Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation

Peter Carr

Bloomberg LP and Courant Institute, New York University

Liuren Wu

Zicklin School of Business, Baruch College

This version: September 26, 2006; First draft: April 15, 2004

We thank Gurdip Bakshi, Philip Brittan, Dajiang Guo, Pat Hagan, Harry Lipman, Sheikh Pancham, Louis Scott, and participants at Bloomberg, MIT, the 2005 Credit Risk Conference at Wharton, the 13th annual conference on Pacific Basin Finance, Economics, and Accounting at Rutgers University, the 2006 North American Winter Meeting of the Econometric Society at Boston, and the Credit Derivative Symposium at Fordham University, for comments. We welcome comments, including references that we have inadvertently missed. Peter Carr is at Bloomberg LP and Courant Institute, New York University, 731 Lexington Avenue, New York, NY 10022; tel: (212) 617-5056; fax: (917) 369-5629; email: pcarr4@bloomberg.com; homepage: http://www.math.nyu.edu/research/carrp/. Liuren Wu is at Zicklin School of Business, Baruch College, One Bernard Baruch Way, Box B10-225, New York, NY 10010; tel: (646) 312-3509; fax: (646) 312-3451; email: Liuren_Wu@baruch.cuny.edu; homepage: http://faculty.baruch.cuny.edu/lwu.

Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation

ABSTRACT

We propose a dynamically consistent framework that allows joint valuation and estimation of stock options and credit default swaps written on the same reference company. We model default as controlled by a Poisson process with a stochastic default arrival rate. When default occurs, the stock price drops to zero. Prior to default, the stock price follows a continuous process with stochastic volatility. The instantaneous default rate and instantaneous diffusion variance rate follow a bivariate continuous Markov process, with its dynamics specified to capture the empirical evidence on stock option prices and credit default swap spreads. Under this joint specification, we derive tractable pricing solutions for stock options and credit default swaps. We estimate the joint dynamics using stock option prices and credit default swap spreads for four of the most actively traded reference companies. The estimation highlights the interaction between market risk (diffusion variance) and credit risk (default arrival) in pricing stock options and credit default swaps. While the credit risk factor dominates credit spreads at long maturities, the stock return volatility also enters credit spreads at short maturities due to positive co-movements between the diffusion variance rate and the default arrival rate. Furthermore, while the diffusion variance rate influences the implied volatility uniformly across moneyness, the impact of the credit risk factor becomes much larger on options at lower strikes. The impact of the credit risk factor on stock options also increases with option maturity. For options maturing in six months, the contribution of the credit risk factor to option pricing is comparable in magnitude to the contribution of the diffusion variance rate.

JEL Classification: C13; C51; G12; G13.

Keywords: Stock options; credit default swaps; default arrival rate; return variance dynamics; option pricing; time-changed Lévy processes.

Stock Options and Credit Default Swaps: A Joint Framework for Valuation and Estimation

Markets for both stock options and credit derivatives have experienced dramatic growth in the past few years. Along with the rapid growth, it has become increasingly clear to market participants that stock option implied volatilities and credit default swap (CDS) spreads are inherently linked. Many academic studies have also empirically documented the positive link between credit spreads and stock volatility at both the firm level and the aggregate level.¹ Interestingly, this empirical relationship has been presaged by classical asset pricing theory. According to the classical structural model of Merton (1974), corporate bond credit spreads are functions of financial leverage and firm asset volatility, which both contribute to volatility in the underlying company's stock and hence to stock option implied volatilities.

Furthermore, when a company defaults, the company's stock price inevitably drops by a sizeable amount. As a result, the possibility of default on a corporate bond generates negative skewness in the risk-neutral probability distribution of stock returns. This negative skewness is manifested in the relative pricing of stock options across different strikes. When the Black and Scholes (1973) implied volatility is plotted against some measure of moneyness at a fixed maturity, the slope of the plot is positively related to the risk-neutral skewness of the stock return distribution. Dennis and Mayhew (2002) and Bakshi, Kapadia, and Madan (2003) examine the negative skew of the implied volatility plot for individual stock options. Recent empirical work, e.g., Cremers, Driessen, Maenhout, and Weinbaum (2004), shows that CDS spreads are positively correlated with both stock option implied volatility levels and the steepness of the negative slope of the implied volatility plot against moneyness.

In this paper, we propose a dynamically consistent framework that allows joint valuation and estimation of stock options and credit default swaps written on the same reference company. We model company default as controlled by a Poisson process with a stochastic arrival rate. When default occurs, the stock price drops to zero. Prior to default, we model the stock price by a continuous process with stochastic volatility. The instantaneous default rate and instantaneous diffusion variance rate follow a bivariate continuous Markov

¹Examples include Bevan and Garzarelli (2000), Pedrosa and Roll (1998), Collin-Dufresne, Goldstein, and Martin (2001), Bangia, Diebold, Kronimus, Schagen, and Schuermann (2002), Capmbell and Taksler (2003), Altman, Brady, Resti, and Sironi (2004), Bakshi, Madan, and Zhang (2004), Ericsson, Jacobs, and Oviedo-Helfenberger (2004), Hilscher (2004), Consigli (2004), and Zhu, Zhang, and Zhou (2005).

process, with its joint dynamics specified to capture the empirical evidence on stock option prices and CDS spreads.

Under this joint specification, we derive tractable pricing solutions for stock options and credit default swaps. We estimate the joint dynamics of the default rate and the diffusion variance rate using stock option prices and CDS spreads for four actively traded companies. Our estimation shows that for all four companies, the default rate is more persistent than the diffusion variance rate under both statistical and risk-neutral measures. The statistical persistence difference suggests different degrees of predictability. The risk-neutral difference suggests that the default rate has a more long-lasting impact on the term structure of option volatilities and CDS spreads than does the diffusion variance.

The estimation also highlights the interaction between market risk (diffusion variance) and credit risk (default arrival) in pricing stock options and credit default swaps. We find that while credit risk dominates the CDS spreads at long maturities, diffusion variance can also affect CDS spreads at short maturities due to positive co-movements between diffusion variance and default arrival. On the other hand, the default arrival rate affects stock option pricing through both its correlation with the diffusion variance rate and its direct impact on the risk-neutral drift of the return process. The impact of the diffusion variance rate on the implied volatility is relatively uniform across different moneyness levels, but the impact of the default arrival rate is mainly on options at low strikes. Furthermore, the impact of the credit risk factor on stock option prices increases with the option maturity. For options maturing in six months, the contribution of the credit risk factor to option pricing is comparable in magnitude to the contribution of the diffusion variance rate.

The positive empirical relation between CDS spreads and stock option implied volatilities has been recognized only very recently in the academic community. As a result, efforts to theoretically capture this link are only in an embryonic stage. In a recent working paper, Hull, Nelken, and White (2004) link CDS spreads and stock option prices by proposing a new implementation and estimation method for the classical structural model of Merton (1974). As is well known, this early model is highly stylized as it assumes that the only source of uncertainty is the firm's asset value. As a result, stock option prices and CDS spreads have changes that are perfectly correlated locally. Thus, the empirical observation that implied volatilities and swap spreads sometimes move in opposite directions can only be accommodated by adding additional sources of uncertainty to the model. In this paper, we assume that prior to default, the stock price process

is continuous. The drift and diffusion coefficients of this process are both stochastic as we assume that the default arrival rate and diffusion variance rate obey a bivariate stochastic process. As a result, we are able to capture the imperfect positive correlation between stock volatility and default risk. Thus, when compared to efforts based on the structural model of Merton (1974), our contribution amounts to adding consistent, inter-related, but separate dynamics to the relation between volatility and default. The CDS market and the stock options market contain overlapping information on the market and credit risk of the company. Our joint valuation and estimation framework exploits this overlapping informational structure to provide better identification of the dynamics of the stock return variance and default arrival rate. The estimation results highlight the inter-related and yet distinct impacts of the two risk factors on the two markets.

The rest of the paper is organized as follows. The next section proposes a joint valuation framework for stock options and credit default swaps. Section 2 describes the data set and summarizes the stylized evidence that motivates our specification. Section 3 describes the joint estimation procedure. Section 4 presents the results and discusses the implications. Section 5 concludes.

1. Joint Valuation of Stock Options and Credit Default Swaps

Consider a reference company which has positive probability of defaulting. Let P_t denote the time-*t* stock price for this company, which we assume falls to zero upon default. Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$ be a complete stochastic basis and let \mathbb{Q} be a risk-neutral probability measure. *Prior to any default*, the risk-neutral stock price dynamics are given by:

$$dP_t/P_t = (r_t - q_t + \lambda_t) dt + \sqrt{v_t} dW_t^P,$$
(1)

where r_t and q_t denote the instantaneous interest rate and dividend yield respectively, which we assume evolve deterministically over time. In (1), $\lambda(t)$ denote the risk-neutral arrival rate of the default event and v_t denotes the instantaneous variance rate for the stock diffusion return component. Both processes evolve stochastically over time. The stock price *P* also evolves stochastically and is driven by standard Brownian motion W_t^P . The incorporation of λ_t in the drift compensates for the possibility of a default, so that the stock price remains a martingale unconditionally under the risk-neutral measure. Thus, the drift and diffusion coefficients of this pre-default stock price process are both stochastic.

1.1. Joint dynamics of diffusion variance rate and default arrival rate

We model the joint dynamics of the default arrival rate and the diffusion return variance rate under the risk-neutral probability measure \mathbb{Q} as follows:

$$dv_t = (\theta_v - \kappa_v v_t) dt + \sigma_v \sqrt{v_t} dW_t^v, \qquad (2)$$

$$\lambda_t = \beta v_t + z_t, \tag{3}$$

$$dz_t = (\theta_z - \kappa_z z_t) dt + \sigma_z \sqrt{z_t} dW_t^z, \quad \mathbb{E}\left[dW^z dW^P\right] = \mathbb{E}\left[dW^z dW^v\right] = 0 \tag{4}$$

$$\rho = \mathbb{E}\left[dW^P dW^\nu\right]/dt. \tag{5}$$

The above specification is motivated by the following empirical evidence and economic justification:

- It is well-documented that stock return volatility is stochastic. We use a square-root process in equation (2) to model the dynamics of the instantaneous variance of the diffusion return component.
- Cremers, Driessen, Maenhout, and Weinbaum (2004) find that implied volatilities covary with CDS spreads. Our own empirical analysis finds similar evidence. Equation (3) captures the positive comovement via a positive loading coefficient β between the default arrival rate λ_t and the diffusion return variance rate v_t.
- Although it is important to recognize the co-movement between the stock market and the credit market, it is also important to accommodate the fact that the credit market can show movements independent of the stock and stock options market. We use z_t to capture this independent credit risk component, with its dynamics controlled by an independent square-root process specified in (4).
- When the stock price falls, its return volatility often increases. A traditional explanation that dates back to Black (1976) is the leverage effect. So long as the face value of debt is not adjusted, a falling stock price increases the company's leverage and hence its risk, which shows up in stock return volatility.² Equation (5) captures this phenomenon via a negative correlation coefficient ρ between diffusion shocks in return and diffusion shocks in return variance.

²Various other explanations have also been proposed in the literature, e.g., Haugen, Talmor, and Torous (1991), Campbell and Hentschel (1992), Campbell and Kyle (1993), and Bekaert and Wu (2000).

1.2. Pricing stock options

Consider the time-*t* value of a European call option $c(P_t, K, T)$ with strike price *K* and expiry date *T*. The terminal payoff of the option is $(P_T - K)^+$ if the company has not defaulted by that time, and is zero otherwise. The value of the call option can be written as,

$$c(P_t, K, T) = \mathbb{E}_t \left[\exp\left(-\int_t^T (r_s + \lambda_s) ds \right) (P_T - K)^+ \right]$$
(6)

where the expectation operator $\mathbb{E}_t[\cdot]$ is under the risk-neutral measure \mathbb{Q} and conditional on the filtration \mathcal{F}_t . Given the deterministic interest rate assumption, we have,

$$c(P_t, K, T) = B(t, T) \mathbb{E}_t \left[\exp\left(-\int_t^T \lambda_s ds\right) (P_T - K)^+ \right],$$
(7)

with B(t,T) denoting the time-*t* value of a default-free zero-coupon bond paying one dollar at its maturity date *T*. The expectation can be solved by inverting the following discounted generalized Fourier transform,

$$\phi(u) \equiv \mathbb{E}_t \left[\exp\left(-\int_t^T \lambda_s ds \right) e^{iu \ln P_T / P_t} \right], \quad u \in \mathcal{D} \subset \mathbb{C},$$
(8)

where \mathcal{D} denotes the subset of the complex plane under which the expectation is well-defined. Under the dynamics specified in (1) to (5), the Fourier transform is exponential affine in the bivariate risk factor $x_t \equiv [v_t, z_t]^\top$:

$$\phi(u) = \exp\left(iu(r(t,T) - q(t,T))\tau - a(\tau) - b(\tau)^{\top}x_t\right), \quad \tau = T - t,$$
(9)

where r(t,T) and q(t,T) denote the continuously compounded spot interest rate and dividend yield at time t and maturity date T, respectively, and the time-homogeneous coefficients $[a(\tau), b(\tau)]$ are given by,

$$a(\tau) = \frac{\theta_{\nu}}{\sigma_{\nu}^{2}} \left[2\ln\left(1 - \frac{\eta_{\nu} - \kappa_{\nu}^{\mathbb{M}}}{2\eta_{\nu}}\left(1 - e^{-\eta_{\nu}\tau}\right)\right) + \left(\eta_{\nu} - \kappa_{\nu}^{\mathbb{M}}\right)\tau \right] \\ + \frac{\theta_{z}}{\sigma_{z}^{2}} \left[2\ln\left(1 - \frac{\eta_{z} - \kappa_{z}}{2\eta_{z}}\left(1 - e^{-\eta_{z}\tau}\right)\right) + (\eta_{z} - \kappa_{z})\tau \right],$$
(10)

$$b(\tau) = \left[\frac{2b_{\nu}(1-e^{-\eta_{\nu}\tau})}{2\eta_{\nu}-(\eta-\kappa_{\nu}^{\mathbb{M}})(1-e^{-\eta_{\nu}\tau})}, \frac{2b_{z}(1-e^{-\eta_{z}\tau})}{2\eta_{z}-(\eta_{z}-\kappa_{z})(1-e^{-\eta_{z}\tau})}\right]^{\top},$$
(11)

with $\kappa_{\nu}^{\mathbb{M}} = \kappa_{\nu} - iu\sigma_{\nu}\rho$, $\eta_{\nu} = \sqrt{(\kappa_{\nu}^{\mathbb{M}})^2 + 2\sigma_{\nu}^2 b_{\nu}}$, $\eta_z = \sqrt{(\kappa_z)^2 + 2\sigma_z^2 b_z}$, $b_{\nu} = (1 - iu)\beta + \frac{1}{2}(iu + u^2)$, and $b_z = 1 - iu$. Appendix A provides details of the derivation. Given $\phi(u)$, option prices can be obtained via fast Fourier inversion (Carr and Wu (2004a)).

1.3. Pricing credit default swap spreads

For a credit default swap initiated at time t and with maturity date T, we let S(t,T) denote the premium (the "CDS spread") paid by the buyer of default protection. Assuming continuous payments for simplicity, the present value of the premium leg of the contract is,

$$\mathsf{Premium}(t,T) = \mathbb{E}_t \left[S(t,T) \int_t^T \exp\left(-\int_t^s (r_u + \lambda_u) du\right) ds \right]. \tag{12}$$

Assuming that the fractional loss given default is constant at w, the present value of the protection leg is,

Protection
$$(t,T) = \mathbb{E}_t \left[w \int_t^T \lambda_s \exp\left(-\int_t^s (r_u + \lambda_u) du\right) ds \right].$$
 (13)

Hence, by equating the present values of the two legs, we can solve for the CDS spread as,

$$S(t,T) = \frac{\mathbb{E}_t \left[w \int_t^T \lambda_s \exp\left(-\int_t^s (r_u + \lambda_u) du\right) ds \right]}{\mathbb{E}_t \left[\int_t^T \exp\left(-\int_t^s (r_u + \lambda_u) du\right) ds \right]},$$
(14)

which can be regarded as a weighted average of the expected default loss.

Under the dynamics specified in (2) to (5), we can solve for the present values of the two legs of the CDS. The value of the premium leg is,

$$\begin{aligned} \mathsf{Premium}(t,T) &= S(t,T) \int_{t}^{T} \mathbb{E}_{t} \left[\exp\left(-\int_{t}^{s} (r_{u} + \lambda_{u}) du\right) ds \right] \\ &= S(t,T) \int_{t}^{T} B(t,s) \mathbb{E}_{t} \left[\exp\left(-\int_{t}^{s} b_{\lambda 0}^{\top} x_{u} du\right) \right] ds, \end{aligned} \tag{15}$$

with $b_{\lambda 0} = [\beta, 1]^{\top}$. The affine dynamics for the bivariate risk factors *x* and the linear loading function $b_{\lambda 0}$ dictate that the present value of the premium leg is an exponential affine function of the state vector (Duffie, Pan, and Singleton (2000)):

$$\mathsf{Premium}(t,T) = S(t,T) \int_{t}^{T} B(t,s) \exp\left(-a_{\lambda}(s-t) - b_{\lambda}(s-t)^{\top} x_{t}\right) ds, \tag{16}$$

where the affine coefficients can be solved analytically:

$$a_{\lambda}(\tau) = \frac{\theta_{\nu}}{\sigma_{\nu}^{2}} \left[2\ln\left(1 - \frac{\eta_{\nu} - \kappa_{\nu}}{2\eta_{\nu}}\left(1 - e^{-\eta_{\nu}\tau}\right)\right) + (\eta_{\nu} - \kappa_{\nu})\tau \right] \\ + \frac{\theta_{z}}{\sigma_{z}^{2}} \left[2\ln\left(1 - \frac{\eta_{z} - \kappa_{z}}{2\eta_{z}}\left(1 - e^{-\eta_{z}\tau}\right)\right) + (\eta_{z} - \kappa_{z})\tau \right],$$
(17)

$$b_{\lambda}(\tau) = \left[\frac{2\beta(1-e^{-\eta_{\nu}\tau})}{2\eta_{\nu}-(\eta-\kappa_{\nu})(1-e^{-\eta_{\nu}\tau})}, \frac{2(1-e^{-\eta_{z}\tau})}{2\eta_{z}-(\eta_{z}-\kappa_{z})(1-e^{-\eta_{z}\tau})}\right]^{\top},$$
(18)

with $\eta_{\nu} = \sqrt{(\kappa_{\nu})^2 + 2\sigma_{\nu}^2\beta}$ and $\eta_z = \sqrt{(\kappa_z)^2 + 2\sigma_z^2}$.

The present value of the protection leg is,

Protection
$$(t,T) = \mathbb{E}_t \left[w \int_t^T B(t,s) \lambda_s \exp\left(-\int_t^s \lambda_u du\right) ds \right]$$

$$= w \int_t^T B(t,s) \mathbb{E}_t \left[\left(b_{\lambda 0}^\top x_s \right) \exp\left(-\int_t^s b_{\lambda 0}^\top x_u du\right) \right] ds,$$
(19)

which also allows for an affine solution:

$$\mathsf{Protection}(t,T) = w \int_{t}^{T} B(t,s) \left(c_{\lambda}(s-t) + d_{\lambda}(s-t)^{\top} x_{t} \right) \exp\left(-a_{\lambda}(s-t) - b_{\lambda}(s-t)^{\top} x_{t} \right) ds, \quad (20)$$

where the coefficients $(a_{\lambda}(\tau), b_{\lambda}(\tau))$ are the same as in (16), and the coefficients $(c_{\lambda}(\tau), d_{\lambda}(\tau))$ can also be solved analytically by taking partial derivatives against $(a_{\lambda}(\tau), b_{\lambda}(\tau))$ with respect to maturity τ :

$$c_{\lambda}(\tau) = \partial a_{\lambda}(\tau) / \partial \tau, \quad d_{\lambda}(\tau) = \partial b_{\lambda}(\tau) / \partial \tau.$$
 (21)

Combining the solutions for the present values of the two legs in equations (15) and (20) leads to the CDS spread S(t,T). When we estimate the model, we discretize the above equation so as to accommodate quarterly premium payments.

1.4. Market prices of risks and time-series dynamics

Our joint estimation identifies both the time-series dynamics and the risk-neutral dynamics of the bivariate state vector $x_t = [v_t, z_t]^{\top}$. To derive the time-series dynamics for the bivariate vector x_t under the statistical measure \mathbb{P} , we assume that the market prices of risks are proportional to the corresponding risk level. Under this assumption, the time-series dynamics are,

$$dv_t = \left(\theta_v - \kappa_v^{\mathbb{P}} v_t\right) dt + \sigma_v \sqrt{v_t} dW_t^{v\mathbb{P}}, \quad dz_t = \left(\theta_z - \kappa_z^{\mathbb{P}} z_t\right) dt + \sigma_z \sqrt{z_t} dW_t^{z\mathbb{P}}, \tag{22}$$

with $\kappa_{\nu}^{\mathbb{P}} = \kappa_{\nu} - \sigma_{\nu} \gamma_{\nu}$ and $\kappa_{z}^{\mathbb{P}} = \kappa_{z} - \sigma_{z} \gamma_{\nu}$.

2. Data and Evidence

Both the stock option prices and the CDS spreads are functions of the two risk factors $x_t = [v_t, z_t]^{\top}$, which jointly determine the stock diffusion variance and the default arrival rate. Therefore, we can use data on stock option prices and CDS spreads to infer the joint dynamics.

2.1. Data description

We estimate the model using CDS spreads and stock option prices on four reference companies. Bloomberg provides CDS spread quotes from several broker dealers. We use quotes from different broker dealers in order to cross-validate them. Then, we take the quotes on each series from the most reliable sources. We choose four companies for which CDS quotes have both a long history and frequent updates. The four companies are: Ford (F), General Motors (GM), Altria Group Inc (MO), and Duke Energy Corp (DUK). For each company, we have CDS spread series at five fixed maturities of one, three, five, seven, and ten years.

The corresponding stock options data is from OptionMetrics. Exchange-traded options on individual stocks are American-style and hence the price reflects an early exercise premium. OptionMetrics uses a binomial tree to back out the option implied volatility that explicitly accounts for this early exercise premium.

For each stock, OptionMetrics provides a standardized implied volatility surface at fixed Black-Scholes forward deltas from 20 to 80 with a five-delta interval for both call and put options, and fixed option maturities of 30, 60, and 91 days. OptionMetrics estimates the implied volatility surface via a kernel smoothing approach whenever the underlying quotes are available and leave as missing values when there are not enough quotes to make the smoothing estimation. Data at longer maturities are also available but only very sparsely. Hence we only use the first three maturities. The implied volatility estimates from OptionMetrics are often different from calls and puts at similar strikes, which is to be expected when options are American and the Black Scholes model is not holding. Our interest is in testing the validity of our model for European options and so we adopt a standard practice for estimating market prices of European options. The practice is to take the average of the two implied volatilities at each strike and convert them into out-of-the-money European option prices using the Black-Scholes formula.

To price the CDS contracts and to convert the implied volatility into option prices, we also need the underlying interest rate curve. Again following standard industry practice, we use the interest rate curve defined by the Eurodollar LIBOR and swap rates. We download LIBOR rates at maturities of one, two, three, six, nine, and 12 months and swap rates at two, three, four, five, seven, and ten years. We use a piece-wise constant forward function in bootstrapping the discount rate curve.

2.2. Summary statistics

Our model estimation uses the common samples of the three data sets from January 2, 2002 to April 30, 2004. The data are available on a daily basis, but we estimate the model using weekly-sampled data on every Wednesday to avoid the impacts of weekday effects. Table 1 reports the summary statistics of the CDS spreads on the four reference companies. The mean term structures of the spreads are relatively flat for all four companies, but the standard deviations of the spreads for all four companies decline with increasing maturities. The weekly autocorrelation estimates for the spreads range from 0.90 to 0.97, showing that the CDS spreads are highly persistent.

Table 2 reports the summary statistics of stock option implied volatilities at the three fixed maturities and 13 fixed put-option deltas for each of the four reference companies. For each company and at each option maturity, the implied volatilities at low strikes (low put deltas) are on average higher than the implied volatilities at high strikes, generating a negatively sloped average implied volatility smirk across moneyness. The standard deviations of the implied volatility series are also larger for out-of-the-money puts than for out-of-the-money calls, but the difference is smaller than the difference in the mean estimates. The weekly autocorrelation for the volatility series range from 0.69 to 0.93, indicating that the implied volatilities are persistent, but less so than the CDS spreads.

Figure 1 plots the average implied volatility smirk at the three fixed maturities as a function of the put option delta. For all four reference companies and for all three fixed maturities, the average implied volatility smirk is negatively skewed, corresponding to a negatively skewed risk-neutral stock return distribution. The three lines in each panel, which correspond to the three option maturities, stay closely to one another, suggesting that the conditional risk-neutral distribution of the stock return retains similar shapes at the three conditioning horizons. Generically, our model specification can generate the negative skewness from two sources: (1) a positive probability of default ($\lambda > 0$) and (2) a negative correlation between the return Brownian motion component and its instantaneous variance rate ($\rho < 0$).

[Figure 1 about here.]

2.3. Co-movements between option implied volatilities and credit default swap spreads

Figure 2 overlays the time series of the CDS spreads (solid lines) with the daily time series of at-the-money (50 delta) stock option implied volatilities at the three fixed option maturities (dashed lines) for the four chosen reference companies. We observe apparent common movements for the two types of time series for each company. The co-movements are the most obvious during periods of financial distress for the company, as the two sets of time series both spike up.

[Figure 2 about here.]

To quantify the co-movements, at each date we fit a second order polynomial on the three-month implied volatilities across moneyness (d_1) ,

$$IV_t(d) = a_t + b_t d_1 + c_t d_1^2,$$

where the moneyness d_1 is defined as $d_1 \equiv (\ln P/K + (r-q)\tau + IV^2\tau/2)/(IV\sqrt{\tau})$. We use the intercept estimate \hat{a} as a smoothed estimate for the at-the-money implied volatility $(ATMV_t)$ at $d_1 = 0$, and use the normalized slope estimate $SKEW_t = \hat{b}_t/\hat{a}_t$ as a proxy for the risk-neutral skewness of the return distribution. Then, we use the five-year CDS spread to proxy the credit spread (CDS_t) and run restricted and unrestricted versions of the following regression:

$$CDS_t = a + bATMV_t + cSKEW_t + e_t.$$
(23)

Table 3 reports the parameter estimates, *t*-statistics, and R^2 of the regressions for the four reference companies. When the at-the-money volatility level is the only explanatory variable, the estimates for its slope coefficient are positive and highly significant for all four companies. When the skewness measure is the only explanatory variable, its slope coefficients are negative and highly significant for all four companies. Thus, an increase in credit spreads is often associated with an increase in the option volatility level and a steepening in the negative slope of the implied volatility smirk. When we incorporate both the volatility level and the skewness measure as explanatory variables, the slope coefficient estimates on the skewness measure are no longer statistically significant, suggesting that the link with the credit market is driven by one source of risk. The R-squares of the joint regressions differ across different companies, as high as 83% for Ford, but as low as 36% for Altria Group. The variations across different companies and the low R^2 estimates in some instances suggest that although return variance and default arrival share common movements, they also have their own independent movements. From a modeling perspective, it is important to capture not only the common movements between the two markets, but also the idiosyncratic movements in each market. Our bivariate risk dynamics in equations (2) to (5) can accommodate different degrees of common and idiosyncratic movements.

Given the persistence of both CDS spreads and implied volatilities, we also study how the weekly changes of one series is correlated with the weekly changes of the other series. Figure 3 plots the cross-correlation estimates at different leads and lags between weekly changes in the five-year CDS spread and

the three-month at-the-money implied volatility. The dash-dotted lines in each panel denote the 95 percent confidence band. For all four reference companies, we identify significantly positive contemporaneous correlations between the weekly changes of the two series, with the estimates ranging from 0.52 to 0.61. The cross-correlation estimates at other leads and lags are largely insignificant.

[Figure 3 about here.]

3. Joint Estimation of Return Variance and Default Arrival Dynamics

We estimate the bivariate risk dynamics jointly using both CDS spreads and stock options. We cast the model into a state-space form and estimate the model using a quasi-maximum likelihood method.

In the state-space form, we regard the bivariate risk vector as the unobservable states and specify the state propagation equation using an Euler approximation of the time-series dynamics in equation (22):

$$x_{t} = \begin{bmatrix} \theta_{v} \\ \theta_{z} \end{bmatrix} \Delta t + \begin{bmatrix} e^{-\kappa_{v}^{\mathbb{P}}\Delta t} & 0 \\ 0 & e^{-\kappa_{z}^{\mathbb{P}}\Delta t} \end{bmatrix} x_{t-1} + \sqrt{\begin{bmatrix} \sigma_{v}^{2}v_{t-1}\Delta t & 0 \\ 0 & \sigma_{z}^{2}z_{t-1}\Delta t \end{bmatrix}} \varepsilon_{t},$$
(24)

where ε denotes an iid bivariate standard normal innovation and $\Delta t = 7/365$ denotes the sampling frequency.

We construct the measurement equations based on CDS spreads and stock options, assuming additive, normally-distributed measurement errors:

$$y_t = h(x_t; \Theta) + e_t, \tag{25}$$

where y_t denotes the observed series and $h(x_t; \Theta)$ denotes the corresponding model value as a function of the state vector x_t and model parameters Θ . Specifically, the measurement equation contains five CDS spread series and 39 option series,

$$h(x_t;\Theta) = \begin{bmatrix} S(x_t, t + \tau_s;\Theta) \\ O(x_t, t + \tau_O, \delta;\Theta) \end{bmatrix}, \quad \begin{aligned} \tau_s = 1, 3, 5, 7, 10 \text{ years} \\ \tau_O = 30, 60, 91 \text{ days}; \delta = 20, 25, \cdots, 80, \end{aligned}$$
(26)

where $S(x_t, t + \tau_s)$ denotes the model value of the CDS spreads at time *t* and maturity τ_s as a function of the state vector x_t and model parameters Θ , $O(x_t, t + \tau_O, \delta; \Theta)$ denotes the model value for out-of-the-money options at time *t*, time-to-maturity τ_O , and delta δ , as a function of the state vector x_t and model parameters Θ . To deal with the predictable variation in option premia across strikes and maturity, we followed the standard industry practice of dividing out-of-the-money option prices by their Black-Scholes vega. There are missing values on both the CDS data and the implied volatility surface. Our estimation algorithm readily handles missing observations. The term e_t in (25) denotes the measurement errors. We assume that the five CDS series generate iid normal pricing errors with the same error variance σ_s^2 . We also assume that the pricing errors on all the options (scaled by their vega) are also iid normal with error variance σ_Q^2 .

When both the state propagation equation and the measurement equations are Gaussian and linear, the Kalman (1960) filter generates efficient forecasts and updates on the conditional mean and covariance of the state vector and the measurement series. In our application, the state propagation equation in (24) is Gaussian and linear, but the measurement equation in (25) is nonlinear. We use the unscented Kalman filter (Wan and van der Merwe (2001)) to handle the nonlinearity. The unscented Kalman filter approximates the posterior state density using a set of deterministically chosen sample points (sigma points). These sample points completely capture the true mean and covariance of the Gaussian state variables, and when propagated through the nonlinear functions in the measurement equations, capture the posterior mean and covariance of the CDS spreads and option prices accurately to the second order for any nonlinearity. Let \overline{y}_{t+1} and \overline{V}_{t+1} denote the time-*t* ex ante forecasts of time-(*t* + 1) values of the measurement series and the covariance of the measurement series, respectively obtained from the unscented Kalman filter. We construct the log-likelihood value assuming normally distributed forecasting errors,

$$l_{t+1}(\Theta) = -\frac{1}{2} \log \left| \overline{V}_{t+1} \right| - \frac{1}{2} \left((y_{t+1} - \overline{y}_{t+1})^\top \left(\overline{V}_{t+1} \right)^{-1} (y_{t+1} - \overline{y}_{t+1}) \right).$$
(27)

The model parameters are chosen to maximize the log likelihood of the data series,

$$\Theta \equiv \arg\max_{\Theta} \mathcal{L}(\Theta, \{y_t\}_{t=1}^N), \quad \text{with} \quad \mathcal{L}(\Theta, \{y_t\}_{t=1}^N) = \sum_{t=0}^{N-1} l_{t+1}(\Theta), \tag{28}$$

where N denotes the number of weeks in our sample.

4. Joint Dynamics and Pricing of Return Variance and Default Arrival Risks

First, we summarize the performance of our joint valuation model on CDS spreads and stock options on the four reference companies. Then, from the structural parameter estimates, we discuss the joint dynamics and pricing of the diffusion variance risk and default arrival risk.

4.1. Performance analysis

Table 4 reports the sample mean in the first panel and the standard deviation in the second panel of the pricing errors on the stock options. We define pricing errors as the difference between the implied volatility quotes (in percentage points) and the corresponding model values. The mean pricing errors are fairly small and show no obvious patterns across moneyness and maturities. The standard deviation ranges from one to four percentages points. Comparing these estimates to the mean implied volatility estimates in Table 2 points to an average pricing error of less than ten percent. The last panel of the table reports the explained variation, defined as one minus the variance of the pricing errors over the variance of the original implied volatility series. The explained variations are over 90 percent for most series, showing that the model is relatively successful in capturing the behavior of stock options on all four companies.

Table 5 reports the sample mean and standard deviation of the pricing errors as well as the explained variation on the CDS spreads. The pricing errors are larger on the swap spreads. The explained variations are over 80 percent for Ford and Duke Energy, but the model's performance is relatively poor for General Motors and Altria Group. The model explains over 50 percent of variation in the CDS spreads on General Motors, and just about 30 percent on Altria Group.

Inspecting the time series plots in Figure 2, we observe that for General Motors, the five CDS spread series diverge dramatically after January 2003 to generate a very steep term structure from a virtually flat term structure before 2003. This dramatic term structure change either comes from economic forces or from the mere fact that there was more frequent quote updating in the second half of the data. Irrespective of the underlying reasons, our two-factor model seems to have difficulties fitting the whole term structure of CDS spreads and the options data. The model performs well on all the options series, and also reasonably well on short-term CDS spreads, but it performs poorly on the long-term CDS spreads. For Altria Group, the CDS

quotes are not updated as frequently before 2003, whereas the options data are actively quoted and traded. It is potentially due to this difference that the model parameters are geared to price the options market better than the CDS spreads.

4.2. The joint dynamics of return variance and default arrival rates

Table 6 reports the maximum likelihood estimates and t-statistics of the structural parameters that control the joint dynamics of the diffusion variance rate and the default arrival rate. The joint dynamics differ across different companies. Nevertheless, several common features emerge from the estimates.

First, the estimates for the risk-neutral mean-reverting coefficients (κ_{ν}, κ_{z}) and their statistical counterparts ($\kappa_{\nu}^{\mathbb{P}}, \kappa_{z}^{\mathbb{P}}$) show that the default arrival rate is more persistent than the diffusion variance rate under both the risk-neutral measure \mathbb{Q} and the statistical measure \mathbb{P} . The difference in statistical persistence suggests that the diffusion return variance rates are strongly mean-reverting and hence predictable, but it is difficult to predict changes in the independent credit risk factor based on its past values. The difference in risk-neutral persistence dictates that the two factors have different impacts across the term structure of options and CDS spreads. Shocks on the diffusion variance rate affect the short-term options and CDS spreads, but dissipate quickly as the option and CDS maturity increases. Shocks on the more persistent credit risk factor last longer across the term structure of options and credit spreads.

For each risk factor, the difference in persistence under the two probability measures defines the market price of that factor's risk:

$$\gamma_{\nu} = (\kappa_{\nu} - \kappa_{\nu}^{\mathbb{P}}) / \sigma_{\nu}, \quad \gamma_{z} = (\kappa_{z} - \kappa_{z}^{\mathbb{P}}) / \sigma_{z}.$$
⁽²⁹⁾

We compute the market prices (γ_v, γ_z) based on the parameter estimates and report them in the bottom panel of Table 6. The estimates for all four companies show positive market price for the diffusion variance risk, but negative market price for the independent credit risk.

Several studies, e.g., Bakshi and Kapadia (2003a,b) and Carr and Wu (2004b), use stock and stock index options and the underlying time series returns to study the total return variance risk premia. They find that the risk premia are negative for some stocks, and highly negative for stock indexes. Our model decomposes the total risk on an individual stock into two components: risk in the diffusion variance rate and risk in the

default arrival rate. By using both the CDS data and stock options data, we are able to separate the two sources of risks and identify their respective market prices. Our estimation suggests that for the four stocks, negative risk premia only come from the default arrival rate, but not from the diffusion variance rate.

Under our specification, market prices not only dictate the persistence difference of the risk factors under the two measures, but also create differences in the long-run means of the risk factors under the two measures. In particular, the statistical mean and the risk-neutral mean of the default arrival rate are given by,

$$\mathbb{E}^{\mathbb{P}}[\lambda] = \mathbb{E}^{\mathbb{P}}[\beta v + z] = \beta(\kappa_{v}^{\mathbb{P}})^{-1}\theta_{v} + (\kappa_{z}^{\mathbb{P}})^{-1}\theta_{z},$$
(30)

$$\mathbb{E}^{\mathbb{Q}}[\lambda] = \mathbb{E}^{\mathbb{Q}}[\beta \nu + z] = \beta(\kappa_{\nu})^{-1} \theta_{\nu} + (\kappa_{z})^{-1} \theta_{z}.$$
(31)

The bottom panel of Table 6 also reports the two mean estimates based on the parameter estimates. The mean default arrival rate is much lower under the statistical measure \mathbb{P} than under the risk-neutral \mathbb{Q} for all four companies. These estimates are consistent with the empirical findings in the corporate bond literature that the historical average default probabilities are much lower than those implied from the corporate bond credit spreads.³

If we define the credit spread at a maturity τ as the difference between the continuously compounded spot rate on a reference company and the corresponding spot rate in the benchmark Eurodollar market, this spread is affine in the two risk factors under our model specification,

$$CS(t,\tau) = \left[\frac{a_{\lambda}(\tau)}{\tau}\right] + \left[\frac{b_{\lambda}(\tau)}{\tau}\right]^{\top} x_t, \qquad (32)$$

where the solutions to $a_{\lambda}(\tau)$ and $b_{\lambda}(\tau)$ are given in equations (17) and (18). Hence, $b_{\lambda}(\tau)/\tau$ measures the contemporaneous response of the credit spread term structure to unit shocks on the two risk factors. Figure 4 plots this response as a function of the credit spread maturity. The solid lines denote the response to the independent credit risk factor z and the dashed lines denote the response to the diffusion variance factor v. As the time to maturity approaches zero, the loading coefficient $b_{\lambda}(\tau)/\tau$ converges to the instantaneous coefficient $b_{\lambda 0}$, which is normalized to unity for the credit risk factor z and is β for the diffusion variance rate v. The decay rate due to increases in time to maturity are controlled by the risk-neutral persistence of

³See, for example, Huang and Huang (2003), Eom, Helwege, and Huang (2004), Elton, Gruber, Agrawal, and Mann (2001), and Collin-Dufresne, Goldstein, and Helwege (2003).

the two risk factors. The higher persistence in z dictates that its impact declines more slowly as maturity increases than does the impact of the more transient factor v.

[Figure 4 about here.]

Another common finding among the four reference companies is that the default arrival rates all covary positively with the diffusion variance rate, as the estimates for the loading coefficient β are all positive. Furthermore, for all four companies, the estimates for the instantaneous correlation between stock return and return variance ρ are negative, consistent with the classic leverage effect.

Finally, the literature has often found it difficult to separately identify the recovery rate and the default arrival rate using credit spread data alone (Houweling and Vorst (2005), Hull and White (2000), and Longstaff, Mithal, and Neis (2005)). As a result, researchers often assume a fixed recovery rate, usually between 30 to 50 percent, instead of estimating it along with other model parameters. By exploiting the overlapping information from the stock options market and the CDS market, we are able to separately identify the recovery rate (1 - w) and the default arrival rate dynamics with high statistical significance. Our recovery rate estimates are between 47 and 81 percent, higher than the normally assumed values. Nevertheless, the estimates are in line with the high actual recovery rates during recent years reported in Altman (2006). They are also similar to the average recovery estimates by Das and Hanouna (2006) using corporate CDS spreads and sovereign recovery rate estimates by Pan and Singleton (2005) based on sovereign CDS term structures.

Figure 5 plots the extracted time series on the variance rate (solid line) and the default arrival rate (dashed line), with scales on the left and right hand sides of the *y*-axis, respectively. The extracted time series show co-movements that match the time series plots of the CDS spreads and implied volatilities in Figure 2. The plots for all four companies show a spike for both the variance rate and the default arrival rate in late 2002, a reflection of the financial stress during that period.

[Figure 5 about here.]

4.3. The term structure of credit default swap spreads

Given the model parameter estimates in Table 6, we can compute the term structures of the CDS spreads at different levels of the risk factors (v,z). In Figure 6, we plot the model-implied mean term structure of the CDS spreads in solid lines, where we set the risk levels to their respective sample averages. The two dashed lines in each panel are constructed by setting the diffusion variance rate v to its sample mean and the independent credit risk factor to one standard deviation away from its sample mean. The two dotted lines in each panel reflect the impact of one standard deviation movements of the diffusion variance rate while holding the independent credit risk factor to its sample mean.

[Figure 6 about here.]

The estimated model parameters on the four companies generate different mean term structures on the CDS spreads. Nevertheless, the impacts of the two risk factors show similar patterns. First, a one standard deviation move of the independent credit risk factor has a much larger impact on the CDS spreads than a one standard deviation move of the diffusion variance rate, supporting the hypothesis that the CDS market is mainly a market for credit risk. Furthermore, the impact of the diffusion variance rate is mainly at short maturities. Its impact declines rapidly as maturity increases. In contrast, the impact of the independent credit risk factor is much more persistent.

4.4. The implied volatility smirk and term structure

To understand how the two risk factors contribute to the pricing of stock options, we compute and plot the one-month implied volatility smirks across different moneyness in Figures 7 at different risk levels. In computing the option values and constructing the implied volatility smirks, we assume zero interest rates and dividend yields, and define the moneyness as $\ln(K/S)/\sqrt{v\tau}$, which can be approximately interpreted as the number of standard deviations that log spot is below log strike. The solid lines are the mean implied volatility smirks evaluated at the sample means of the two risk factors. The two dashed lines in each panel are generated with the diffusion variance rate at its sample mean and the independent credit risk factor one standard deviation away from its sample mean. Hence, they capture the impact of shocks in the independent credit risk factor. The two dotted lines in each panel are generated by setting the independent credit risk factor at its sample mean and the diffusion variance rate at one standard deviation away from its sample mean. Hence, the dotted lines capture the impact of shocks in the diffusion variance rate.

[Figure 7 about here.]

The implied volatility smirks show similar patterns across the four companies. Furthermore, variations in the diffusion variance rate level lead to relatively uniform shifts in the implied volatility smirk across moneyness. In contrast, the impact of the independent credit risk factor is mainly at low strikes. The impact of the credit risk factor on far out-of-the-money call option implied volatilities (at high strikes) is negligible.

To see how the impact changes at different maturities, we also plot in Figure 8 the corresponding implied volatility smirk for six-month options. As for the one-month implied volatility smirk, the impacts of the diffusion variance rate (dotted lines) are relatively uniform across all moneyness levels, whereas the impacts of the independent credit risk factor (dotted lines) are stronger at lower strikes. Comparing Figures 7 and 8 also brings out visible differences: The impact of the independent credit risk factor is larger at longer maturities.

[Figure 8 about here.]

Figure 9 plots the term structure of the at-the-money implied volatilities at different risk levels. Again, we use the solid line to denote the mean term structure, the dashed lines to capture the impact of one standard deviation moves on the independent credit risk factor, and the dotted lines to capture the impact of the diffusion variance rate. At short option maturities, we find that for all four companies, the impact of the diffusion variance rate is much larger than the impact of the independent credit risk factor. However, as maturity increases, the influence of the diffusion variance rate declines, whereas the influence of the credit risk factor increases. For six-month options on GM, the impacts of the two risk factors become comparable in magnitude.

[Figure 9 about here.]

5. Summary and Conclusions

Based on documented evidence on the joint movements between CDS spreads and stock option implied volatilities, we propose a dynamically consistent framework for the joint valuation and estimation of stock options and CDS spreads written on the same reference company. We model the possible default of a company by a Poisson process with stochastic arrival rate, and we assume that the stock price falls to zero upon default. We model the pre-default stock price as following a continuous process with stochastic volatility. We assume that the default arrival rate and diffusion variance rate follow a bivariate process with dynamic interactions that match the empirical evidence linking stock option implied volatilities and CDS spreads. Importantly, our dynamic specification allows both common movements and independent variations between the two markets.

Under this joint specification, we derive tractable pricing solutions for stock options and credit default swaps. We then estimate the joint dynamics of the diffusion variance rate and the default arrival rate using data on stock option implied volatilities and CDS spreads for four of the most actively traded reference companies. Estimation of the model parameters shows that the default arrival rate is much more persistent than the diffusion variance rate under both the statistical measure and the risk-neutral measure. The statistical persistence difference suggests different degrees of predictability. The risk-neutral difference in persistence suggests that the default arrival rate has a more long-lasting impact on the term structure of option volatilities and CDS spreads than does the diffusion variance.

The estimation also highlights the interaction between market and credit risk in pricing stock options and credit default swaps. We find that the independent credit risk factor dominates CDS spreads at long maturities, but stock return volatility can also affect CDS spreads at short maturities, due to positive comovements between diffusion variance and default arrival. On the other hand, the default arrival rate affects stock option pricing through both its correlation with the diffusion variance rate and its direct effect on the risk-neutral drift of the return process. We find that the impact of the diffusion variance rate on the implied volatility is relatively uniform across different moneyness levels, while the impact of the credit risk factor is mainly on options at low strikes. Furthermore, the impact of the credit risk factor on stock options prices increases with the option maturity. When the option has about six months to maturity, the contribution of the credit risk factor to option pricing is comparable in magnitude to the contribution of the diffusion variance rate.

We conclude that one can learn more about the stock options and the CDS market by developing a model that integrates both markets, rather than having separate models for each market. In particular, one can identify the recovery rate on the bond insured by CDS much more effectively by adjoining stock option prices to CDS data.

Appendix. Generalized Fourier transform of stock returns

To derive the generalized Fourier transform:

$$\phi(u) \equiv \mathbb{E}_t \left[\exp\left(-\int_t^T \lambda_s ds \right) e^{iu \ln P_T / P_t} \right], \quad u \in \mathcal{D} \subset \mathbb{C},$$
(A1)

we use the language of stochastic time change of Carr and Wu (2004a) and define

$$\mathcal{T}_t \equiv \int_t^T v_s ds, \quad \mathcal{T}_t^z \equiv \int_t^T z_s ds, \quad \mathcal{T}_t^\lambda \equiv \int_t^T \lambda_s ds = \mathcal{T}_t^z + \beta \mathcal{T}_t$$

Then, conditional on no default during the time horizon [t, T], with $\tau = T - t$, we can write the log stock return as

$$\ln\left(P_T/P_t\right) = \left(r(t,T) - q(t,T)\right)\tau + \mathcal{T}_t^{\lambda} + W_{\mathcal{T}_t}^P - \frac{1}{2}\mathcal{T}_t,\tag{A2}$$

where r(t,T) and q(t,T) denote the continuously compounded spot interest rates and dividend yields of the relevant maturity.

The discounted generalized Fourier transform becomes,

$$\begin{split} \Phi(u) &= \mathbb{E}_t \left[\exp\left(-\mathcal{T}_t^{\lambda} + iu\left(r(t,T) - q(t,T)\right)\tau + iu\mathcal{T}_t^{\lambda} + iuW_{\mathcal{T}_t}^{P} - \frac{1}{2}iu\mathcal{T}_t \right) \right] \\ &= \mathbb{E}_t \left[\exp\left(iuW_{\mathcal{T}_t}^{P} + \frac{1}{2}u^2\mathcal{T}_t \right) \exp\left(-\mathcal{T}_t^{\lambda} + iu\left(r(t,T) - q(t,T)\right)\tau + iu\mathcal{T}_t^{\lambda} - \frac{1}{2}iu\mathcal{T}_t - \frac{1}{2}u^2\mathcal{T}_t \right) \right] \\ &= \exp\left(iu\left(r(t,T) - q(t,T)\right)\tau \right) \mathbb{E}_t^{\mathbb{M}} \left[\exp\left(- \left(1 - iu \right)\mathcal{T}_t^{\lambda} - \frac{1}{2}\left(iu + u^2 \right)\mathcal{T}_t \right) \right] \\ &= \exp\left(iu\left(r(t,T) - q(t,T)\right)\tau \right) \mathbb{E}_t^{\mathbb{M}} \left[\exp\left(- \left(1 - iu \right)\mathcal{T}_t^{z} - \left(\left(1 - iu \right)\beta + \frac{1}{2}\left(iu + u^2 \right) \right)\mathcal{T}_t \right) \right], \end{split}$$

where the new measure $\ensuremath{\mathbb{M}}$ is defined by

$$\left. \frac{d\mathbb{M}}{d\mathbb{Q}} \right|_t = \exp\left(iuW_{\mathcal{T}_t}^P + \frac{1}{2}u^2\mathcal{T}_t \right)$$

under which the drift of the two dynamic processes change to:

$$\begin{split} \mu_{\nu}^{\mathbb{M}} &= \theta_{\nu} - (\kappa_{\nu} - iu\sigma_{\nu}\rho)\nu(t) = \theta_{\nu} - \kappa_{\nu}^{\mathbb{M}}\nu(t), \\ \mu_{z}^{\mathbb{M}} &= \theta_{z} - \kappa_{z}z(t). \end{split}$$

We have

$$\phi(u) = \exp\left(iu\left(r(t,T) - q(t,T)\right)\tau\right)\mathbb{E}_t^{\mathbb{M}}\left[\exp\left(-\int_t^T b_0^\top x_s ds\right)\right],$$

with $x_t = [v_t, z_t]^\top$, $b_0 = [b_v, b_z]^\top$, $b_v = (1 - iu)\beta + \frac{1}{2}(iu + u^2)$, and $b_z = 1 - iu$.

Since the risk factors x follow affine dynamics, the solution is exponential affine in x_t ,

$$\phi(u) = \exp\left(iu\left(r(t,T) - q(t,T)\right)\tau\right)\exp\left(-a(\tau) - b(\tau)^{\top}x_{t}\right)$$

where the coefficients can be solved analytically as in (10) and (11).

References

Altman, E. I., 2006, "The Link Between Default and Recovery Rates," working paper, New York University.

- Altman, E. I., B. Brady, A. Resti, and A. Sironi, 2004, "The Link Between Default and Recovery Rates: Theory, Empirical Evidence and Implications," *Journal of Business*, forthcoming.
- Bakshi, G., and N. Kapadia, 2003a, "Delta-Hedged Gains and the Negative Market Volatility Risk Premium," *Review* of *Financial Studies*, 16(2), 527–566.
- Bakshi, G., and N. Kapadia, 2003b, "Volatility Risk Premium Embedded in Individual Equity Options: Some New Insights," *Journal of Derivatives*, 11(1), 45–54.
- Bakshi, G., N. Kapadia, and D. Madan, 2003, "Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options," *Review of Financial Studies*, 16(1), 101–143.
- Bakshi, G., D. Madan, and F. Zhang, 2004, "Investigating the Role of Systematic and Firm-Specific Factors in Default Risk: Lessons From Empirically Evaluating Credit Risk Models," *Journal of Business*, forthcoming.
- Bangia, A., F. X. Diebold, A. Kronimus, C. Schagen, and T. Schuermann, 2002, "Ratings Migration and the Business Cycle, With Application to Credit Portfolio Stress Testing," *Journal of Banking and Finance*, 26(2-3), 445–474.
- Bekaert, G., and G. Wu, 2000, "Asymmetric Volatilities and Risk in Equity Markets," *Review of Financial Studies*, 13(1), 1–42.
- Bevan, A., and F. Garzarelli, 2000, "Corporate Bond Spreads and the Business Cycle: Introducing GS-Spread," *Journal of Fixed Income*, 9(4), 8–18.
- Black, F., 1976, "The Pricing of Commodity Contracts," Journal of Financial Economics, 3, 167–179.
- Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637–654.
- Campbell, J. Y., and L. Hentschel, 1992, "No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns," *Review of Economic Studies*, 31, 281–318.
- Campbell, J. Y., and A. S. Kyle, 1993, "Smart Money, Noise Trading and Stock Price Behavior," *Review of Economic Studies*, 60(1), 1–34.
- Capmbell, J. Y., and G. B. Taksler, 2003, "Equity Volatility and Corporate Bond Yields," *Journal of Finance*, 63(6), 2321–2349.
- Carr, P., and L. Wu, 2004a, "Time-Changed Lévy Processes and Option Pricing," *Journal of Financial Economics*, 71(1), 113–141.

Carr, P., and L. Wu, 2004b, "Variance Risk Premia," working paper, Bloomberg and Baruch College.

- Collin-Dufresne, P., R. S. Goldstein, and J. Helwege, 2003, "Is Credit Event Risk Priced? Modeling Contagion via the Updating of Beliefs," working paper, Carnegie-Mellon University, Washington University, and Ohio State University.
- Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin, 2001, "The Determinants of Credit Spread Changes," *Journal* of Finance, 56(6), 2177–2207.
- Consigli, G., 2004, "Credit Default Swaps and Equity Volatility: Theoretical Modelling and Market Evidence," working paper, University Ca'Foscari.
- Cremers, M., J. Driessen, P. J. Maenhout, and D. Weinbaum, 2004, "Individual Stock Options and Credit Spreads," Yale ICF Working Paper 04-14, Yale School of Management.
- Das, S. R., and P. Hanouna, 2006, "Implied Recovery," working paper, Santa Clara University and Villanova University.
- Dennis, P., and S. Mayhew, 2002, "Risk-neutral Skewness: Evidence from Stock Options," *Journal of Financial and Quantitative Analysis*, 37(3), 471–493.
- Duffie, D., J. Pan, and K. Singleton, 2000, "Transform Analysis and Asset Pricing for Affine Jump Diffusions," *Econometrica*, 68(6), 1343–1376.
- Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann, 2001, "Explaining the Rate Spread on Corporate Bonds," *Journal of Finance*, 56, 247–277.
- Eom, Y. H., J. Helwege, and J.-z. Huang, 2004, "Structural Models of Corporate Bond Pricing: An Empirical Analysis," *Review of Financial Studies*, 17(2), 499–544.
- Ericsson, J., K. Jacobs, and R. Oviedo-Helfenberger, 2004, "The Determinants of Credit Default Swap Premia," working paper, McGill University.
- Haugen, R. A., E. Talmor, and W. N. Torous, 1991, "The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns," *Journal of Finance*, 46(3), 985–1007.
- Hilscher, J., 2004, "Is the Corporate Bond Market Forward Looking?," working paper, Harvard University.
- Houweling, P., and T. Vorst, 2005, "Pricing Default Swaps: Empirical Evidence," *Journal of International Money and Finance*, 24(8), 1200–1225.
- Huang, J.-z., and M. Huang, 2003, "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," working paper, Penn State University.

- Hull, J., I. Nelken, and A. White, 2004, "Mertons Model, Credit Risk and Volatility Skews," working paper, University of Toronto.
- Hull, J., and A. White, 2000, "Valuing Credit Default Swaps I: No Counterparty Default Risk," *Journal of Derivatives*, 8(1), 29–40.
- Kalman, R. E., 1960, "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME–Journal of Basic Engineering*, 82(Series D), 35–45.
- Longstaff, F. A., S. Mithal, and E. Neis, 2005, "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit-Default Swap Market," *Journal of Finance*, forthcoming.
- Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29(1), 449–470.
- Newey, W. K., and K. D. West, 1987, "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55(3), 703–708.
- Pan, J., and K. J. Singleton, 2005, "Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads," working paper, Stanford University and MIT.
- Pedrosa, M., and R. Roll, 1998, "Systematic Risk in Corporate Bond Yields," Journal of Fixed Income, 8(1), 7-2.
- Wan, E. A., and R. van der Merwe, 2001, "The Unscented Kalman Filter," in *Kalman Filtering and Neural Networks*, ed. by S. Haykin. Wiley & Sons Publishing, New York.
- Zhu, H., Y. Zhang, and H. Zhou, 2005, "Equity Volatility of Individual Firms and Credit Spreads," working paper, Bank for International Settlements.

Summary Statistics on credit default swap spreads

Entries report the sample estimates of the mean, standard deviation, and weekly autocorrelation on the credit default swap spreads (in percentages) at five fixed maturities for each of the four reference companies. The statistics are based on weekly sampled data from January 2, 2002 to April 28, 2004.

Maturity	1	3	5	7	10	
Mean:						
F	2.19	2.89	2.97	2.95	2.87	
GM	1.51	2.03	2.19	2.28	2.15	
MO	1.79	1.78	1.75	1.69	1.79	
DUK	2.31	2.14	1.99	1.92	1.27	
Standard Devia	tion:					
F	1.31	1.38	1.16	1.06	0.96	
GM	0.89	0.82	0.72	0.69	0.67	
MO	1.15	0.84	0.72	0.62	0.32	
DUK	1.93	1.60	1.31	1.17	0.31	
Autocorrelation	:					
F	0.97	0.97	0.96	0.95	0.95	
GM	0.96	0.95	0.94	0.92	0.93	
МО	0.91	0.92	0.92	0.90	0.94	
DUK	0.96	0.97	0.96	0.96	0.96	

Summary statistics on stock option impled volatilities

Entries report the sample estimates of the mean, standard deviation, and weekly autocorrelation on the implied volatilities (in percentages) at 13 fixed deltas and three fixed maturities for four reference companies. The statistics are based on weekly sampled data from January 2, 2002 to April 28, 2004.

Delta		20	25	30	35	40	45	50	55	60	65	70	75	80
Mean:														
F	1m	50.07	49.09	48.02	46.82	45.84	44.74	43.68	43.12	42.52	42.25	41.95	41.98	42.28
F	2m	50.49	48.87	47.45	46.18	45.19	44.14	43.19	42.62	42.13	41.63	41.45	41.47	41.67
F	3m	49.98	47.97	46.64	45.46	44.53	43.61	42.73	42.14	41.46	40.93	40.63	40.46	40.53
GM	1m	40.87	39.39	38.16	37.14	36.33	35.60	34.95	34.42	33.96	33.54	33.19	32.94	32.95
GM	2m	41.45	39.92	38.64	37.49	36.51	35.70	34.97	34.35	33.81	33.29	32.81	32.41	32.17
GM	3m	41.58	39.84	38.46	37.29	36.24	35.34	34.58	33.92	33.31	32.71	32.14	31.64	31.27
MO	1m	33.73	32.07	30.84	29.89	29.24	28.69	28.36	28.06	27.74	27.50	27.42	27.55	28.07
MO	2m	33.23	31.79	30.71	29.85	29.17	28.63	28.21	27.81	27.42	27.09	26.86	26.78	26.91
MO	3m	33.09	31.78	30.80	29.97	29.25	28.63	28.12	27.66	27.24	26.85	26.50	26.23	26.04
DUK	1m	46.40	44.28	42.65	41.37	39.94	38.66	37.67	37.04	36.56	36.12	35.94	35.79	36.14
DUK	2m	46.09	43.96	42.26	40.85	39.48	38.22	37.25	36.58	35.94	35.30	34.69	34.30	34.38
DUK	3m	44.97	43.08	41.42	39.94	38.65	37.39	36.42	35.70	34.92	34.17	33.46	32.97	32.80
Standa	ard De	eviation:												
F	1m	15.93	15.29	14.71	14.31	13.88	13.40	12.95	12.39	11.90	11.72	11.47	11.17	10.59
F	2m	15.55	15.03	14.39	13.57	12.97	12.40	11.86	11.64	11.37	10.62	10.33	10.15	10.03
F	3m	15.24	14.41	13.69	12.85	12.27	11.79	11.31	11.14	10.63	10.04	9.75	9.49	9.16
GM	1m	15.37	14.64	13.95	13.39	12.86	12.23	11.68	11.26	10.83	10.39	10.00	9.61	9.20
GM	2m	14.60	13.88	13.14	12.51	11.95	11.35	10.79	10.30	9.85	9.46	9.04	8.61	8.14
GM	3m	13.97	13.16	12.37	11.68	11.08	10.51	9.98	9.49	9.05	8.63	8.20	7.76	7.29
MO	1m	10.99	10.38	9.96	9.62	9.25	8.98	8.71	8.46	8.18	7.93	7.77	7.70	7.74
MO	2m	9.65	9.10	8.71	8.39	8.07	7.82	7.60	7.36	7.12	6.95	6.81	6.66	6.52
MO	3m	9.18	8.65	8.20	7.85	7.57	7.33	7.11	6.89	6.69	6.56	6.42	6.26	6.13
DUK	1m	19.25	18.36	17.76	17.05	16.44	15.82	15.20	14.65	14.23	13.91	13.47	13.04	12.54
DUK	2m	17.43	16.52	15.94	15.36	14.72	14.06	13.47	12.94	12.55	12.15	11.64	11.16	10.64
DUK	3m	16.54	15.61	14.80	14.16	13.50	12.86	12.29	11.77	11.38	10.96	10.50	10.05	9.63
Autoc	orrela	tion:												
F	1m	0.83	0.86	0.87	0.89	0.87	0.84	0.84	0.85	0.84	0.85	0.85	0.85	0.87
F	2m	0.89	0.87	0.88	0.91	0.88	0.86	0.87	0.86	0.88	0.90	0.91	0.91	0.90
F	3m	0.93	0.91	0.91	0.94	0.91	0.89	0.89	0.89	0.91	0.93	0.92	0.91	0.91
GM	1m	0.92	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.92	0.92	0.92	0.92	0.90
GM	2m	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.94	0.94	0.93	0.92
GM	3m	0.96	0.96	0.96	0.96	0.96	0.96	0.95	0.95	0.95	0.95	0.95	0.95	0.94
MO	1m	0.78	0.78	0.79	0.80	0.81	0.82	0.82	0.81	0.81	0.81	0.80	0.76	0.69
MO	2m	0.84	0.84	0.85	0.85	0.86	0.86	0.86	0.86	0.87	0.87	0.86	0.84	0.80
MO	3m	0.87	0.88	0.88	0.88	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.88
DUK	1m	0.90	0.90	0.90	0.89	0.90	0.90	0.91	0.91	0.90	0.90	0.89	0.89	0.88
DUK	2m	0.93	0.93	0.93	0.92	0.92	0.93	0.93	0.92	0.92	0.91	0.91	0.91	0.91
DUK	3m	0.95	0.94	0.94	0.94	0.94	0.94	0.93	0.93	0.93	0.93	0.93	0.93	0.93

Table 3Regressing CDS spreads on stock option implied volatility levels and skews

Entries report the parameter estimates, *t*-statistics (in parentheses), and R^2 for different versions of the following regressions on four reference companies:

$$CDS_t = a + bATMV_t + cSKEW_t + e_t$$

where CDS_t denotes the five-year credit default swap spreads in percentage points, $ATMV_t$ denotes a smoothed estimate of the three-month at-the-money implied volatility in percentage points, and $SKEW_t$ denotes a normalized slope estimate on the implied volatility skew against moneyness. Data are weekly from January 2, 2002 to April 28, 2004. To compute the *t*-statistics, we cast the regression into a GMM framework, and estimate the covariance matrix following Newey and West (1987) with four lags.

Companies	а			b		С		
F	-0.997	(-2.786)	0.092	(10.62)			0.82	
GM	0.267	(1.139)	0.055	(9.46)			0.58	
MO	0.088	(0.242)	0.059	(3.98)			0.34	
DUK	-1.231	(-5.391)	0.088	(10.63)			0.69	
F	0.996	(4.796)			-15.989	(-8.03)	0.53	
GM	0.900	(3.672)	—		-7.988	(-5.75)	0.46	
MO	0.978	(3.282)	—		-5.465	(-2.43)	0.15	
DUK	-0.055	(-0.143)		—	-10.795	(-4.27)	0.30	
F	-0.928	(-2.523)	0.083	(6.44)	-2.584	(-1.30)	0.83	
GM	0.289	(1.255)	0.049	(4.68)	-1.173	(-0.65)	0.59	
MO	-0.060	(-0.156)	0.052	(3.71)	-2.491	(-1.55)	0.36	
DUK	-1.226	(-5.194)	0.088	(7.09)	0.081	(0.04)	0.69	

Summary statistics on the pricing errors in stock option impled volatilities

Entries report the sample mean and standard deviation of the pricing errors in stock option implied volatilities, defined as the difference between observations and model-implied values in percentage points, at 13 fixed deltas and three fixed maturities for four reference companies. The last panel reports the explained variation, defined as one minus the ratio of the pricing error variance to the variance of the original implied volatility series. The statistics are based on weekly sampled data from January 2, 2002 to April 28, 2004.

Delta		20	25	30	35	40	45	50	55	60	65	70	75	80	
Mean:															
F	1m	-0.26	0.68	0.86	0.75	0.60	0.15	-0.36	-0.83	-0.86	-0.90	-0.91	-0.71	-0.43	
F	2m	-1.05	-0.07	0.41	0.57	0.67	0.47	0.16	0.09	-0.04	-0.32	-0.37	-0.36	-0.31	
F	3m	-1.87	-0.91	-0.10	0.35	0.67	0.71	0.55	0.52	0.22	-0.09	-0.29	-0.50	-0.66	
GM	1m	0.01	0.30	0.32	0.25	0.16	0.03	-0.12	-0.22	-0.31	-0.40	-0.45	-0.43	-0.18	
GM	2m	-0.74	0.02	0.40	0.51	0.51	0.49	0.42	0.35	0.29	0.18	0.05	-0.04	-0.01	
GM	3m	-1.56	-0.66	-0.13	0.14	0.23	0.24	0.23	0.21	0.13	0.01	-0.16	-0.31	-0.38	
MO	1m	0.67	0.48	0.24	0.00	-0.11	-0.23	-0.21	-0.21	-0.28	-0.32	-0.22	0.05	0.66	
MO	2m	-0.70	-0.30	-0.10	-0.02	0.02	0.05	0.09	0.06	-0.02	-0.10	-0.13	-0.08	0.13	
MO	3m	-1.38	-0.63	-0.16	0.11	0.22	0.27	0.28	0.24	0.15	0.03	-0.12	-0.25	-0.35	
DUK	1m	-0.53	-0.49	-0.45	-0.38	-0.65	-0.96	-1.10	-1.01	-0.85	-0.73	-0.46	-0.25	0.33	
DUK	2m	-1.05	-0.55	-0.19	0.08	0.10	0.02	0.05	0.25	0.33	0.29	0.14	0.08	0.35	
DUK	3m	-2.02	-1.11	-0.56	-0.24	-0.03	-0.04	0.05	0.23	0.21	0.08	-0.14	-0.28	-0.22	
Stondo	ad Da	vistion													
<u>Standa</u> E	1m	2 02	2 97	2 60	274	2 57	2 20	2.02	276	2 65	2 57	2 5 2	2 45	2 72	
Г Г	1111 2m	5.92 2.52	2.07	2.50	5.74 2.66	5.57 2.57	5.50 2.27	2.95	2.70	2.03	2.37	2.35	2.43	2.75	
Г F	2111 3m	1.90	2.32	2.30	2.00	2.37	2.57	2.00	1.92	1.00	1.27 1.54	1.50	1.59	1.04	
GM	1m	1.90	1.00	1.67	2.00	2.08	2.05	1.00	0.85	0.76	0.84	0.05	1.56	1.72	
GM	2m	1.70	0.99	1.08	1.01	1.47	1.20	0.96	0.85	0.70	0.04	0.95	1.14	1.05	
GM	2111 3m	1.25	0.55	0.69	0.87	1.17	1.00	1.06	1.08	1.07	1.07	1 13	1.12	1.40	
MO	1m	2 57	2 37	2.02	2.01	1.00	1.04	1.00	1.00	1.07	1.07	1.15	1.23	2.65	
MO	2m	1 39	1.27	1 17	1 11	1.04	1.02	1.01	1.07	1.19	1.02	1.10	1.72	1.82	
MO	2m	1.09	0.93	0.94	0.97	1.00	1.02	1.01	1.07	1.02	1.02	1.11	1.11	1.38	
DUK	1m	3 48	3.26	3 33	3.26	3 25	2.98	2 56	2 58	2 47	2 37	2 71	2.81	2.90	
DUK	2m	2.86	2.45	2.29	2.19	2.04	1.87	1.62	1 49	1 54	1.58	1.66	1.81	$\frac{2.90}{2.20}$	
DUK	3m	2.72	2.46	2.25	2.18	2.08	1.96	1.95	1.88	1.78	1.77	1.76	1.75	1.89	
2011	0111			2.20		2.00	1170	1.70	1.00	11/0	1.,,	11/0	1170	1105	
Explai	ned V	ariation	1:												
F	1m	0.94	0.94	0.94	0.93	0.93	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.93	
F	2m	0.97	0.97	0.97	0.96	0.96	0.96	0.97	0.97	0.98	0.99	0.98	0.98	0.97	
F	3m	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97	0.98	0.98	0.97	0.97	0.96	
GM	1m	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	0.99	0.99	0.99	0.97	
GM	2m	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.97	
GM	3m	0.99	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.96	
MO	1m	0.95	0.95	0.95	0.96	0.96	0.97	0.98	0.98	0.98	0.98	0.98	0.95	0.88	
MO	2m	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.96	0.92	
MO	3m	0.99	0.99	0.99	0.98	0.98	0.98	0.97	0.97	0.97	0.96	0.96	0.96	0.95	
DUK	1m	0.97	0.97	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.96	0.95	0.95	
DUK	2m	0.97	0.98	0.98	0.98	0.98	0.98	0.99	0.99	0.99	0.98	0.98	0.97	0.96	
DUK	3m	0.97	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.97	0.97	0.97	0.96	

Summary statistics of pricing errors on credit default swap spreads

Entries report the sample mean and standard deviation of the pricing errors on credit default swap spreads, defined as the difference between observations and model-implied values in percentage points, at five fixed maturities for each of the four reference companies. The last panel reports the explained variation, defined as one minus the ratio of the pricing error variance to the variance of the original implied volatility series. The statistics are based on weekly sampled data from January 2, 2002 to April 28, 2004.

Maturity	1	3	5	7	10
Mean:					
F	-0.58	0.12	0.17	0.14	0.13
GM	-0.20	0.01	0.05	0.04	-0.07
MO	-0.41	0.01	0.09	0.08	0.28
DUK	0.02	-0.04	-0.05	-0.01	0.03
Standard Devi	ation:				
F	0.51	0.27	0.31	0.32	0.32
GM	0.40	0.40	0.44	0.46	0.47
MO	0.93	0.68	0.60	0.53	0.27
DUK	0.20	0.11	0.13	0.20	0.12
Explained Var	iation:				
F	0.85	0.96	0.93	0.91	0.89
GM	0.79	0.76	0.63	0.55	0.50
MO	0.34	0.35	0.30	0.27	0.31
DUK	0.99	1.00	0.99	0.97	0.84

Maximum likelihood estimates of model parameters

Entries in panel A report the model parameter estimates and absolute values of the *t*-statistics (in parentheses), estimated for each of the four reference companies. The estimation is based on weekly sampled data from January 2, 2002 to April 30, 2004. Panel B reports the estimates and *t*-statistics for the market price of risk for the two risk factors (z and v), computed from the model parameter estimates and covariance matrix.

Companies	F	GM	МО	DUK		
κ_v	4.0788 (47.34)	7.8085 (121.24)	5.7515 (163.79)	6.5862 (69.90)		
κ_z	0.0067 (0.40)	0.0065 (0.12)	0.0067 (0.08)	0.0485 (2.84)		
$\kappa_v^{\mathbb{P}}$	1.1878 (1.52)	1.6451 (25.48)	1.2558 (5.34)	3.4894 (2.46)		
$\kappa_{7}^{\mathbb{P}}$	0.1745 (3.71)	1.8806 (1.81)	0.1811 (0.81)	0.2966 (1.43)		
$\hat{\boldsymbol{\theta}_{v}}$	0.4153 (33.03)	0.5536 (83.33)	0.2604 (82.12)	0.6873 (96.49)		
Θ_z	0.0050 (9.34)	0.0421 (19.15)	0.0068 (2.29)	0.0058 (15.36)		
σ_v	1.3738 (84.84)	0.8675 (36.05)	0.7512 (112.46)	2.0178 (38.30)		
σ_z	0.1740 (21.70)	0.5749 (23.64)	1.5864 (47.56)	0.3685 (26.18)		
β	0.3062 (21.58)	0.3303 (22.88)	0.4776 (40.52)	0.0993 (14.34)		
ρ	-0.1354 (15.35)	-0.2690 (53.27)	-0.1804 (28.83)	-0.4256 (52.77)		
1-w	0.6417 (85.54)	0.8090 (134.09)	0.4688 (13.89)	0.5657 (126.65)		
γ_{ν}	2.1044 (3.56)	7.1046 (35.35)	5.9851 (17.49)	1.5347 (2.04)		
γ_{z}	-0.9645 (4.20)	-3.2600 (1.80)	-0.1099 (0.71)	-0.6733 (1.20)		
$\mathbb{E}^{\mathbb{P}}_{-}[\lambda]$	0.1355 (2.13)	0.1335 (11.59)	0.1367 (4.42)	0.0392 (1.91)		
$\mathbb{E}^{\mathbb{Q}}[\lambda]$	0.7751 (0.42)	6.4610 (0.12)	1.0416 (0.08)	0.1302 (3.13)		



The average implied volatility smirk on stock options

Lines are the average implied volatility plotted against put option delta at three fixed maturities: one month (solid lines), two months (dashed lines), and three months (dash-dotted lines). Each panel is for one company.



Time series of CDS spreads and at-the-money stock option implied volatilities.

The solid lines are the time series of CDS spreads at fixed maturities of one, three, five, seven, and ten years, with scales on the left hand size. The dashed lines are the time series of the at-the-money (50 delta) stock option implied volatilities at fixed maturities of 30, 60, and 91 days, with the scales on the right hand side.



Cross-correlations between weekly changes in the five-year CDS spread and the three-month at-themoney implied volatility.

The bars show the cross-correlation estimates between weekly changes in the five-year CDS spread and weekly changes in the three-month at-the-money implied volatility at different leads and lags. The two dash-dotted lines in each panel define the 95 percent confidence band.



The contemporaneous response of the credit spread to unit shocks in the two risk factors

Lines denote the contemporaneous response of the credit spread, defined as the difference between continuously compounded spot rate of a reference company and the corresponding spot rate for the libor/swap market, to unit shocks to the two sources of risks z (solid lines) and v (dashed lines).



The time series of return variance rates and default arrival rates.

Solid lines are the extracted time series of the instantaneous variance rate on the diffusion component of the stock return, with the scales on the left hand side. Dashed lines are the extracted time series of the default arrival rate on the reference companies, with the scales on the right hand side.



The term structure of credit default swap spreads.

The solid lines represent the mean term structures computed from the estimated model and the sample mean levels of the two risk factors. Dashed lines are computed by setting v_t to the sample average and z_t to one standard deviation away from its sample mean. Dotted lines are computed by setting z_t to the sample mean and varying v_t one standard deviation away from its sample mean.



The one-month implied volatility smirks.

Moneyness is defined as $\ln(K/S)/\sqrt{v\tau}$. The solid lines are the mean implied volatility smirks at one-month maturity computed from the estimated model and the sample mean levels of the two risk factors. Dashed lines are computed by setting v_t to its sample average and z_t to one standard deviation away from its sample mean. Dotted lines are computed by setting z_t to the sample mean and v_t to one standard deviation away from its sample mean.



The six-month implied volatility smirks.

Moneyness is defined as $\ln(K/S)/\sqrt{v\tau}$. The solid lines are the mean implied volatility smirk at one-month maturity computed from the estimated model and the sample mean levels of the two risk factors. Dashed lines are computed by setting v_t to its sample average and z_t to one standard deviation away from its sample mean. Dotted lines are computed by setting z_t to the sample mean and v_t to one standard deviation away from its sample mean.



The term structure of at-the-money implied volatilities.

The solid lines are the mean term structure of the at-the-money forward implied volatility computed from the estimated model and the sample mean levels of the two risk factors. Dashed lines are computed by setting v_t to the sample average and z_t one standard deviation away from its sample mean. Dotted lines are computed by setting z_t to the sample mean and v_t to one standard deviation away from its sample mean.