Finance and Corporate Governance Conference*.

Bogomolov, T. (2013). Pairs trading based on statistical variability of the spread process. *Quantitative Finance*, 13(9):1411–1430.

Bowen, D., Hutchinson, M. C., and O'Sullivan, N. (2010). High frequency equity pairs trading: transaction costs, speed of execution and patterns in returns. *Journal of Trading*, 5(3):31–38.

Bowen, D. A. and Hutchinson, M. C. (2014). Pairs trading in the uk equity market: risk and return. *The European Journal of Finance*, pages 1–25.

Broussard, J. P. and Vaihekoski, M. (2012). Profitability of pairs trading strategy in an illiquid market with multiple share classes. *Journal of International Financial Markets, Institutions and Money*, 22(5):1188–1201.

Caldeira, J. and Moura, G. V. (2013). Selection of a portfolio of pairs based on cointegration: A statistical arbitrage strategy. *Available at SSRN 2196391*.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, 52(1):57–82.

Chan, L. K., Chen, H.-L., and Lakonishok, J. (2002). On mutual fund investment styles. *Review of financial studies*, 15(5):1407–1437.

Cummins, M. and Bucca, A. (2012). Quantitative spread trading on crude oil and refined products markets. *Quantitative Finance*, 12(12):1857–1875.

Daniel, W. W. et al. (1990). Applied nonparametric statistics.

Do, B. and Faff, R. (2010). Does simple pairs trading still work? *Financial Analysts Journal*, 66(4):83–95.

Do, B. and Faff, R. (2012). Are pairs trading profits robust to trading costs? *Journal of Financial Research*, 35(2):261–287.

Do, B., Faff, R., and Hamza, K. (2006). A new approach to modeling and estimation for pairs trading. In *Proceedings of 2006 Financial Management Association European Conference*, pages 87–99. Citeseer.

Dunis, C., Laws, J., and Evans, B. (2006). Trading futures spreads: an application of correlation and threshold filters. *Applied Financial Economics*, 16(12):903–914.

Dunis, C. L., Giorgioni, G., Laws, J., and Rudy, J. (2010). Statistical arbitrage and high-frequency data with an application to eurostoxx 50 equities. *Liverpool Business School, Working paper*.

Eling, M. (2008). Does the measure matter in the mutual fund industry? *Financial Analysts Journal*, 64(3):54–66.

Emery, G. W. and Liu, Q. W. (2002). An analysis of the relationship between electricity and natural-gas futures prices. *Journal of Futures Markets*, 22(2):95–122.

Engle, R. F. and Granger, C. W. (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica: journal of the Econometric Society*, pages 251–276.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, 33(1):3–56.

Ferreira, L. (2008). New tools for spread trading. *Futures*, 37(12):38–41.

Gatev, E., Goetzmann, W. N., and Rouwenhorst, K. G. (2006). Pairs trading: Performance of a relative-value arbitrage rule. *Review of Financial Studies*, 19(3):797–827.

Genest, C., Ghoudi, K., and Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika*, 82(3):543–552.

Girma, P. B., Paulson, A. S., et al. (1999). Risk arbitrage opportunities in petroleum futures spreads. *Journal of Futures Markets*, 19(8):931–955.

Gutierrez, J. A. and Tse, Y. (2011). Illuminating the profitability of pairs trading: A test of the relative pricing efficiency of markets for water utility stocks. *The Journal of Trading*, 6(2):50–64.

Hu, L. (2006). Dependence patterns across financial markets: a mixed copula approach. *Applied financial economics*, 16(10):717–729.

Huck, N. and Afawubo, K. (2015). Pairs trading and selection methods: is cointegration superior? *Applied Economics*, 47(6):599–613.

Jacobs, H. (2015). What explains the dynamics of 100 anomalies? *Journal of Banking & Finance*, 57:65–85.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, 48(1):65–91.

Jobson, J. D. and Korkie, B. M. (1981). Performance hypothesis testing with the sharpe and treynor measures. *The Journal of Finance*, 36(4):889–908.

Jondeau, E., Poon, S.-H., and Rockinger, M. (2007). *Financial modeling under non-Gaussian distributions*. Springer Science & Business Media.

Kat, H. M. (2003). The dangers of using correlation to measure dependence. *The Journal of Alternative Investments*, 6(2):54–58.

Kendall, M. G. (1948). Rank correlation methods.

Kishore, V. (2012). Optimizing pairs trading of us equities in a high frequency setting.

Kole, E., Koedijk, K., and Verbeek, M. (2007). Selecting copulas for risk management. *Journal of Banking & Finance*, 31(8):2405–2423.

Krauss, C. (2015). Statistical arbitrage pairs trading strategies: Review and outlook. Technical report, IWQW Discussion Paper Series.

Krauss, C. and Stübinger, J. (2015). Nonlinear dependence modeling with bivariate copulas: Statistical arbitrage pairs trading on the s&p 100. Technical report, IWQW Discussion Paper Series.

Li, M. L., Chui, C. M., and Li, C. Q. (2014). Is pairs trading profitable on china ah-share markets? *Applied Economics Letters*, 21(16):1116–1121.

Liew, R. Q. and Wu, Y. (2013). Pairs trading: A copula approach. *Journal of Derivatives & Hedge Funds*, 19(1):12–30.

Lopez-Paz, D., Hennig, P., and Schölkopf, B. (2013). The randomized dependence coefficient. In *Advances in Neural Information Processing Systems*, pages 1–9.

Manner, H. and Reznikova, O. (2012). A survey on time-varying copulas: specification, simulations, and application. *Econometric Reviews*, 31(6):654–687.

Maverick (2015, January 8). What is a good sharpe ratio? | Investopedia. Retrieved June 17,2016 from http://www.investopedia.com/ask/answers/010815/what-good-sharpe-ratio.asp.

Memmel, C. (2003). Performance hypothesis testing with the sharpe ratio. *Finance Letters*, 1(1).

Meyer, C. (2013). The bivariate normal copula. *Communications in Statistics-Theory and Methods*, 42(13):2402–2422.

Miao, G. J. (2014). High frequency and dynamic pairs trading based on statistical arbitrage using a two-stage correlation and cointegration approach. *International Journal of Economics and Finance*, 6(3):96.

Mori, M. and Ziobrowski, A. J. (2011). Performance of pairs trading strategy in the us reit market. *Real Estate Economics*, 39(3):409–428.

Nath, P. (2003). High frequency pairs trading with us treasury securities: Risks and rewards for hedge funds. *Available at SSRN 565441*.

Nelsen, R. (2006). An introduction to copulas, 2nd. *New York: SpringerScience Business Media*.

Patton, A. (2008). Copula models for financial time series, forthcoming in handbook of financial time series.

Rad, H., Low, R. K. Y., and Faff, R. W. (2015). The profitability of pairs trading strategies: Distance, cointegration, and copula methods. *Cointegration, and Copula Methods (June 3, 2015)*.

Ross, S. A. (1976). The arbitrage theory of capital asset pricing. *Journal of economic theory*, 13(3):341–360.

Simon, D. P. (1999). The soybean crush spread: Empirical evidence and trading strategies. *Journal of Futures Markets*, 19(3):271–289.

Sklar, M. (1959). *Fonctions de répartition à n dimensions et leurs marges*. Université Paris 8.

Stander, Y., Marais, D., and Botha, I. (2013). Trading strategies with copulas.

Stock, J. H. and Watson, M. W. (1988). Testing for common trends. *Journal of the American statistical Association*, 83(404):1097–1107.

Trivedi, P. K. and Zimmer, D. M. (2007). *Copula modeling: an introduction for practitioners*. Now Publishers Inc.

Van der Vaart, A. W. (2000). *Asymptotic statistics*, volume 3. Cambridge university press.

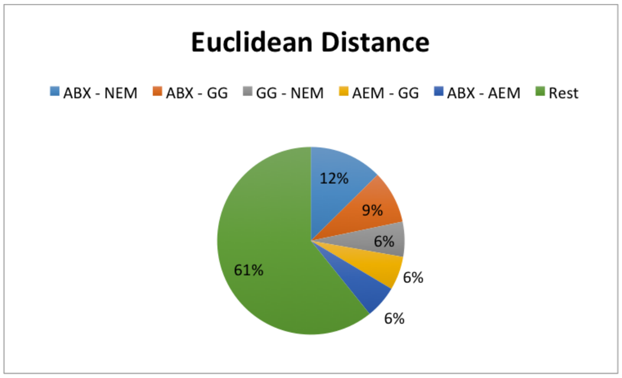
Venter, G. (2001). Tails of copulas. Technical report, Proceedings of the ASTIN Colloquium.

Vidyamurthy, G. (2004). *Pairs Trading: quantitative methods and analysis*, volume 217. John Wiley & Sons.

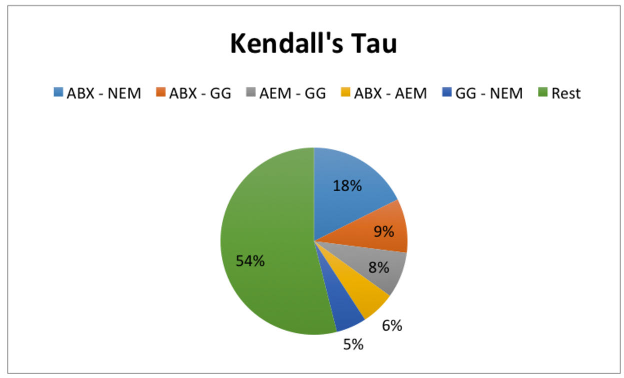
Wahab, M., Cohn, R., and Lashgari, M. (1994). The gold-silver spread: Integration, cointegration, predictability, and ex-ante arbitrage. *Journal of Futures Markets*, 14(6):709–756.

Xie, W., Liew, Q. R., Wu, Y., and Zou, X. (2014). Pairs trading with copulas. *Available at SSRN 2383185*.

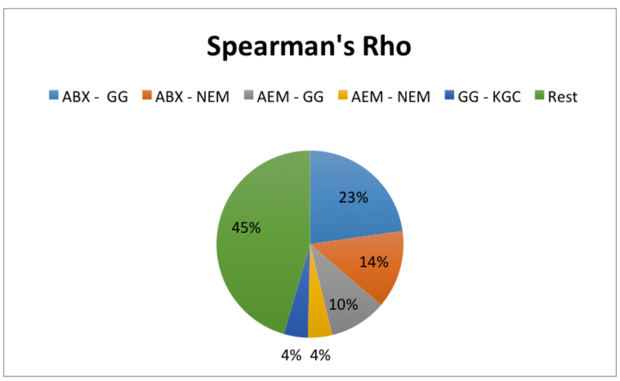
A Most frequently selected pairs



(a)



(b)



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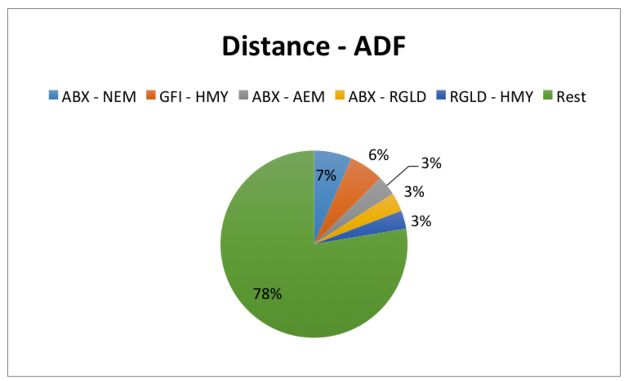
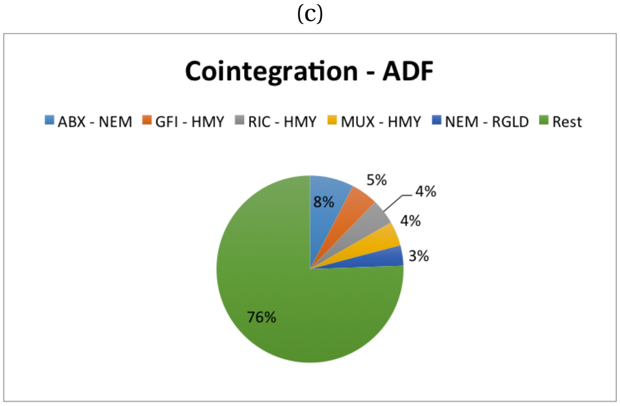


(d)



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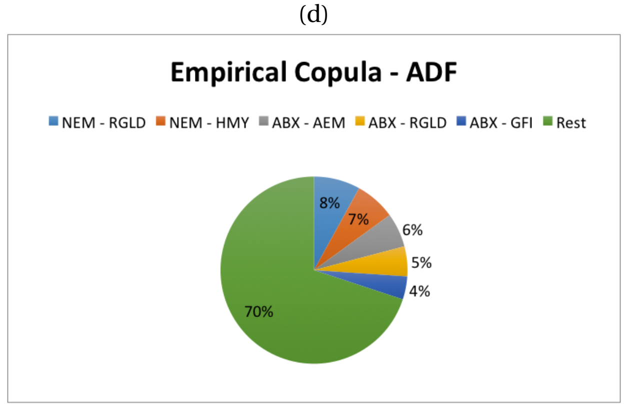


(f)

Figure 11: These pie charts show the five most frequently selected pairs for either selection criterion.

B Copula Estimation Error - An Illustration

As described in Section 4.5.1, wrongly estimated copula parameter might induce false transaction signals. In this Appendix we illustrate this issue.

For illustration purposes, we choose a rather extreme case of parameter estimation error. Let us assume that the true (minutely) marginal distributions of stocks A and B are represented by Generalized Extreme Value (GEV) distributions, with location parameter $\mu = -0.00008$, scale parameter $\sigma = 0.00188$ and shape parameter $k = -0.852$ and $k = -0.685$ respectively. Assuming that we are able to correctly estimate μ and σ, we only fit a Gaussian distribution to the returns and hence are not able to fully capture the correct marginal distributions (for simplicity we fit a Gaussian distribution now - in the thesis we would estimate the additional degrees of freedom to fit a Student-t distribution). Figure 12 illustrates this dilemma, by plotting the probability density functions of the two true GEV distributions and the fitted normal distribution. It becomes apparent, that the Gaussian distribution attaches more probability to large positive returns, while the GEV distributions truly generate no large positive returns at all. Moreover, the Gaussian distribution significantly underestimates small positive returns.

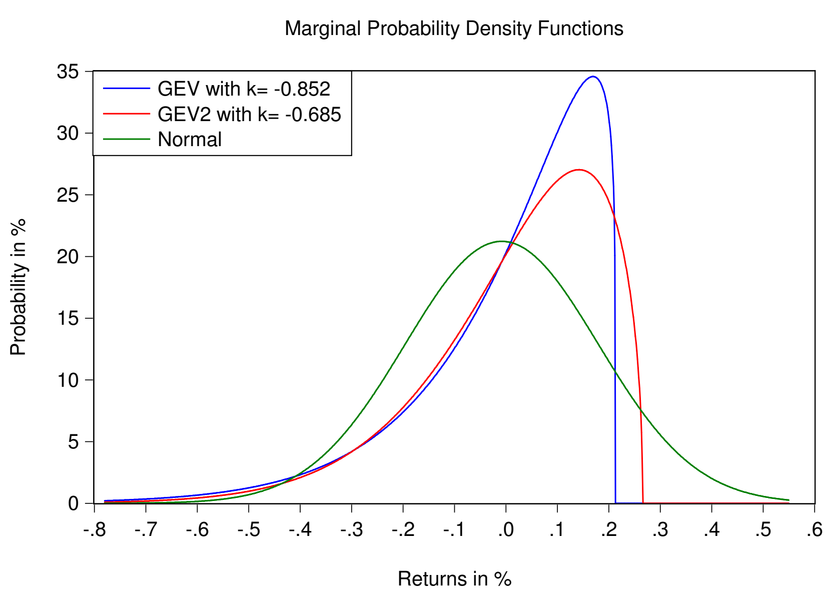
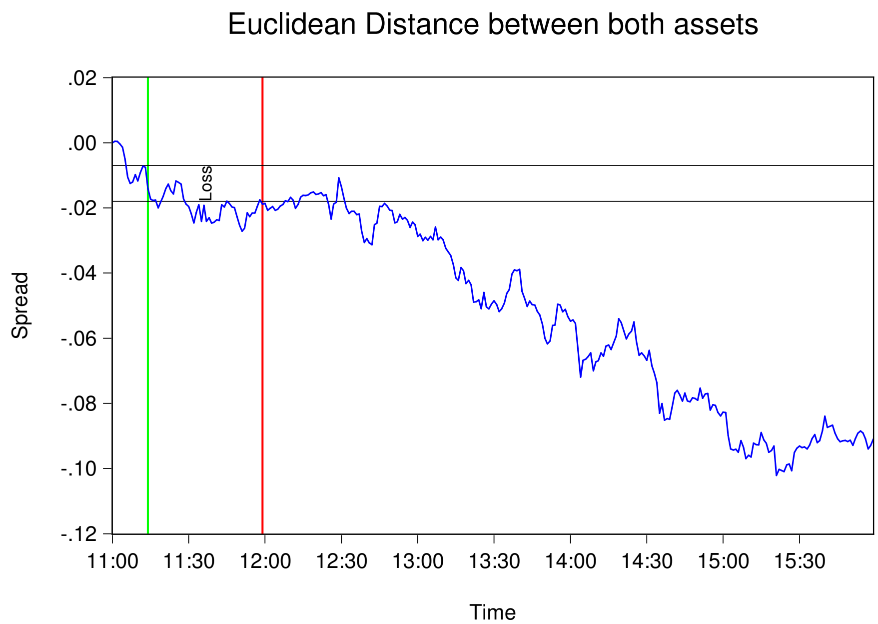


Figure 12: True and fitted marginal PDFs for stocks A and B, with $\mu = -0.00008$ and $\sigma = 0.00188$.

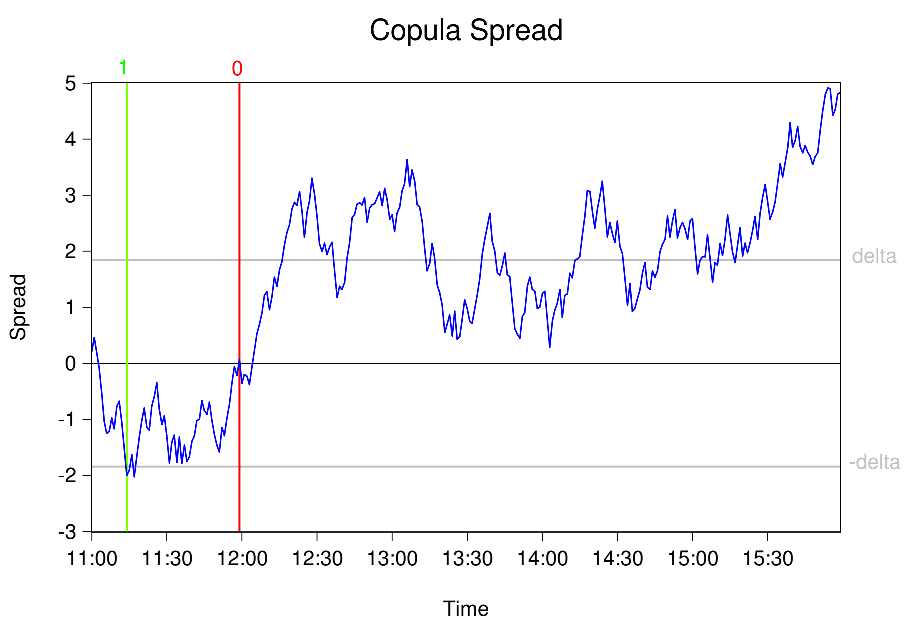
We further assume that both stocks are truly *uncorrelated*, yet we falsely estimate their correlation to be $\rho = 0.35$ during the formation period. We utilize this correlation estimate to fit a Gaussian copula and trade on only one of the two induced spreads for simplicity. Using the same length as trading period, we proceed by simulating two uncorrelated returns $r_{A,t}$ and $r_{B,t}$ from the GEV distributions in every minute. These returns are directly transformed to $u = \Phi^{-1}(r_{A,t})$ and $v = \Phi^{-1}(r_{B,t})$, with Φ^{-1} being the inverse of our fitted Gaussian distribution. In the following, we plug u, v and $\rho = 0.35$ in the conditional Gaussian copula function and construct the copula spread.

Figure 13 plots the Euclidean distance between the two simulated price series in (a) and the resulting copula spread in (b). At 11:14, the copula spread triggered trades and reversed them at 11:59. In (a) it can be observed that this RT trade happens to be negative, due to the fact that both prices further diverged from each other by the time of exiting the position. Clearly, the estimation error of both the marginal distributions and the copula parameter is too large to adequately specify the copula spread. Consequently negative RT trades may occur in the copula approach. Contrary to the copula method, the distance framework would have entered positions as well, yet not exited before market closing (which would have resulted in a far bigger loss in this case).

In order to fully understand the importance of adequate parameter estimates, we further conduct a small simulation study. To this end, we simulate both a series including estimation error and a series without estimation error. The error series is simulated by making use of the same assumptions and settings as described above. The error-free series is gained by assuming a *true* correlation between both assets of $\rho = 0.35$ and by assuming the marginal distributions to be Gaussian, instead of GEV. Hence, we then simulate the returns $r_{A,t}$ and $r_{B,t}$ from a multivariate normal distribution with $\mu = \begin{bmatrix} -0.00008 \\ -0.00008 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 3.53 \times 10^{-6} & 1.24 \times 10^{-6} \\ 1.24 \times 10^{-6} & 3.53 \times 10^{-6} \end{bmatrix}$. Repeating the simulation 1000 times and computing the overall percentage of negative RT trades for either variant yields 15% negative RT trades for the error series and 1.5% (slightly) negative RT trades for the error-free variant- a great improvement. Yet, it becomes clear that even with no estimation error, RT trades may become negative. A reason could be that the simulated returns are not exactly correlated with $\rho = 0.35$, after all.



(a)



(b)

Figure 13: The plot in (a) shows the Euclidean distance between both cumulative price series. The plot in (b) is the Copula spread resulting from the simulated returns.

C Formula Sheet for Performance Evaluation

Let $er_t = r_t - r_{f,t}$ denote the daily excess return, with $r_{f,t}$ as the daily 1 month U.S. T-Bill rate. Furthermore let $\bar{er}$ denote the mean daily excess return and σ denote the sample standard deviation of the daily excess returns. The downside standard deviation is denoted by $\sigma_d = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\min\left(r_t - r_{f,t}, 0\right)\right)^2}$.

$$\text{Annualized Sharpe Ratio} = \frac{\bar{er}}{\sigma} \times \sqrt{252}$$

$$\text{Annualized Sortino Ratio} = \frac{\bar{er}}{\sigma_d} \times \sqrt{252}$$

$$\text{Annualized Omega} = \frac{\sum_{t=1}^{T} er_t \times \mathbb{1}_{\{er_t>0\}}}{\sum_{t=1}^{T} |er_t| \times \mathbb{1}_{\{er_t<0\}}} \times \sqrt{252}$$

$$\text{Annualized Upside Potential} = \frac{\sum_{t=1}^{T} er_t \times \mathbb{1}_{\{er_t>0\}}}{\sigma_d} \times \sqrt{252}$$

Concerning the drawdown measures, let $p(t)$ denote the portfolio value of a strategy at time t, in an interval from [0,T]. Furthermore let the annualized excess return be denoted as $ar = \left(1 + cumret\right)^{1/18.25} - 1$, with $cumret = \prod_{t=1}^{T}(1 + er_t)$.

$$\text{Maximum Drawdown} = \max_{\tau\in(0,T)}\left[\frac{\max_{t\in(0,\tau)} p(t) - p(\tau)}{\max_{t\in(0,\tau)} p(t)}\right]$$

$$\text{Calmar Ratio} = \frac{ar}{\text{Maximum Drawdown}}$$

$$\text{Sterling Ratio} = \frac{ar}{\text{Average yearly Maximum Drawdown}}$$

$$\text{Burke Ratio} = \frac{ar}{\sqrt{\text{Sum of 10 largest(Maximum drawdowns)}^2}}$$

$$\text{Ulcer Index} = \sqrt{\frac{\sum_{t=1}^{T} R_t^2}{T}},$$
$$R_t = 100 \times \frac{p(t) - \max_{\tau \in (0,t)} p(\tau)}{\max_{\tau \in (0,t)} p(\tau)}$$

$$\text{Martin Ratio} = \frac{ar}{\text{Ulcer Index}}$$

D Sharpe Ratio Comparison Test

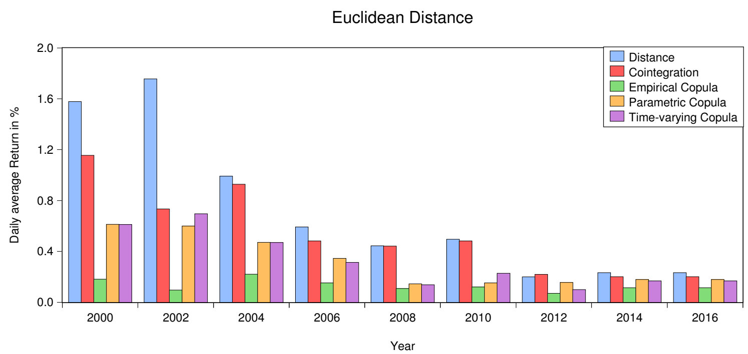
In this thesis we use the by Memmel (2003) extended version of the Jobson and Korkie (1981) Sharpe ratio comparison test. Let (μ_x, μ_y) and (σ_x, σ_y) denote the means and standard deviations of return series x and y respectively. Furthermore, let σ_{xy} denote their covariance. Then, the z-score of the null hypothesis of no difference can be computed as

$$z = \frac{\sigma_x \mu_y - \sigma_y \mu_x}{\sqrt{\theta}},$$

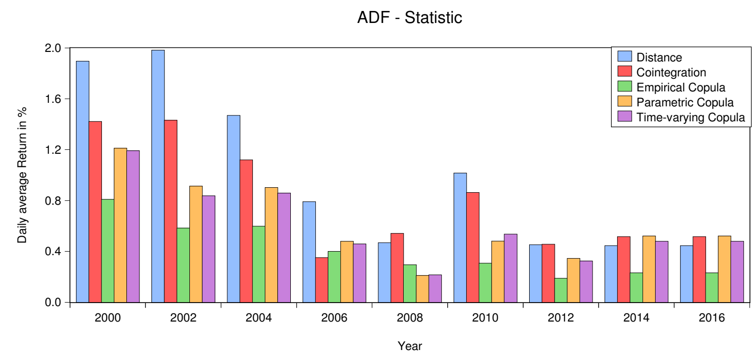
where

$$\theta = \frac{1}{T}\left(2\sigma_x^2\sigma_y^2 - 2\sigma_x\sigma_y\sigma_{xy} + 0.5\mu_x^2\sigma_y^2 + 0.5\mu_y^2\sigma_x^2 - \frac{\mu_x\mu_y\sigma_{xy}^2}{\sigma_x\sigma_y}\right).$$

E Subperiod Analysis - Daily average Returns



(a)



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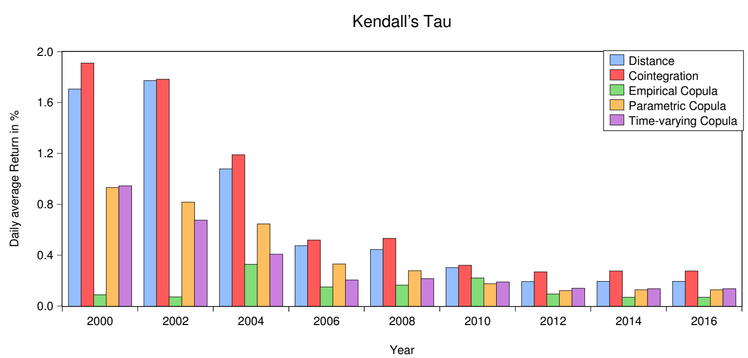

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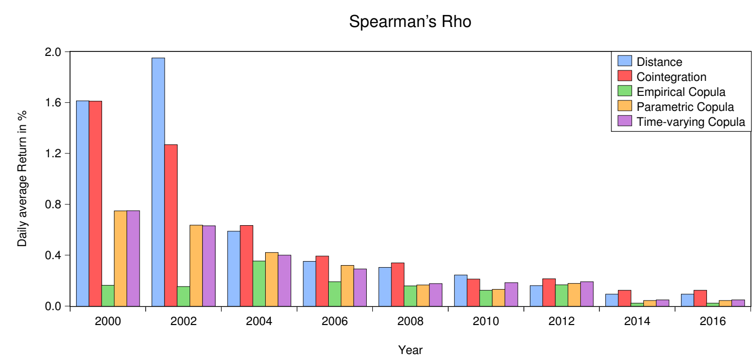

(d)

Figure 14: Daily average Return Bar Charts over two-year horizons.