**How does the daily volatility of foreign exchange rates depend on the time of day at which the daily returns are calculated?**

Małgorzata Doman[1], Ryszard Doman[2]

***Abstract:*** In the paper, we show how the estimates of the daily volatility of major exchange rates, EUR/USD, AUD/USD, GBP/USD, and NZD/USD, depend on the hour at which the daily returns are calculated. FOREX market is open 24 hours a day, but traders from different parts of the world, if some local time is fixed, are most active in different times of day. This is the reason why the dynamics of volatility changes during a trading day. To analyze this feature, we consider daily returns calculated using the exchange rates quoted at each hour of the day. Volatility (the square root of the conditional variance) is described by means of GARCH models. The approach used enables us to scrutinize changes in the volatility, depending on the hour of the day, which can be useful in risk management. We investigate separately bid and ask prices, so we obtain some results concerning microstructure of the FOREX market as well.

***Keywords:*** exchange rates, major currencies, bid, ask, volatility, GARCH, FOREX

***JEL codes:*** *G15, F31, C58, C22*

## 1. Introduction

In the paper, we document how the estimates of the daily volatility of major exchange rates depend on the hour at which the returns are calculated. We investigate the foreign exchange rates: EUR/USD, AUD/USD, GBP/USD, and NZD/USD. The notation XYZ/USD means the price of one unit of currency XYZ in the US dollars. Traditionally, the selected currency pairs are the only major currency pairs that involve the US dollar as the counter currency (Donnelly 2019). According to BIS (2019), the currency pairs accounted for 40.6% of the average daily turnover of the FOREX market in April 2019. The analysis is based on daily returns calculated using the exchange rates quoted at each full hour of a day. Throughout the paper, we use the current local time in London (LDN), taking into account daylight saving time rules. Dynamics

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of volatility (the square root of the conditional variance) is described by means of ARMA-GARCH models.

**Table 1. A rough chart of trading activity in the FOREX market**

<table>
<tr><td colspan="24">London Time</td></tr>
<tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td></tr>
<tr><td colspan="6">New Zealand</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="3"></td></tr>
<tr><td colspan="8">Australia</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="9">Japan</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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<tr><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="11">Continental Europe</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="11">UK</td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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</table>

The FOREX market is open 24 hours a day, with the exception of weekends (Table 1). More precisely, it works from Sunday around 20:00 LDN to Friday around 22:00 LDN (Donnelly 2019). Trading between the FOREX participants is conducted through electronic communication networks in various markets around the world. Decentralization implies that currencies are quoted almost continuously. The level of daily FOREX activity, however, varies, depending on the time of day. Extensive research literature exists, which studies the subject from the perspective of the market microstructure of FOREX markets, using high frequency data (e.g. Andersen and Bollerslev 1998, Baillie and Bollerslev 1991, Breedon and Ranaldo 2013, Ranaldo 2009). Our interests are somewhat different. We are going to examine how the diversity in FOREX activity during a day can impact on estimation of daily volatility of foreign exchange rates, which could be usable in risk management. For instance, a question appears if the choice of daily volatility estimates calculated using exchange rates observed at some specific time of day could produce desirable Value-at-Risk (VaR) estimates.

Results obtained by Doman and Doman (2014) on the basis of mid prices for GBP/USD and EUR/USD show that the dynamics of daily volatility estimates calculated at different hours do not differ considerably. Nevertheless, the differences between the averages of volatility corresponding to some hours are statistically significant. In the present paper, we perform more careful analysis involving both bid and ask prices. We investigate the differences in the daily volatility dynamics corresponding to returns calculated at each hour of the day: 0:00, 1:00,…, 23:00. Moreover, we get an evidence of different volatility behavior of bid and ask prices for some key hours of the day. Since we analyze separately the dynamics of bid and ask prices, our results develop expertise on the behavior of sellers and buyers in the FOREX market.

The analysis presented in this paper is the first part of a wider investigation concerning the structure and dynamics of volatility and dependence in the FOREX market. The results presented here concern the impact of  the time of day, at which the daily returns are calculated, on volatility. The standardized residuals obtained as a by-product in the process of fitting a volatility model are used in the next parts of the analysis, which deal with the dependence structure.

## 2. GARCH models

In this paper, we apply the most popular approach to model volatility, i.e. ARMA-GARCH modeling. Let $\Omega_{t-1}$ denote the set of information available on the daily return process $r_t$ up to time $t-1$. The volatility $\sigma_t$ for the period $t$ is defined as $\sigma_t = \sqrt{\mathrm{var}(r_t \mid \Omega_{t-1})}$. Since $\sigma_t$ is not observable, it has to be estimated. This can be done by different methods. One possible approach is to use a parametric model GARCH(*p*,*q*). Let $y_t$ denote the return innovation process defined by $y_t = r_t - E(r_t \mid \Omega_{t-1})$. In the linear GARCH(*p*,*q*) model originally introduced by Bollerslev (1986), the conditional variance, $\sigma_t^2$, is postulated to satisfy the equations

$$y_t = \sigma_t \varepsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2, \tag{2}$$

where $\varepsilon_t$ are independently and identically distributed with zero mean and unit variance. Assumptions on the parameters are: $\omega > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$. The latter inequality implies that the unconditional variance of $y_t$ is finite. As concerns the distribution of the errors $\varepsilon_t$, it is usually assumed to be the standard normal or the standardized Student *t*. Sometimes, it is also advised to apply the standardized skewed Student *t* distribution (Tsay 2010). One of the most important features of GARCH models is that they can capture volatility clustering and a part of excess kurtosis occurring in financial time series. Among many extensions of the standard GARCH model, very popular are ones, for which negative innovations can have a bigger impact on volatility than positive innovations of the same magnitude. Such effect is usually referred to as the leverage effect (Tsay 2010). In the present paper we use two such models. The first is the GJR-GARCH(*p*,*q*) model (Glosten *et al*., 1993), in which

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \left( \alpha_i y_{t-i}^2 + \gamma_i I\left(y_{t-i} < 0\right) y_{t-i}^2 \right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{3}$$

where $I(\cdot)$ is an indicator function. The next, the APARCH(*p*,*q*) model (Ding *et al*., 1993), can be seen as a generalization of the former. It assumes that

$$\sigma_t^{\delta} = \omega + \sum_{i=1}^{q} \alpha_i \left( \left| y_{t-i} \right| - \gamma_i y_{t-i} \right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{4}$$

for $\delta > 0$ and $-1 < \gamma_i < 1$, $(i = 1,\dots,q)$.

## 3. The data

We consider four from major currency pairs quoted on the FOREX market: EUR/USD, GBP/USD, AUD/USD, and NZD/USD. The notation XYZ/USD means the price of one unit of currency XYZ in the USD. Usually, investigation of dynamics of exchange rates is performed for mid prices. We decided to examine bid and ask prices separately to get information about the behavior of the two groups of market participants: sellers and buyers. The analysis is performed for the daily percentage logarithmic returns, which are separately calculated based on exchange rates quoted at each hour of the day: 0:00, 1:00,…., 23:00. Throughout the paper we use the current local time in London (LDN) following European Daylight Saving Time rules. The period under scrutiny is from March 3, 2003 to October 1, 2019. This gives 4216 daily returns for each hour of the day. The data come from the Historical Data Feed of Dukascopy Bank SA, and were collected on-line via the webpage https://www.dukascopy.com/swiss/english/marketwatch/historical/.

In Figures 1 and 2 we present the periodic daily pattern observed in 1-hour returns of the exchange rates EUR/USD and NZD/USD (bid and ask prices). The plots are obtained by calculating the average absolute value of the returns corresponding to each 1-hour segment. It is easy to observe that during a day there are phases of higher volatility and thereby with higher market activity. The maxima visible in Figures 1 and 2 indicate the periods of the highest traders' activity: 1:00 LDN (Australia and Asia opening), 7:00, 8:00, 9:00 LDN (Europe opening), 13:00, 15:00 LDN (the USA opening), and 22:00 LDN (the USA close). The starting point for our analysis was a supposition that this variation in daily activity should be related with different patterns of conditional volatility estimated on the basis of daily returns corresponding to exchange rates quoted at different hours of the day. Analyzing daily returns obtained in such a manner is quite natural under certain conditions. For instance, the behavior of daily returns calculated based on the quotations observed at 3:00 LDN is inevitably driven by the activity of the Asian traders. On the other hand, the returns corresponding to 9:00 LDN considerably reflect decisions undertaken by the European traders. Moreover, a different level of importance can be attached to some currencies by investors operating in different continents, which should also be manifested in the volatility of the foreign exchange rates.

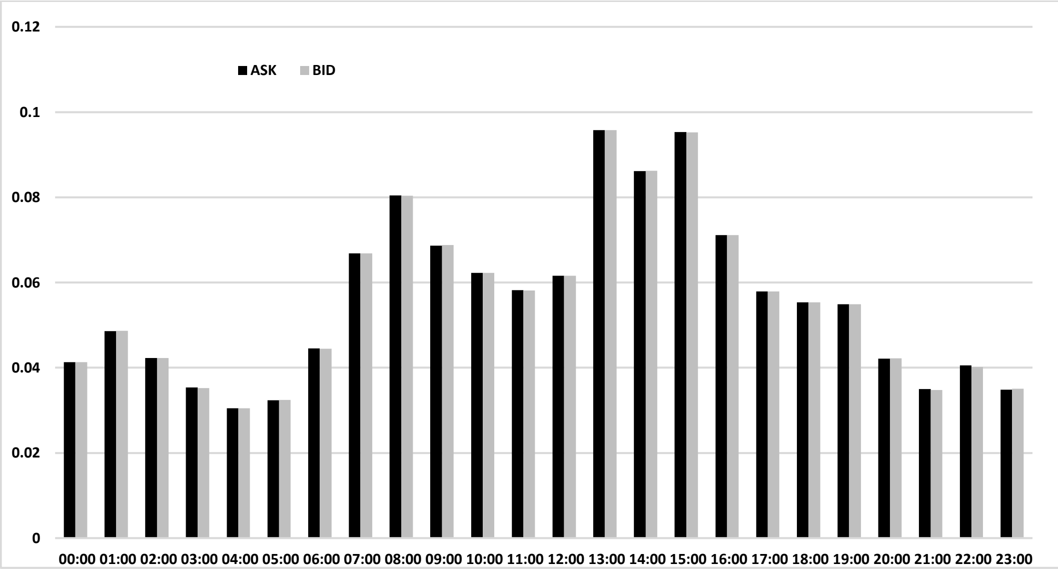


**Figure 1. EUR/USD: the daily periodic pattern in 1-hour returns (London Time). Calculations based on the observations from the period: September 3, 2003 – October 1, 2019**

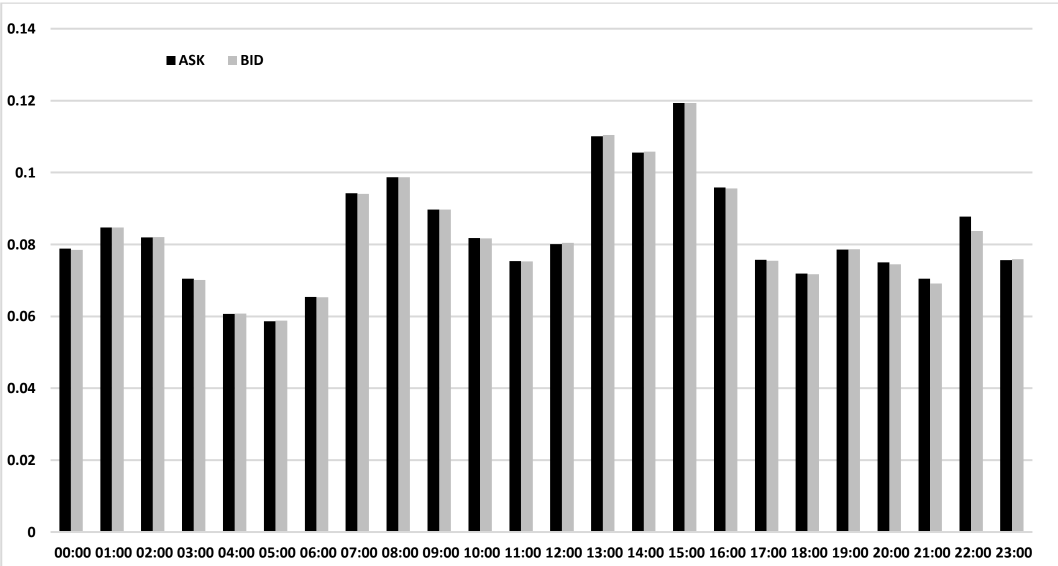


**Figure 2. NZD/USD. The daily pattern in 1-hour returns (LDN). Calculations based on the observations from the period: September 3, 2003 – October 1, 2019**

In Figure 2 we observe an exceptional situation in our analysis where at 21:00 and 22:00 the averages for ask prices are visibly higher than for bid prices. This is due the simultaneous close of the US market and opening of trading in New Zealand, and shows higher uncertainty and activity of buyers. The differences are small but statistically significant.

## 4. Results

The results presented in this Section were obtained with an Ox package G@RCH 8 (Laurent 2018, Doornik 2013). Our analyzes include 192 time series of daily returns, so it is impossible to give here full information about the fitted models. Instead of that we show in Table 2 the types of ARMA-GARCH models fitted to the analyzed data, together with the corresponding error distributions. Since each exchange rate is a result of valuation of two currencies, additional problems in our analysis are connected with introducing daylight saving time (DST). In different countries the local time changes occur on different days. This changes the hours at which the markets open and close and of course impacts our investigations. After careful analysis of the data, we decided to introduce to the mean and volatility equations in ARMA-GARCH models a dummy variable (DST) pointing the days when daylight saving time is in force in the USA but not in the EU and UK.

The models described in Table 2 indicate some changes in the dynamics of volatility when different hours of the day are considered. In each case we chose the best model from plenty specifications offered in GA@RCH 8 package. The chosen types of models are GARCH($p$,$q$) , GJR($p$,$q$) and APARCH( $p$,$q$). Mainly, $p=1$ and $q=1$. As can be seen, most of the fitted models follow the GJR or APARCH specification, which indicates the presence of the leverage effect in the data. In the case of the considered exchange rates, if estimates of the $\gamma_i$ in formulas (3) and (4) are positive, such effect means that the market reacts stronger to the depreciation of the base currency relative to the USD. In our results the $\gamma_i$ parameter estimates are positive. Thus, in view of the above, it is natural that the types of models fitted to bid and ask data are often different. In the case of AUD/USD, the leverage effect disappears for a long period when the US market is most active (15:00-20:00). The models chosen for the returns on GBP/USD show rather complicated dynamics but the asymmetric volatility responses are present for less than half of the considered hours. A different situation is observed for EUR/USD and NZD/USD: almost all fitted models are asymmetric GARCH.

In several problems in finance, for instance, for VaR computation, the modeling of entire conditional distribution of returns is of crucial importance. This is why we use the GARCH models that can capture the asymmetry of the conditional distribution of returns. As concerns the investigated returns, we have observed that in the case of AUD/USD and NZD/USD, in each fitted model the parameter indicating negative skewness is strongly significant. This means that, conditionally on the previous information, the return innovation drop relative to the mode is on each day more probable than the rise. Roughly speaking, this shows that the AUD and NZD are more prone to weaken than strengthen against the USD.

**Table 2. Types of the fitted ARMA-GARCH models** (DST denotes a dummy variable indicating daylight saving time in force in the USA, but not in the EU and UK)

| | **AUDUSD ASK** | **AUDUSD BID** | **EURUSD ASK** | **EURUSD BID** | **GBPUSD ASK** | **GBPUSD BID** | **NZDUSD ASK** | **NZDUSD BID** |
|---|---|---|---|---|---|---|---|---|
| **00:00** | AR(1)-GARCH(1,3) skew Student | AR(1)-GARCH(1,3) skew Student | GJR(1,1) Student | APARCH(1,1) Student | GJR(1,2) Student | GARCH(1,1) Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **01:00** | ARMA(1,1)-APARCH(1,1) skew Student | APARCH(1,1) skew Student | APARCH(1,1)+DST Student | GJR(1,1) Student | GJR(1,1)+DST Student | GARCH(1,4)+DST Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **02:00** | AR(1)-GJR(1,2) skew Student | AR(1)-GJR(1,2) skew Student | APARCH(1,1) Student | APARCH(1,1) Student | GJR(1,1)+DST Student | GARCH(1,4)+DST Student | APARCH(1,1) skew Student | APARCH(1,1) skew Student |
| **03:00** | AR(1)-GJR(1,2) skew Student | GJR(1,1) skew Student | APARCH(1,1) Student | APARCH(1,1) Student | GJR(1,1)+DST Student | MA(1)-APARCH(1,1) Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **04:00** | GJR(1,2) skew Student | GARCH(1,1) skew Student | GARCH(1, 3)+DST Student | GARCH(1, 3)+DST skew Student | GARCH(1,1)+DST Student | APARCH(1,1) Student | APARCH(1,1) skew Student | APARCH(1,1) skew Student |
| **05:00** | GJR(1,2) skew Student | GJR(1,1) skew Student | GARCH(1,1) skew Student | GARCH(1,1) skew Student | APARCH(1,1) Student | APARCH(1,1) Student | GARCH(1,1) skew Student | APARCH(1,1)+DST skew Student |
| **06:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | APARCH(1,1) Student | APARCH(1,1) Student | APARCH(1,1)+DST Student | APARCH(1,1)+DST Student | GARCH(1,1)+DST skew Student | GARCH(1,1) skew Student |
| **07:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | APARCH(1,1) Student | APARCH(1,1) Student | APARCH(1,1)+DST Student | GARCH(1,2) Student | GJR(1,1) skew Student | GARCH(1,1) skew Student |
| **08:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | APARCH(1,1) skew Student | GJR(1,1) skew Student | GARCH(1,5) Student | GARCH(1,5) Student | GJR(1,1) skew Student | MA(1)-APARCH(1,1) skew Student |
| **09:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | GJR(1,1) Student | APARCH(1,1) Student | GARCH(1,5) Student | GARCH(1,1) Student | GARCH(1,1) skew Student | GARCH(1,1) skew Student |
| **10:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | APARCH(1,1) skew Student | APARCH(1,1) skew Student | GJR(1,2) Student | GARCH(1,4) Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **11:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | APARCH(1,1) skew Student | APARCH(1,1) skew Student | GARCH(1,1) Student | GARCH(1,1) Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **12:00** | GARCH(1,1)+DST skew Student | APARCH(1,1) skew Student | GARCH(1,1)+DST skew Student | GARCH(1,1)+DST skew Student | GJR(1,2) Student | GARCH(1,1) Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **13:00** | APARCH(1,1) skew Student | APARCH(1,1) skew Student | GJR(1,1) Student | GJR(1,1) Student | GARCH(1,3) Student | GARCH(1,3) skew Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **14:00** | GJR(1,1) skew Student | GJR(1,1) skew Student | GJR(1,1) Student | GJR(1,1) Student | MA(1)-GJR(1,2)+DST skew Student | GARCH(1,1) skew Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **15:00** | GARCH(1,1) skew Student | GARCH(1,1) skew Student | GJR(1,1) Student | APARCH(1,1) Student | GJR(1,1) skew Student | GARCH(1,1) Student | GJR(1,2) skew Student | GJR(1,2) skew Student |
| **16:00** | GARCH(1,1) skew Student | GARCH(1,1) skew Student | GJR(1,1)+DST Student | GJR(1,1)+DST Student | GARCH(1,1) skew Student | GJR(1, 3) skew Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **17:00** | GARCH(1,1) skew Student | GARCH(1,1) skew Student | GJR(1,1)+DST Student | GJR(1,1) Student | MA(1)-GARCH(1,4) skew Student | GARCH(1,1) skew Student | APARCH(1,1) skew Student | GJR(1,2) skew Student |
| **18:00** | AR(1)-GARCH(1,1) skew Student | MA(1)-GARCH(1,1) skew Student | GJR(1,1) Student | GARCH(1,1)+DST skew Student | GARCH(1,1)+DST Student | GARCH(1,1) Student | APARCH(1,1) skew Student | GJR(1,1) skew Student |
| **19:00** | GARCH(1,1) skew Student | MA(2)-GARCH(1, 3) skew Student | GJR(1,1)+DST Student | GARCH(1,1) skew Student | GARCH(1,1)+DST skew Student | GJR(1,1)+DST Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **20:00** | GARCH(1,1) skew Student | MA(1)-GARCH(1,1) skew Student | GJR(1,1)+DST Student | GARCH(1,1) Student | GARCH(1,1)+DST Student | GJR(1,1)+DST Student | GJR(1,1) skew Student | APARCH(1,1) skew Student |
| **21:00** | GJR(1,2) skew Student | MA(1)-GJR(1,1) skew Student | GJR(1,1)+DST Student | GJR(1,1)+DST Student | GARCH(1,4)+DST skew Student | GARCH(1,1)+DST Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **22:00** | AR(1)-GJR(1,3) skew Student | AR(1)-GJR(1,1) skew Student | AR(1)-GJR(1,1)+DST Student | GJR(1,1)+DST Student | APARCH(1,1)+DST Student | APARCH(1,1)+DST Student | GJR(1,1) skew Student | GJR(1,1) skew Student |
| **23:00** | AR(1)-APARCH(1,1) skew Student | AR(1)-GJR(1,1) skew Student | GJR(1,1)+DST Student | GJR(1,1) Student | MA(1)-GARCH(1,1)+DST Student | APARCH(1,1)+DST Student | GJR(1,1) skew Student | GJR(1,1) skew Student |

In the next four subsections we discuss in detail the results obtained for the analyzed return series. For each currency pair we present the surfaces of daily volatility, depending on the hour at which the data were quoted (Figures 3, 4, 7, 8, 11, 12, 15, and 16). They give a reader a general impression about the volatility dynamics. The period under scrutiny includes three main subperiods of market turmoil: the subprime crisis (2007-2009), the European sovereign debt crisis (2010-2013), and Brexit (2016). A common feature of all plots is the explosion of volatility during the subprime crisis. Even a cursory analysis of the figures reveals the differences in the volatility caused by the choice of the hour. Plots in Figures 5, 9, 13, and 17 are cross-sections of the surfaces presenting the dynamics of volatility at the selected hours. The last figure in each subsection presents daily ranges (the differences between the largest and smallest values) of the daily volatilities corresponding to daily returns calculated at each hour of the day (Figures 6, 10, 14, 18).

### 4.1. EUR/USD

From Figures 3 and 4, it is easy to see that the volatility of EUR/USD depends on the hour at which the data are collected. Figure 5 enables us to assess the appearing differences. In Figure 6, we show the plot of daily ranges of daily volatilities corresponding to daily returns calculated at each hour of the day. The minimum is about 0.03 and the maximum about 0.47, for both bid and ask prices. It is understandable that during a turmoil period the ranges are higher than when the markets are calm.

The surfaces of volatility obtained for bid and ask prices look very similar, but in fact there are clear differences for daily return calculated at some hours of the day. For instance, for 19:00-returns and 20:00-returns, the volatility of bid prices is higher and more variable, showing higher nervousness of the sellers closing their position at the end of the trading day in the US.

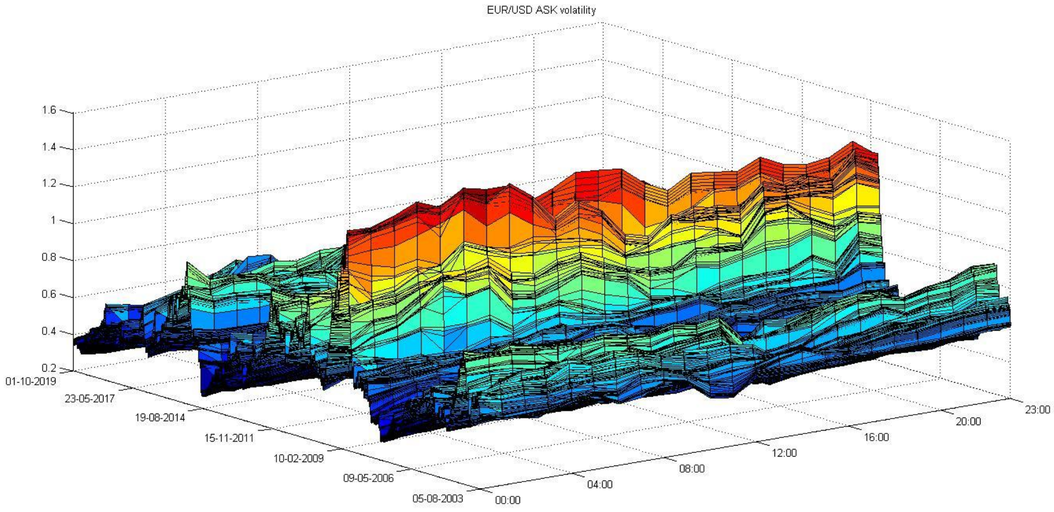


**Figure 3. Daily volatility surface for EUR/USD ASK**

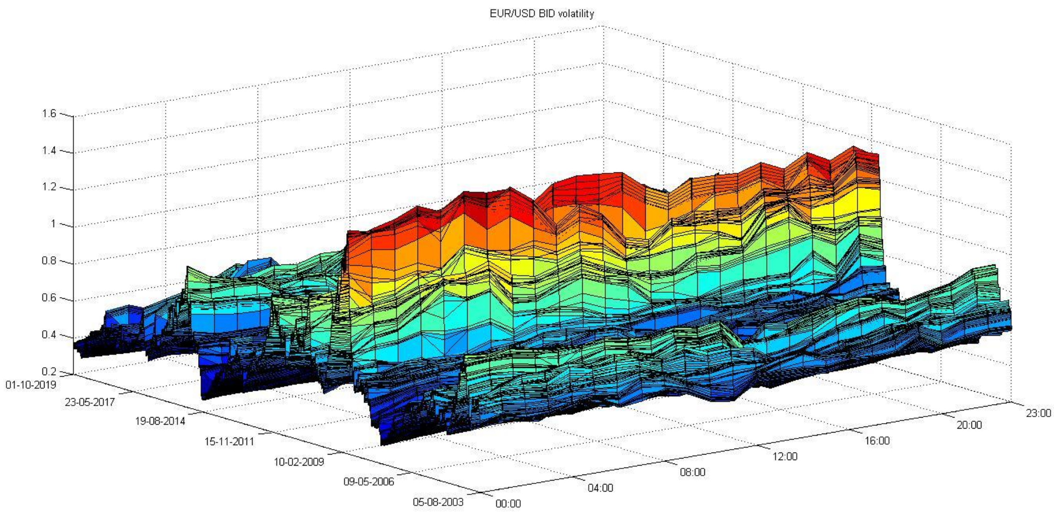


**Figure 4. Daily volatility surface for EUR/USD BID**

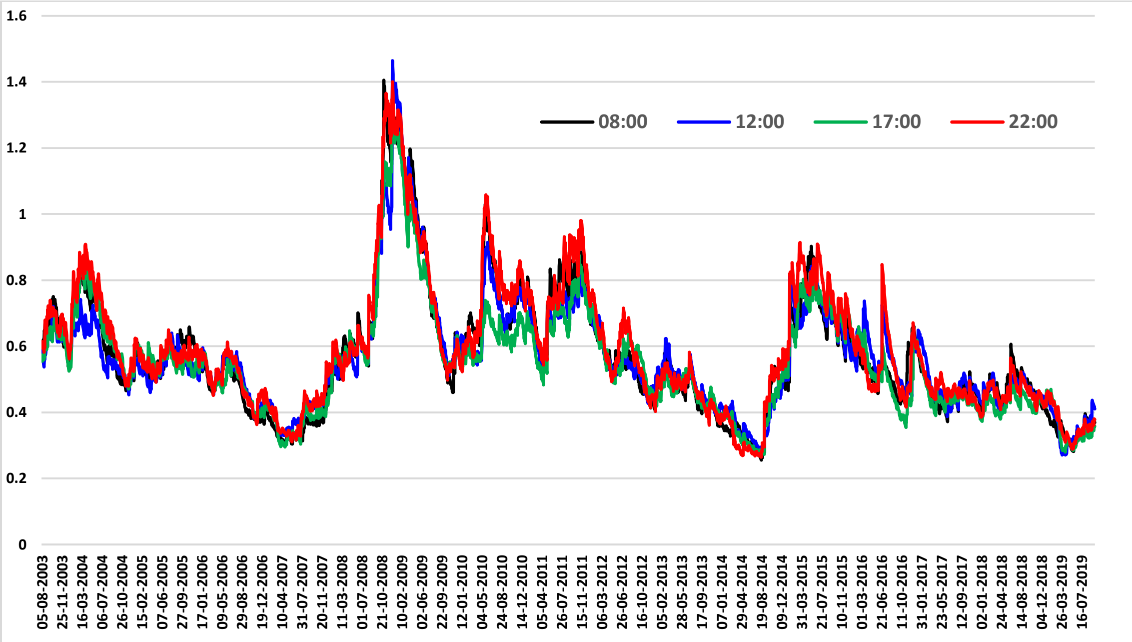


**Figure 5. EUR/USD ASK. Patterns of daily volatility calculated at different hours of the day**

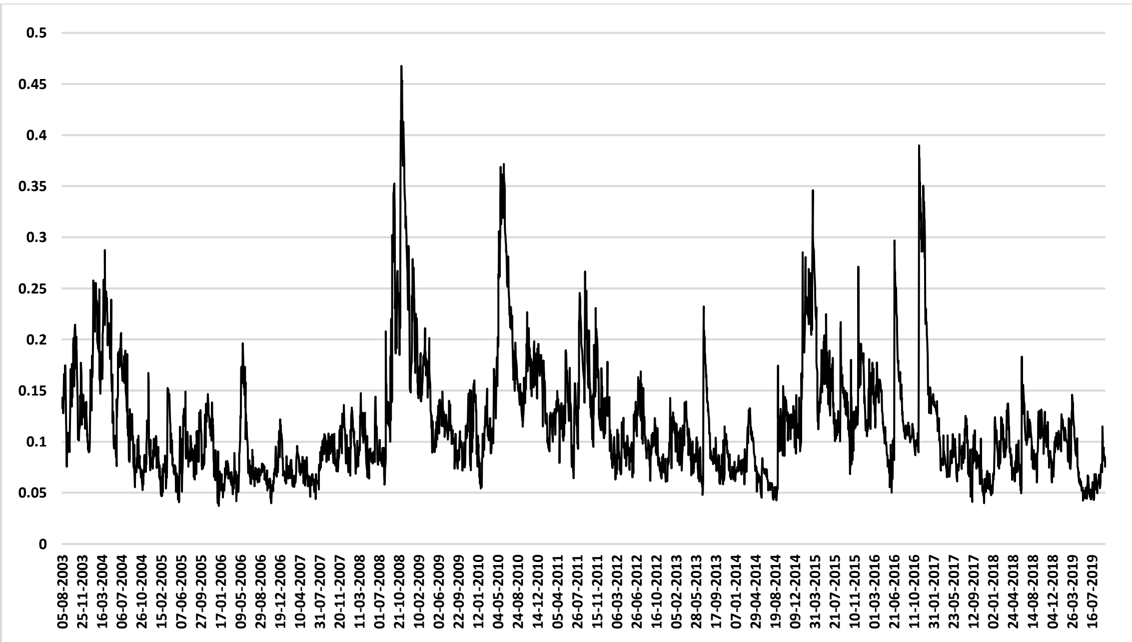

**Figure 6. Daily ranges of daily volatilities for EUR/USD ASK**

## 4.2. GBP/USD

The volatility estimates for GBP/USD exhibit two periods of very high volatility: during the subprime crisis and around the Brexit referendum. Figures 7 and 8 indicate that in the case of GBP/USD the volatility dynamics for bid prices clearly differs from that for ask prices. This is particularly evident at the time of the Brexit referendum, where the volatility of ask prices is much more spiky than that of bid prices. The plots in Figure 9 show the differences in the volatility estimates  caused by the choice of the hour at which the data are quoted.

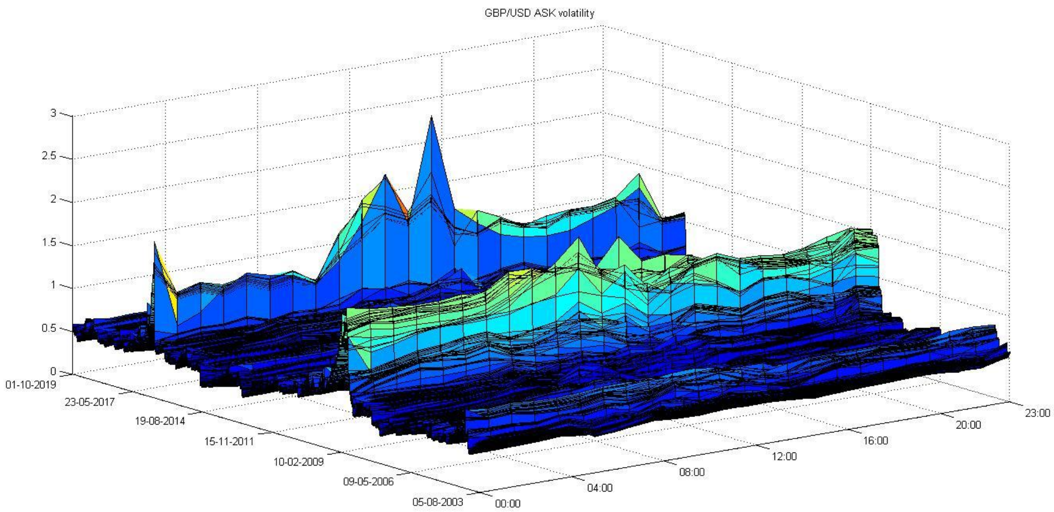


**Figure 7. Daily volatility surface for GBP/USD ASK**

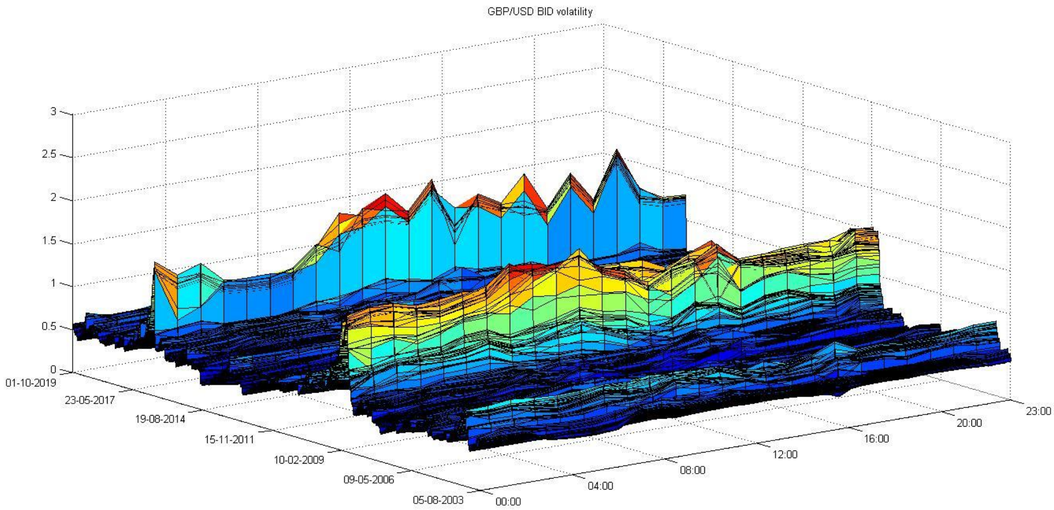


**Figure 8. Daily volatility surface for GBP/USD BID**

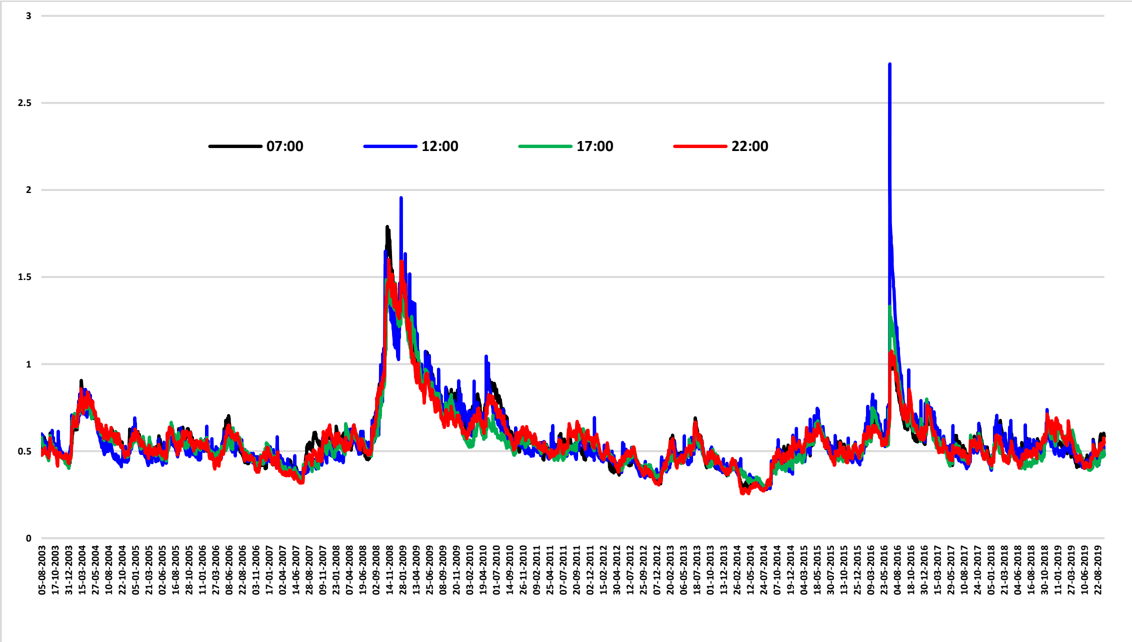


**Figure 9. GBP/USD ASK. Patterns of daily volatility calculated at different hours of the day.**

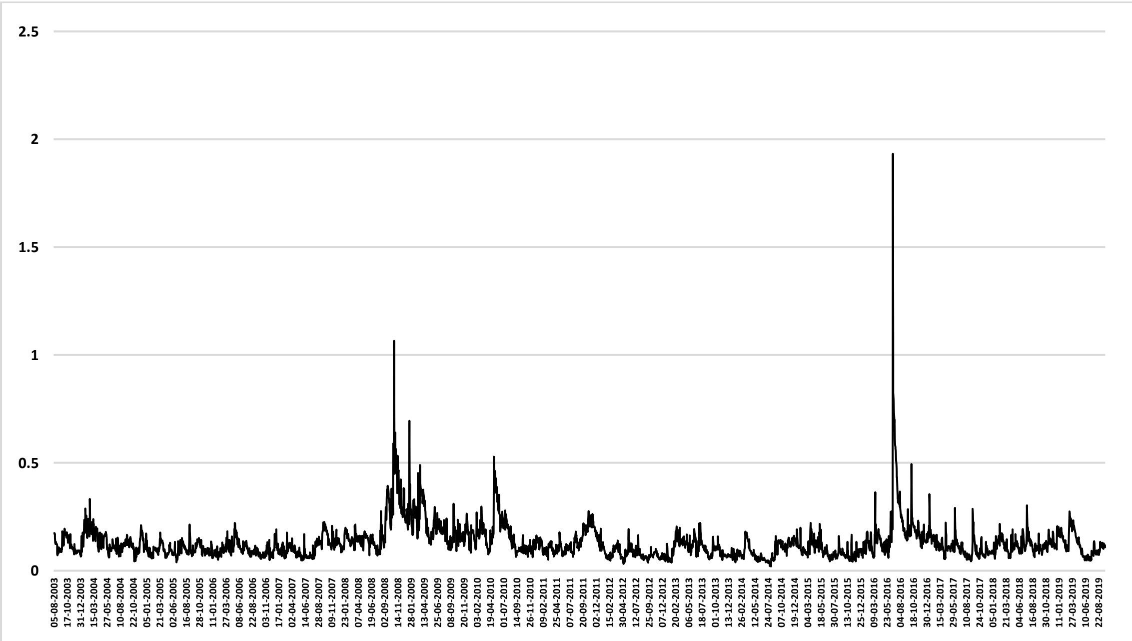

**Figure 10. Daily ranges of daily volatilities for GBP/USD ASK**

In Figure 10, we show the plot of daily ranges of daily volatilities estimated for daily returns calculated at each hour of the day (ask prices). The minimum is about 0.02 and the maximum about 1.93. In the case of bid prices, they are 0.02 and 1.05, respectively.

## 4.3. AUD/USD

The volatility of AUD/USD is low, apart of the period of the subprime crisis (Figures 11 and 12). It is clear that the volatility estimates depend on the hour at which the daily returns are calculated. The plots in Figure 13 confirm this first impression. In Figure 14, we show the plot of daily ranges of daily volatilities estimated using daily returns calculated at each hour of the day. The minimum is about 0.03 and the maximum about 2.18.

At first glance the surfaces presented in Figures 11 and 12 seem to imply that the dynamics of volatility for bid prices does not differ very much from that for ask ones. More careful inspection, however, shows clear differences for daily returns calculated at 3:00, 4:00, and 5:00, where the volatility estimates for ask prices are higher and more variable, and 21:00-returns, where the opposite situation is observed.

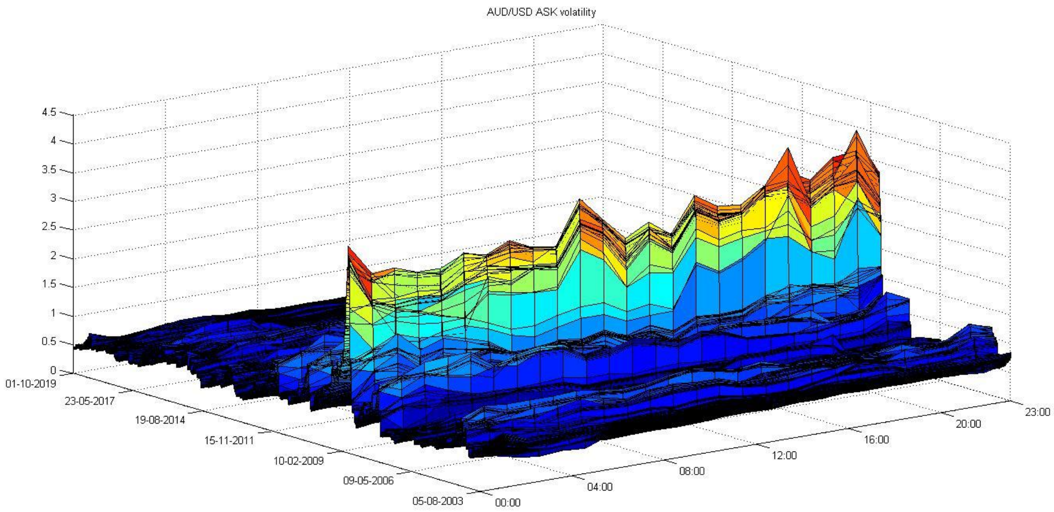


**Figure 11. Daily volatility surface for AUD/USD ASK**

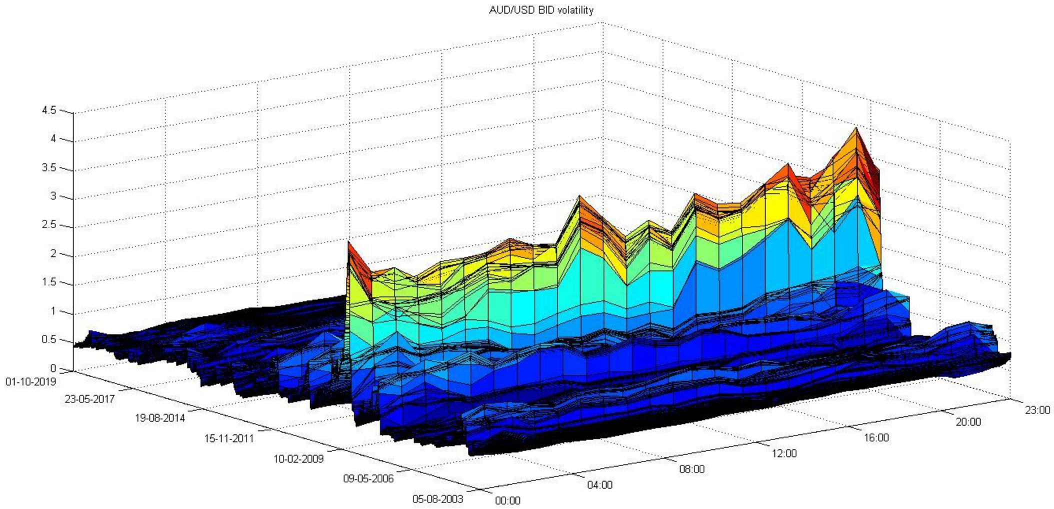


**Figure 12. Daily volatility surface for AUD/USD BID**

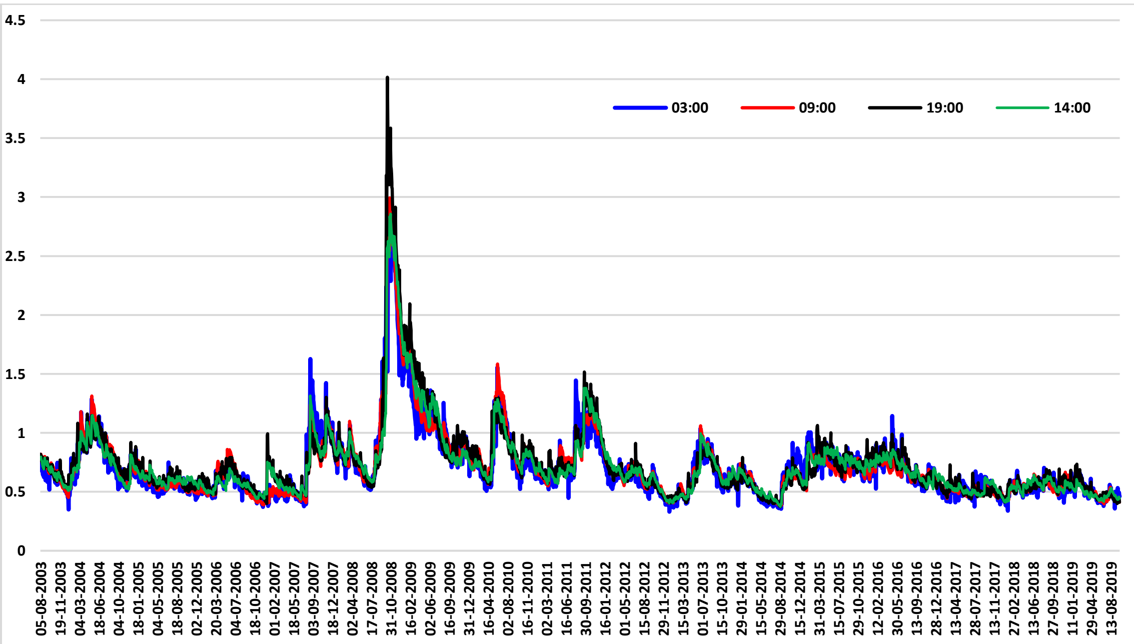


**Figure 13. AUD/USD ASK. Patterns of daily volatility calculated at different hours of the day.**

In Figure 13, the plots of the daily volatility estimates for different hours of the day (for ask prices) are presented. The dynamics looks similar in each case, but the plot of daily ranges (Figure 14) shows that, in fact, the differences are clear. The ranges vary from 0.03 to 2.19. In the case of  bid prices, the corresponding values are 0.04 and 1.79.

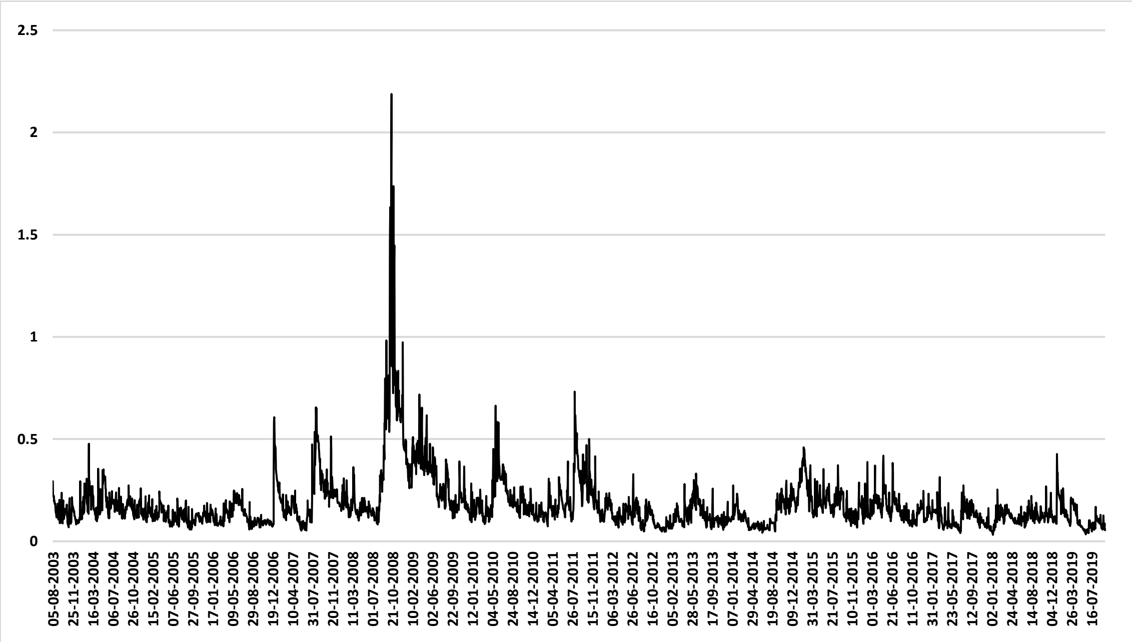


**Figure 14. Daily ranges of daily volatilities for AUD/USD ASK**

### 4.4. NZD/USD

The pattern observed for NZD/USD (Figure 15 and 16) is similar to that for AUD/USD: explosion of volatility during the subprime crisis and visible differences in the dynamics for the data collected at different hours. Plots in Figure 17 confirm this impression. In Figure 18, we show the plot of daily ranges of daily volatilities estimates obtained based on daily returns calculated at each hour of the day (ask prices). The minimum is about 0.04 and the maximum about 1.09. For bid prices, the corresponding quantities are almost the same.

The results for bid and ask prices differ from each other, especially clearly for 17:00-returns, but visible differences are also observed in the case of daily returns calculated at 1:00, 5:00, and 18:00.

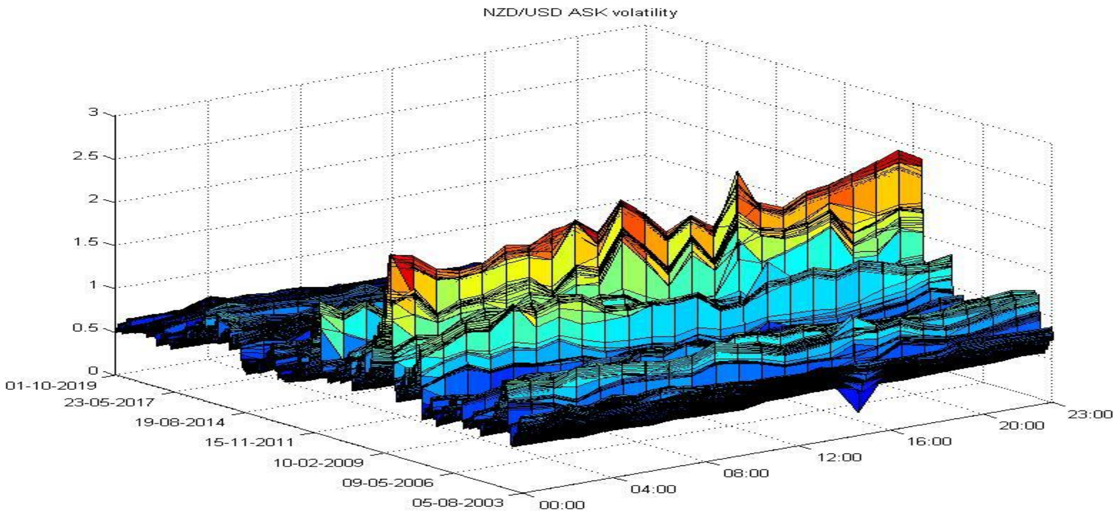


**Figure 15. Daily volatility surface for NZD/USD ASK**

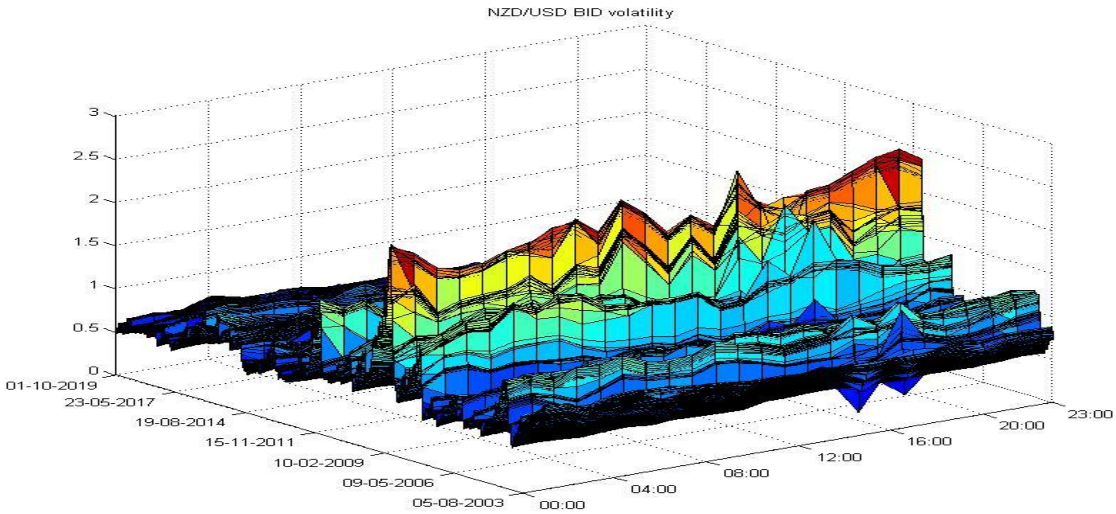


**Figure 16. Daily volatility surface for NZD/USD BID**

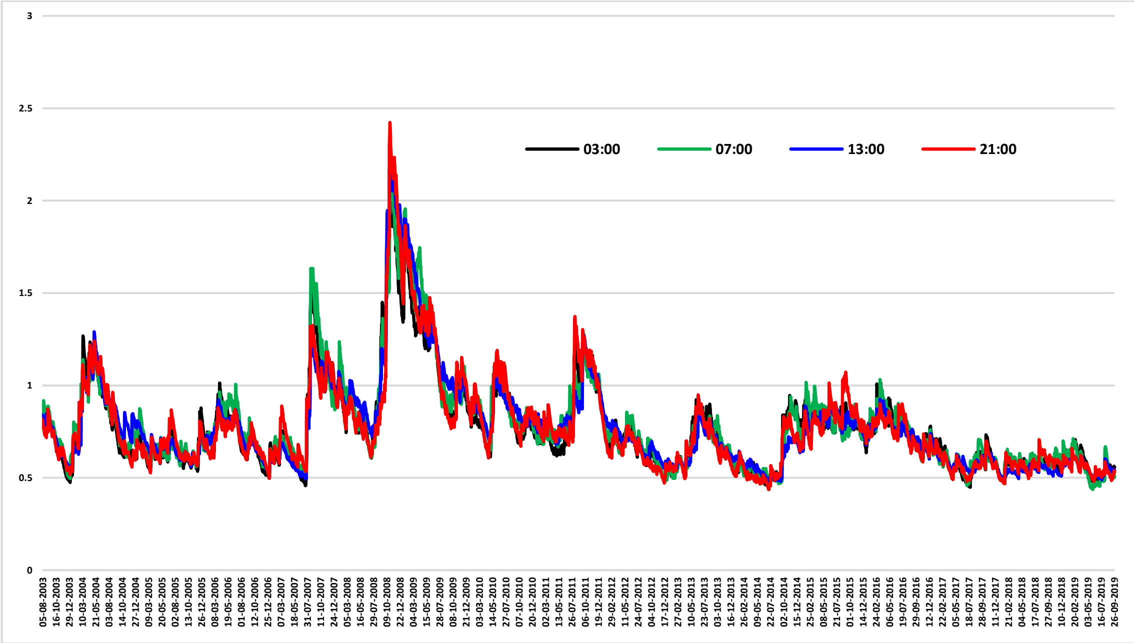


**Figure 17. NZD/USD ASK. Patterns of daily volatility calculated at different hours of the day.**

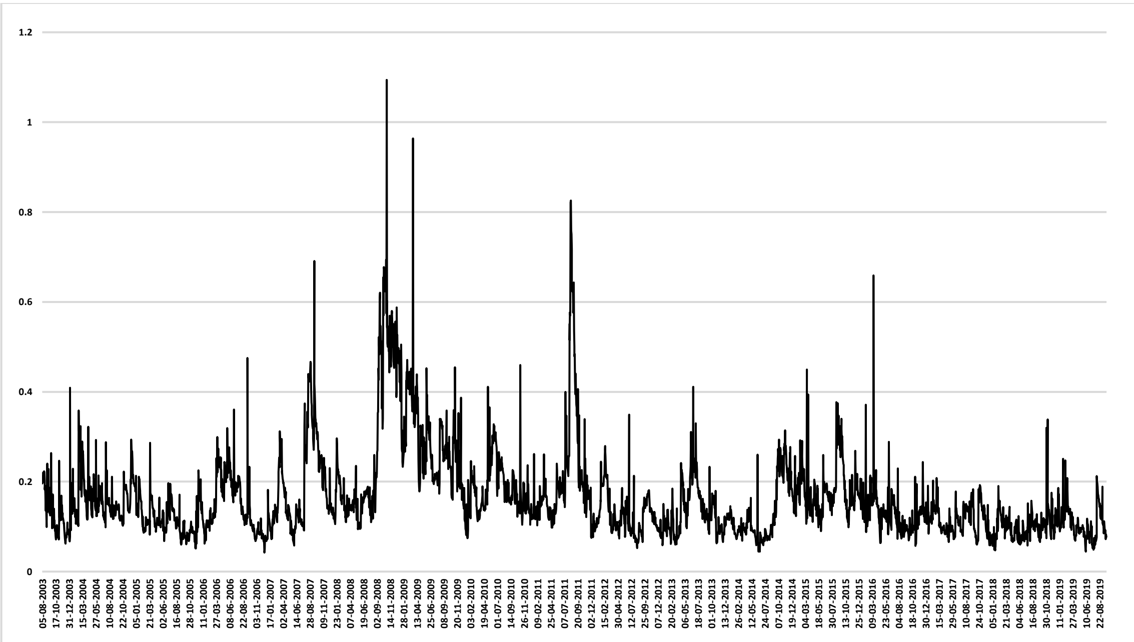

**Figure 18. Daily ranges of daily volatilities for NZD/USD ASK**

The results presented in this section show that the volatility estimates obtained by means of GARCH models depend on the hour at which the daily quotations are collected. The question arises to what extent this impacts estimates of conditional risk measures. Very roughly speaking, the difference of 0.2 on the daily volatility estimate gives for a position worth 100 000 USD a change in 10-day VaR (at tolerance level 0.01) equal to 4650 USD.

## 5. Conclusions

The aim of the paper was to find out if estimates of the daily volatility of major exchange rates depend on the hour at which the daily returns are calculated. The exchange rates under scrutiny were: EUR/USD, AUD/USD, GBP/USD, and NZD/USD. The analysis was based on daily returns calculated using the exchange rates quoted at each hour of the day, and performed separately for bid and ask prices. Volatility was estimated by means of ARMA-GARCH models.

Our findings are as follows. The differences between the volatility estimates corresponding to different hours are visible. For daily returns calculated at some hours of the day (mostly corresponding to the opening or closing of national markets) they are quite clear. Usually, the differences are large enough to impact VaR estimates. This means that the portfolio managers from different parts of the world can perceive exactly the same position as more or less risky since they usually use the daily data quoted at the hours when their local market is active. They are also able to affect VaR estimates, choosing the hour at which they collect the data. Moreover, significant differences in the behavior of the volatilities for bid and ask prices that we discovered for daily returns calculated at some hours, may reflect temporal imbalances between demand and supply side of the markets.

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